VI CONGRESSO NACIONAL DE ENGENHARIA MECÂNICA
VI NATIONAL CONGRESS OF MECHANICAL ENGINEERING
18 a 21 de agosto de 2010 – Campina Grande – Paraíba - Brasil
August 18 – 21, 2010 – Campina Grande – Paraíba – Brazil

PLASTIC STRAIN LOCALIZATION PROMOTED BY THERMOMECHANICAL COUPLING IN METALLIC MATERIALS

Natalie Pereira Rodríguez, natalieprodriguez@gmail.com
Pedro Manuel Calas Lopes Pacheco, calas@cefet-rj.br
CEFET/RJ – PPEMM – Department of Mechanical Engineering – Av. Maracanã, 229 – 20271-110, Rio de Janeiro – RJ – Brazil

Fulvio Enrico Giacomo Chimisso, fulvio@dmc.furg.br
FURG – Department of Materials and Construction – Av. Itália Km 8, Campus Carreiros – 96201-900, Rio Grande – RS – Brazil

Abstract. *Thermomechanical coupling is an important phenomenon in different engineering problems. Inelastic cyclic strain promotes heating of metallic structural elements, and a considerable amount of heat can be generated in situations where high loading rates and/or high amplitudes of inelastic strain are of concern. The temperature rise of mechanical component depends on the loading amplitude, frequency and temperature boundary conditions. Nevertheless, traditional low-cycle fatigue models neglect the material temperature variation due to thermomechanical coupling and unreal life predictions may be obtained. Indeed, there are situations where such couplings cannot be neglected and a physically more realistic model must take it into account. In this paper, a continuum mechanics model with internal variables is proposed to study the thermomechanical coupling effects of metallic components subjected to inelastic loadings. A thermodynamic approach allows a proper identification of the thermomechanical coupling in the mechanical and thermal equations. A numerical procedure is developed based on an operator split technique associated with an iterative numerical scheme in order to deal with the non-linearities in the formulation. With this assumption, coupled governing equations are solved involving three uncoupled problems: thermal, thermoelastic and elastoplastic behaviors. Numerical simulations of steel plates with a stress concentrator subjected to inelastic loadings are presented and analyzed. Results suggest that the proposed model is capable of capturing important localization phenomena related to plastic strain localization due thermomechanical coupling*

Keywords*: Thermomechanical Coupling, Modeling, Numerical Simulation, Elastoplasticity*

1. INTRODUCTION

When metallic structural elements are submitted to inelastic cyclic strain, a very important phenomenon must be considered: the thermomechanical coupling. This kind of solicitation promotes heating of the elements and a considerable amount of heat can be generated in situations where high loading rates and/or high amplitudes of inelastic strain are of concern (Simo and Miehe, 1992; Pacheco, 1994; Barbosa *et al.*, 1995; Pacheco and Costa-Mattos, 1997; Stabler and Baker, 2000, Rosakis *et al*., 2000; Longère and Dragon, 2008; Costa-Mattos and Pacheco, 2009). The temperature rise of mechanical component depends on the loading amplitude, frequency and temperature boundary conditions. It promotes a mechanical properties decrease, which in turn promotes a plastic strain increase. This phenomenon is known as thermomechanical coupling and can accelerate the structural degradation process.

Usually, the material temperature variation due to thermomechanical coupling is not taken into account in traditional low-cycle fatigue models, so unreal life predictions may be obtained. Since there are situations where such couplings cannot be neglected and a physically more realistic model must take it in consideration, this paper presents a continuum mechanics model with internal variables to study the thermomechanical coupling effects of metallic components submitted to inelastic loadings (Pacheco, 1994; Lemaitre and Chaboche, 1990). Figure 1 shows the feedback phenomenon that can be observed in metallic elements subjected to inelastic strain loadings (Nolte, 2007; Nolte *et al.*, 2007).

A thermodynamic approach allows a proper identification of the thermomechanical coupling in the mechanical and thermal equations, while a numerical procedure is developed based on an operator split technique associated with an iterative numerical scheme in order to deal with the non-linearities in the formulation. Three uncoupled problems are involved to solve coupled governing: thermal, thermoelastic and elastoplastic behaviors. Classical finite element method is employed for spatial discretization in all uncoupled problems and numerical simulations of a steel plate with a stress concentrator subjected to inelastic loadings are presented and analyzed. Results suggest that the proposed model is capable of capturing important localization phenomena related to plastic strain localization due thermomechanical coupling.

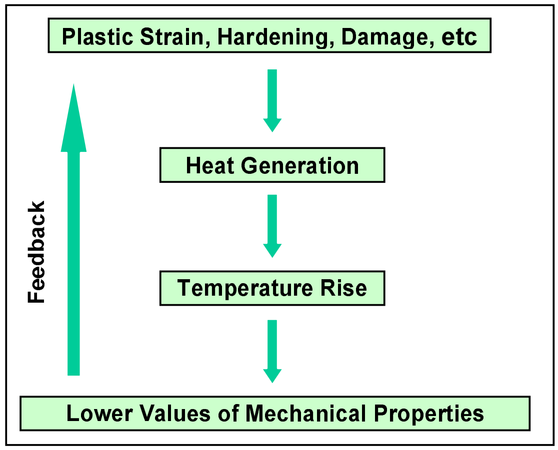


Figure 1. Thermomechanical coupling in metallic elements subjected to inelastic strain loadings.

2. CONSTITUTIVE MODEL

By considering thermodynamic forces, defined from the Helmholtz free energy, ψ, and thermodynamic fluxes, defined from the pseudo-potential of dissipation, ϕ, it is possible to formulate constitutive equations within the framework of continuum mechanics and the thermodynamics of irreversible processes, by (Lemaitre and Chaboche, 1990; Pacheco, 1994).

For this, a Helmholtz free energy is proposed as a function of total strain, ε_{ij}, temperature, T and observable variables. Besides, the following internal variables are considered: plastic strain, ε_{ij}^{p}, kinematic hardening, c_{ij}, and isotropic hardening, p. Therefore, the following free energy is proposed, employing indicial notation where summation convention (i = 1,2,3) is evoked (Eringen, 1967), except when indicated:

$$\rho\psi(\varepsilon_{ij},\varepsilon_{ij}^{p},c_{ij},p,T)=\left[W_e(\varepsilon_{ij}-\varepsilon_{ij}^{p},T)+W_a(c_{ij},p,T)\right]-W_T(T) \tag{1}$$

where ρ is the material density, W_e is the elastic energy density, W_a is the energy density associated to the hardening and W_T is the energy density associated with the temperature, defined as:

$$W_e(\varepsilon_{ij}-\varepsilon_{ij}^{p},T)=\frac{E}{2(1+\nu)}\left[(\varepsilon_{ij}-\varepsilon_{ij}^{p})(\varepsilon_{ij}-\varepsilon_{ij}^{p})+\frac{\nu}{1-2\nu}(\varepsilon_{jj}-\varepsilon_{jj}^{p})^2\right]-\frac{\alpha E}{1-2\nu}(\varepsilon_{jj}-\varepsilon_{jj}^{p})$$

$$W_a(c_{ij},p,T)=\tfrac{1}{2}a\,c_{ij}c_{ij}+b\left[p+(1/d)\,e^{-dp}\right] \quad ; \quad \mathrm{W}_T(T)=\rho\int_{T_0}^{T}C_1\log(\xi)\,\mathrm{d}\xi+\frac{\rho}{2}C_2T^2 \tag{2}$$

where T_0 is a reference temperature, E is the Young modulus, ν is the Poisson ratio, a is a material parameter associated with kinematic hardening, while b and d are material parameters associated with isotropic hardening. C_1 and C_2 are positive constants. The increment of elastic strain is defined as $d\varepsilon_{ij}^{e}=d\varepsilon_{ij}-d\varepsilon_{ij}^{p}-\alpha_T\,dT\delta_{ij}$. The last term is associated with thermal expansion and the parameter α_T is the coefficient of linear thermal expansion.

The general formulation of this model was developed and previously applied to the study of various related problems (Pacheco, 1994; Pacheco and Mattos, 1997; Pacheco *et al.*, 2001; Oliveira *et al.*, 2003; Oliveira, 2004; Silva *et al.*, 2004). A detailed description of this constitutive model may be obtained in the cited references.

This contribution considers life prediction of metallic plane truss structures subjected to cyclic inelastic loadings. From the mechanical point of view, it is assumed that the specimen is submitted to uniaxial strain. Concerning thermal characteristics, it is assumed that the specimen experiments a heat conveccion through its surface. Under these assumptions, a one-dimensional model is formulated and tensor quantities presented in the general formulation may be replaced by scalar quantities. For this situation the thermodynamics forces (σ_{ij}, P_{ij}, B_{ij}^{c} ,B^c, B^p, s), respectively associated with state variables $(\varepsilon_{ij},\varepsilon_{ij}^{p},c_{ij},p,T)$, are defined as follows:

$$\sigma_{ij}=\rho\frac{\partial\psi}{\partial\varepsilon_{ij}}=\mathrm{E}(\varepsilon_{ij}-\varepsilon_{ij}^{p})-E\delta_{ij}\alpha_T(T-T_0)\ ;\ P_{ij}=-\rho\frac{\partial\psi}{\partial\varepsilon_{ij}^{p}}=\sigma_{ij}\ ;\ B_{ij}^{c}=-\rho\frac{\partial\psi}{\partial c_{ij}}=-(2/3)\,X_{ij}=-a\,c_{ij}$$

$$B^{p}=-\rho\frac{\partial\psi}{\partial p}=-R=-(1-D)b\left[1-e^{-dp}\right]\ ;\ B^{D}=-\rho\frac{\partial\psi}{\partial D}=W_e(\varepsilon_{ij}-\varepsilon_{ij}^{p},T)+W_a(c_{ij},p,T)\ ;\ \mathrm{s}=-\rho\frac{\partial\psi}{\partial T} \tag{3}$$

where X and R are auxiliary variables directly related to kinematic and isotropic hardenings, respectively. In order to describe dissipation processes, it is necessary to introduce a potential of dissipation $\phi(\dot{\varepsilon}^p, \dot{c}, \dot{p}, q)$, which can be split into two parts: $\phi(\dot{\varepsilon}_{ij}^p, \dot{c}_{ij}, \dot{p}, q) = \phi_I(\dot{\varepsilon}_{ij}^p, \dot{c}_{ij}, \dot{p}) + \phi_T(q)$. This potential can be written through its dual $\phi^*(P_{ij}, X_{ij}, R, g) = \phi_I^*(P_{ij}, X_{ij}, R,) + \phi_T^*(g)$, as $\phi_I^* = I_f^*(P_{ij}, X_{ij}, R)$ and $\phi_T^* = \frac{T}{2}\Lambda\, g^2$, where $g = (1/T)\ \partial T/\partial x$ and Λ is the coefficient of thermal conductivity; $I_f^*(P_{ij}, X_{ij}, R)$ is the indicator function associated with elastic domain (Lemaitre and Chaboche, 1990),

$$f(\sigma_{ij}, X_{ij}, R) = \sqrt{\frac{3}{2}(\sigma_{ij}^d - X_{ij}^d)(\sigma_{ij}^d - X_{ij}^d)} - (S_Y + R) \le 0 \tag{4}$$

where S_Y is the material yield stress, $\sigma_{ij}^d = \sigma_{ij} - \delta_{ij}(\sigma_{kk}/3)$ and $X_{ij}^d = X_{ij} - \delta_{ij}(X_{kk}/3)$. A set of evolution laws obtained from ϕ^* characterizes dissipative processes,

$$\dot{\varepsilon}_{ij}^p = \frac{\partial\phi^*}{\partial P_{ij}} = \lambda\,\mathrm{sign}(\sigma_{ij} - X_{ij})\ ;\ \dot{c}_{ij} = \frac{\partial\phi^*}{\partial B_{ij}^c} = \dot{\varepsilon}_{ij}^p + \frac{\varphi}{a}B_{ij}^c\dot{p}\quad ;\ \dot{p} = \frac{\partial\phi^*}{\partial B^p} = \lambda\ ;\ q = -\frac{\partial\phi^*}{\partial g} = -\Lambda T\, g = -\Lambda\frac{\partial T}{\partial x} \tag{5}$$

where λ is the plastic multiplier (Lemaitre and Chaboche, 1990) from the classical theory of plasticity, sign(x) = x / |x|, φ is a material parameter associated with kinematic hardening and q is the heat flow. By assuming that the specific heat is $c_p = -(T/\rho)\,\partial^2 W/\partial T^2$ and also considering the set of constitutive Eqs. (3) and (5), the energy equation can be written as (Pacheco, 1994):

$$\frac{\partial}{\partial x_i}\left(\Lambda\frac{\partial T}{\partial x_i}\right) - h\frac{Per}{A}(T - T_\infty) - \rho c_p\dot{T} = -a_I - a_T \qquad \text{where}\ \begin{cases} a_I = \sigma_{ij}\dot{\varepsilon}_{ij}^p - X_{ij}\dot{c}_{ij} - R\dot{p} \\ a_T = T\left(\frac{\partial\sigma_{ij}}{\partial T}\left(\dot{\varepsilon}_{ij} - \dot{\varepsilon}_{ij}^p\right) + \frac{\partial X_{ij}}{\partial T}\dot{c}_{ij} + \frac{\partial R}{\partial T}\dot{p}\right) \end{cases} \tag{6}$$

where h is the convection coefficient, T_∞ is the surrounding temperature, Per is the perimeter and A is the cross section area. Terms a_I and a_T are, respectively, internal and thermal coupling. The first one appears in the right hand side of the energy equation and is called internal coupling. It is always positive and has a role in the energy equation similar to a heat source in the classical heat equation for rigid bodies. The last term in the right hand side of the energy equation can be positive or negative and is called the thermal coupling.

3. NUMERICAL SIMULATIONS

The proposed model is applied to study the thermomechanical coupling effects of metallic components submitted to inelastic loadings. A non-linear finite element model with temperature dependent properties is presented to study the effect of thermomechanical coupling in mechanical components subjected to inelastic deformation. Numerical simulations are performed with commercial finite element code ANSYS (ANSYS, 2006), employing coupled thermal and mechanical fields element PLANE13 (4 nodes bidimensional element with displacement and temperature degrees of freedom) for spatial discretization. The final meshes are defined after a convergence analysis.

The numerical procedure here proposed is based on the operator split technique (Ortiz *et al.*, 1983; Pacheco, 1994) in order to deal with nonlinearities in the formulation. With this assumption, coupled governing equations are solved from two uncoupled problems: thermal and thermo-elastoplastic. In this article, finite element method is employed to perform spatial discretization of governing equations. Therefore, the following moduli are considered:

Thermal Problem - Comprises a conduction problem with surface convection. Themomechanical coupling is considered as a heat source. Material properties depend on temperature and, therefore. Classical finite element method is employed for spatial discretization.

Thermo-elastoplastic Problem - Stress and strain fields are evaluated from temperature distribution obtained in the thermal problem and from the mechanical loading. Classical finite element method is employed for discretization.

To implement the operator split technique and the themomechanical coupling as a heat source, a program developed in APDL (ANSYS Parametric Design Language) is used. Through this approach the internal coupling associated to plastic deformation and kinematic hardening is calculated for each time step from stress and plastic strain fields obtained from the thermo-elastoplastic problem results of the previous step. A small step and a convergence analysis guarantee the convergence of the results. A plate with a central circular hole is considered to assess de effect of thermomechanical coupling in mechanical components with stress concentrators. The plate has a length of 150 mm, a width of 50 mm and a 10 mm radius circular central hole. A plane stress condition is considered and symmetry conditions are adopted to reduce the computational cost.

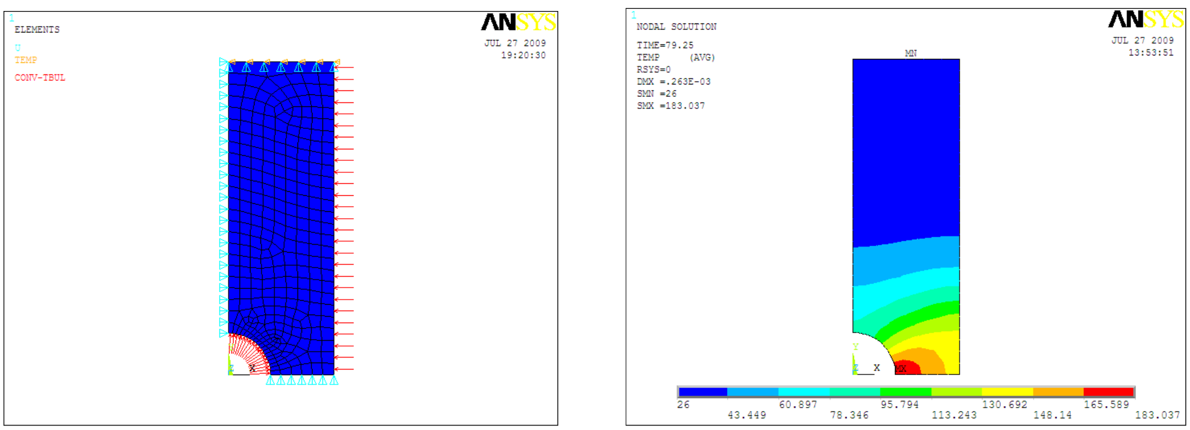
A prescribed senoidal displacement loading with a frequency of 1 Hz is adopted. A linear kinematic hardening is considered. It is assumed that the specimen is at an initial temperature of 26 ºC. This temperature is similar to the ambient temperature and a convection coefficient, h, of 10 W/m^2 °C is used. Convection boundary condition is prescribed at the specimen surface. It is considered that the grips temperature remains constant and a constant temperature condition is adopted at the specimen end. Temperature dependent thermomechanical properties are adopted (Oliveira, 2004, 2008). In order to allow the evaluation of the thermomechanical effects in the plastic strain localization, two models are considered:

Uncoupled model: neglects the thermomechanical coupling terms present in Eq. (6) and therefore, the thermal problem is solved as a rigid body;

Coupled model: considers the thermomechanical coupling terms.

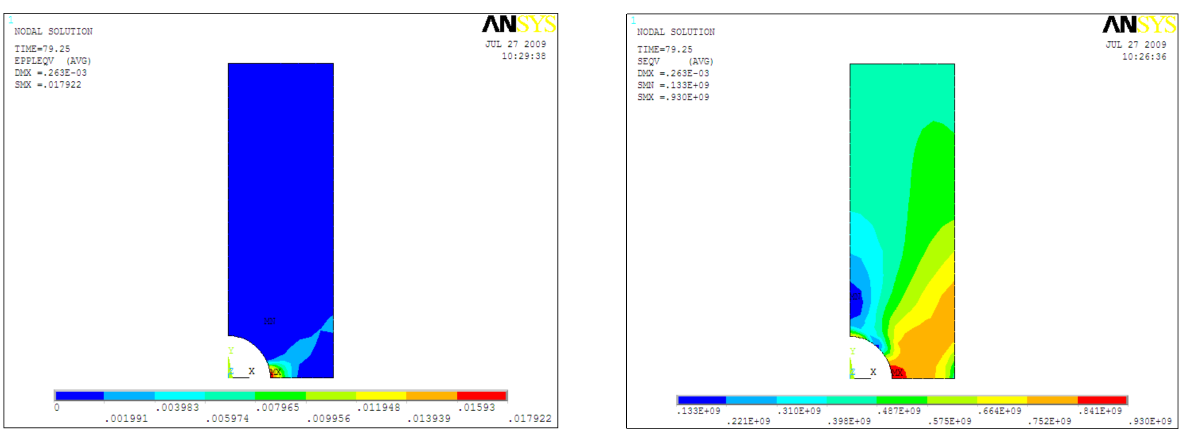
Figure 2*a* shows the mesh obtained after a convergence analysis with the boundary conditions and the applied loading. Figure 2*b* shows temperature distribution at the last loading cycle for the *coupled* model.

Figure 3 presents plastic strain and *von Mises* equivalent stress distribution for *uncoupled* and *coupled models* at the last loading cycle. Plastic strain localization can be observed at the middle of the specimen where a stress concentrator exists. The localization phenomenon is promoted by the feedback effect due to thermomechanical coupling. The thermal boundary conditions and the temperature depended mechanical properties promotes the localization of thermal and plastic strain processes at the stress concentrator.



(*a*) (*b*)

Figure 2. Mesh with boundary conditions and loadings (*a*) and temperature distribution for the *coupled model*.



(*a*) (*b*)

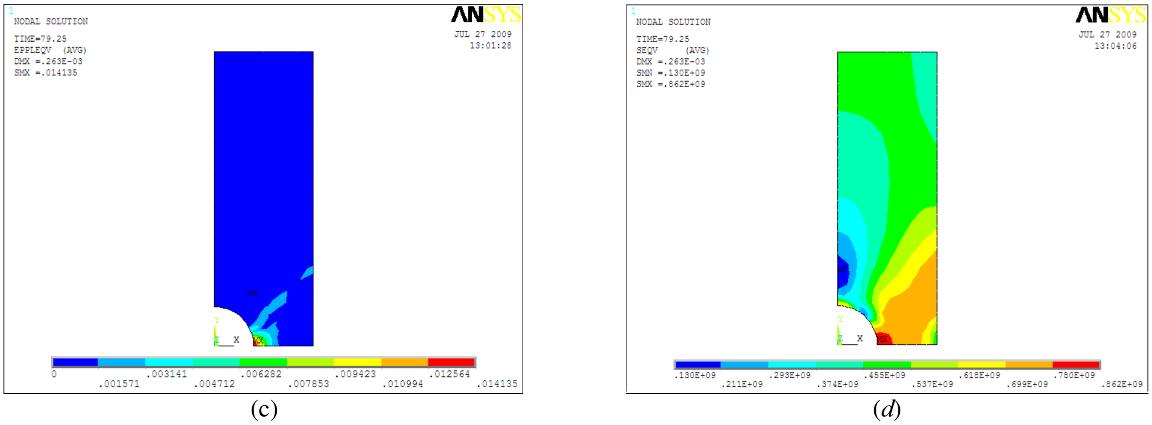


Figure 3. Equivalent plastic strain distribution (*a*) and *von Mises* equivalent stress distribution (*b*). *Uncoupled model*. Equivalent plastic strain distribution (*c*) and *von Mises* equivalent stress distribution (*d*). *Coupled model*.

Figure 4 presents the evolution of the stress-strain curve (*y* axis), temperature, equivalent plastic strain and internal coupling term (a_I) for *uncoupled* and *coupled models*. Results show that the *uncoupled model* predicts a stabilized cycle for all variables whereas for the *coupled model* maximum temperature and maximum plastic strain presents a continuous rise. A detailed view of the last 10 cycles is presented in Figs. 4*b-d*. Temperature presents a cyclic behavior that follows the loading cycle. A temperature rise is observed in both tension and compression phases, whenever plastic strain is present. In the absence of plastic strain, cooling is observed promoted by convection and conduction mechanisms. Internal coupling term (a_I) presents a similar behavior as it depends on the plastic strain evolution. It is important to observe that only positive values are observed as pointed in section 2. Equivalent plastic strain presents larger values for the *coupled model*.

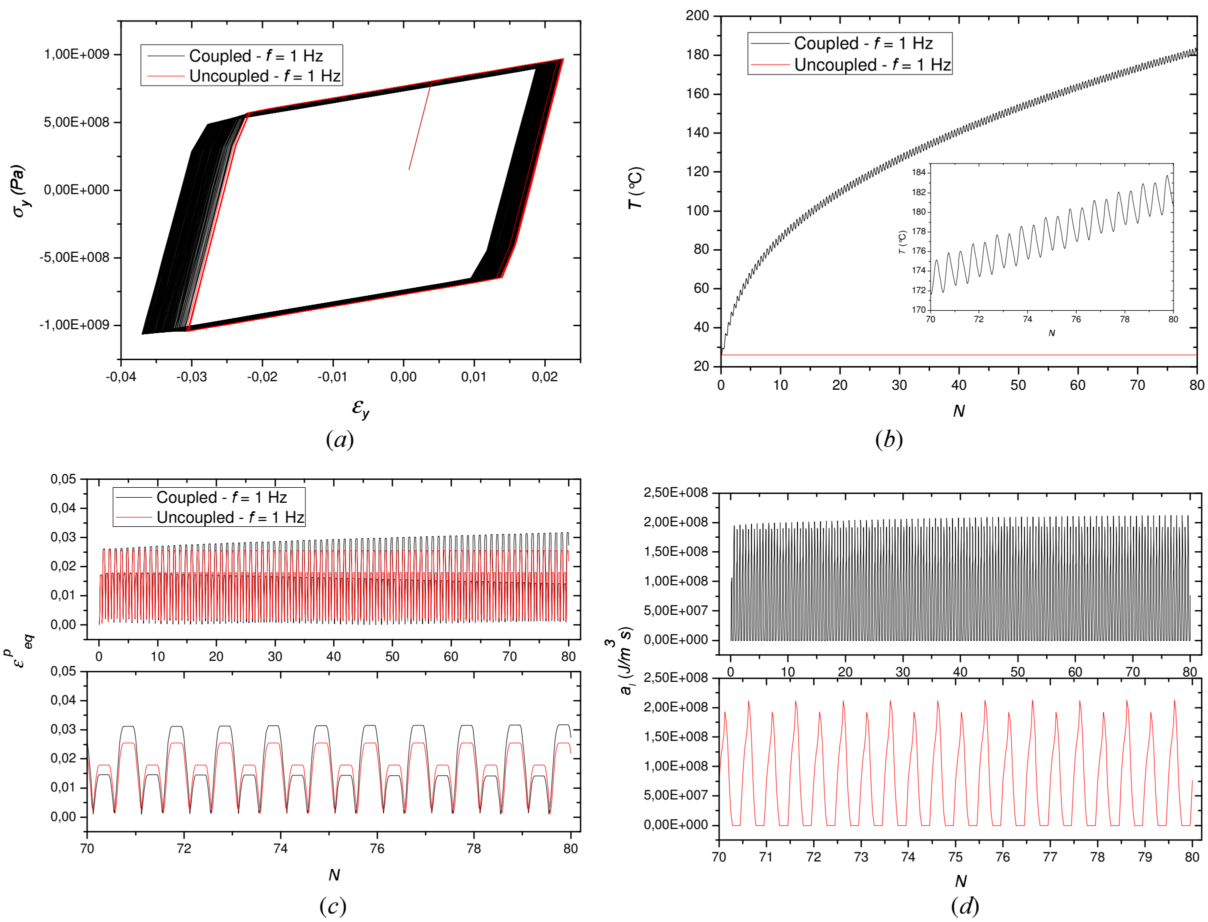


Figure 4. Stress-strain curve (*a*). Temperature (*b*), equivalent plastic strain (*c*) and the internal coupling term (*d*) evolution.

4. CONCLUSION

In this paper an anisothermal constitutive model with internal variables based on continuum damage mechanics is proposed to study the thermomechanical coupling effects in elastoplastic round specimens subjected to inelastic mechanical loadings. This formulation provides a rational methodology to study complex phenomena like the amount of heat generated during plastic strain of metals and how it affects its structural integrity. The numerical procedure developed is based on the operator split technique and allows one to deal with the nonlinearities in the formulation using traditional tested classical numerical methods, as the finite element method which is used for spatial discretization. In order to allow the evaluation of the thermomechamical effects in the plastic strain localization, two models are considered: *uncoupled model*, where the thermal problem is solved as a rigid body, and the *coupled model* that considers the thermomechanical coupling terms. Numerical simulations considering a plate with a central circular hole subjected to inelastic loadings are presented and analyzed. Results show that the *uncoupled model* predicts a stabilized cycle for all variables whereas for the *coupled model* maximum temperature and maximum plastic strain presents a continuous rise. Results suggest that the proposed model is capable of capturing important localization phenomena related to plastic strain evolution.

5. ACKNOWLEDGEMENTS

The authors would like to acknowledge the support of the Brazilian Research Agencies CAPES and CNPq.

6. REFERENCES

ANSYS, 2006; *Structural Analysis Guide,* ANSYS Inc.

Barbosa, J.M.A., Pacheco, P.M.C.L. and Mattos, H.C., 1995, "On the role of Temperature in the Mechanical Vibration of Elasto-Viscoplastic Bars", COBEM-95/CIDIM-95, *13th Brazilian Congress of Mech. Engineering*, Belo Horizonte, Brazil, in Portuguese.

Costa-Mattos, H. and Pacheco, P.M.C.L., 2009, "Non-isothermal low-cycle fatigue analysis of elasto-viscoplastic materials", *Mechanics Research Communications*, Vol. 36, Issue 4, pp.428-436 .

Eringen, A.C., 1967, "*Mechanics of Continua*", John Wiley, New York.

Lemaitre, J. e Chaboche, J.L., 1990, "*Mechanic of Solid Materials",* Cambridge Press.

Longère, P. and Dragon, A., 2008, "Plastic work induced heating evaluation under dynamic conditions: Critical assessment". *Mech. Res. Commun.*, 35, 135–141.

Nolte, C., 2007, *"Analysis of the Thermomechanical Coupling in Elastoplastic Trusses Subjected to Cycling Loadings"*, M.Sc. Dissertation, Master in Technology, CEFET/RJ, in Portuguese.

Nolte, C., Pacheco, P.M.C.L. e Savi, M.A., 2007; "Damage Localization in Elastoplastic Plane Trusses Promoted by Thermomechanical Coupling", *COBEM-2007, 19th International Congress of Mechanical Engineering*, Brasília.

Oliveira, W.P., Souza, L.F.G., Pacheco, P.M.C.L. e Savi, M.A., 2003; "Quenching Process Modeling in Steel Cylinders Using a Multi-Phase Constitutive Model", *COBEM-2003, 17th International Congress of Mechanical Engineering*, São Paulo.

Oliveira, W.P., 2004, *"Modeling the Quenching Process of Steel Cylinders using a Constitutive Multi-Phase Model"*, M.Sc. Dissertation, Master in Technology, CEFET/RJ, in Portuguese.

Oliveira, W. P., 2008, "Modeling and Simulation of Quenching in Axisimmetrical Geometries Using a Multi-Phase Constitutive Model", Ph.D. Thesis, COPPE/UFRJ, in Portuguese.

Ortiz, M., Pinsky, P.M. and Taylor, R.L., 1983, Operator Split Methods for the Numerical Solution of the Elastoplastic Dynamic Problem, *Computer Methods of Applied Mechanics and Engineering*, v. 39, pp.137-157.

Pacheco, P.M.C.L., 1994, "Analysis of the Thermomechanical Coupling in Elaso-Viscoplastic Materials", Ph.D. Thesis, Department of Mechanical Engineering, PUC-RJ, in Portuguese.

Pacheco, P.M.C.L., Camarão, A.F. and Savi, M.A., 2001, "Analysis of Residual Stresses Generated by Progressive Induction Hardening of Steel Cylinders", *Journal of Strain Analysis for Engineering Design*, v.36, n.5, pp.507-516.

Pacheco, P.M.C.L. and Costa-Mattos, H, 1997; "Modeling the Thermomechanical Coupling Effects on Low-Cycle Fatigue of Metallic Materials", *5th ICBMFF, 5th International Conference on Biaxial/Multiaxial Fatigue and Fracture*, pp.291-301, Cracow, Poland.

Rosakis, P., Rosakis, A. J., Ravichandran, G. and Hodowany J., 2000. "A thermodynamic internal variable model for the partition of plastic work into heat and stored energy in metals". *J. Mech. Phys. Solid*, 48, 581 – 607.

Silva, E.P., Pacheco, P.M.C.L. and Savi, M.A., 2004, *"On the Thermo-Mechanical Coupling in Austenite-Martensite Phase Transformation Related to the Quenching Process", International Journal of Solids and Structures*, ISSN 0020-7683, Vol. 41, pp. 1139-1155.

Simo, J.C. and Miehe, C., 1992, "Associative Coupled Thermoplasticity at Finite Strains: Formulation, Numerical Analysis and Implementation", *Computer Methods in Applied Mechanics and Engineering*, v.98, pp.41-104.

Smithell, C.J., 1976, *"Metal Reference Book",* 5th. Ed., Buttersworth, London, pp. 960.
Stabler J., Baker, G., 2000; "On the form of free energy and specific heat in coupled thermo-elasticity with isotropic damage". *Int. J. Solid Struct.*, V.37, 34, 4691-4713.

7. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.