PROJECT OF CONTROLLER PROPORTIONAL-INTEGRAL FOR A COORDINATE TABLE XY THROUGH GUILLEMIN-TRUXAL'S METHOD

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Abstract. *The tables coordinate has great performance in modern industry such as, for example, the positioning of workpieces and tools for machining, soldering printed circuit boards and measurement of complex geometries. In order to these machines be efficient is necessary that the movements carried out by their shafts have a low error. This work aims to design controllers for a table of coordinates with two degrees of freedom to be able to describe a circular path and another polynomial with the lowest possible error. The system consists of two bases perpendicular to each other and triggered by continuous current motors. It was used the design procedure Guillemin - Truxal in order to determine the controllers of bases from their transfer functions. The controllers achieved were of type PI. The simulated and experimental results showed that the table followed the proposed trajectories with low error and without saturation of the control variable.*

Keywords: *table coordinates, position control, method of Guillemin Truxal*

1. INTRODUCTION

The automatic control has played an important role every day that passes in engineering and science. It uses goes from control of industrial operations, such as pressure, temperature and humidity, to the direction of spacecraft (Ogata, 2003).

Since it emerged until today, the automatic control has evolved greatly, in part due to advances in technology, such as computers.

In the branch of the industry especially, the automatic control has become an essential tool to improve the quality of a product and maximization of resources.

An example of that was mentioned is associated with coordinate tables XY and XYZ, used in machine tool industries. Even today exist in the market that ones that work with manual actuation through steering wheels, which makes them more affordable and dismisses specialized training of staff, but the quality of the product becomes restricted exclusively to the operator skill and its experience acquired for product quality (Menezes Filho, 2010). The advancement in industrial sectors and the need for greater accuracy in positioning system of these machines, made them been replaced by tables driven by electric motors, hydraulics and pneumatic (JÚLIO, 2010). With this, it became possible to apply the automatic position control instead of manual control, gaining greater efficiency, speed and reliability.

Most tables of coordinates on the market uses two types of drive: stepper motor, which works on a structure of open loop and servo mechanism, which uses continuous current motors or induction loop closed. Thus there is a need for position sensors (encoders) that serve to provide the speed and angular position of the motor shaft (Menezes, 2007).

For good performance the positioning error and the smoothness of the movement are d accuracy determinants of form and roughness to the part intended to be manufactured, being the error one of the critical points of these types of machine tools (Jesus, 1999). The controllers used in these machines has the function of minimizing the error, in another words, decreasing the the relationship between the desired measurement or set point and the measurement made.

The objective of this work is to design controllers for a table of coordinates using the method Guillemin Truxal, so that it can describe a circular path and polynomial with the smallest possible error and without saturation of the control variable. For this purpose the mathematical models are used obtained by parametric identification of a real prototype.

2. TABLE COORDINATES AND TRANSFER FUNCTIONS

The system consists of two bases, independently activated, which move in perpendicular directions in the horizontal plane, as shown in Figure (1).

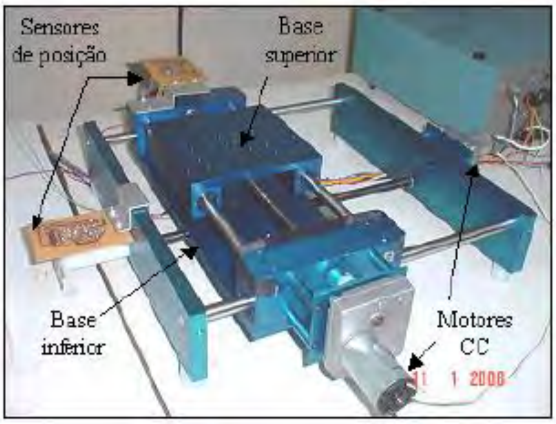


Figure 1. Table Coordinates

The tables are triggered by continuous current motors and their positions are detected by encoders, located at the ends of each axis. The sensor signals are recorded by a computer through a plate with input and output of data and the software named Lab VIEW. The control variable is the strain on the engines, which are the elements responsible for the position control of the table.
Mathematical models of the upper and lower coordinate tables were obtained using parametric identification technique, in open loop mode, through identification model BJ (Box Jenkins), using a square wave as excitation signal with an amplitude of ± 2.5 V and sampling time 10 ms, as can be seen in equations (1) and (2). (BRAGA, 2006)

$$_{Psuperior}(s) = \frac{24{,}44}{S^3+68{,}18^2+863s} \quad \text{(Base upper)} \quad (1)$$

$$_{Pinferior}(s) = \frac{77{,}8}{S^3+125{,}3^2+3665s} \quad \text{(Base bottom)} \quad (2)$$

3. PROJECT OF CONTROLLER BASED ON A MODEL OF REFERENCE FOR TRAJECTORY TRACKING OF A POLYNOMIAL

The method of Guillemin Truxal is trampled in the design of a controller GC (s) that leads to a relationship of control desired in closed loop (D'Azzo & HOUPIS, 1978). This relationship is chosen by the designer and is represented in Eq (3) by the transfer function (reference model):

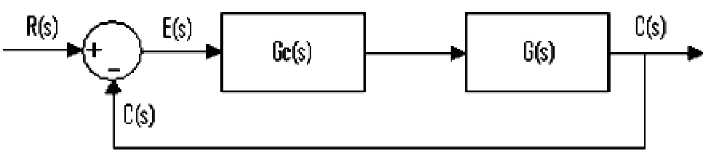


Figure 2 - control system with cascade controller

$$_{R}(s) = M(s) = \frac{C(s)}{R(s)} = \frac{N(s)}{D(s)} = \frac{G_C(s)G(s)}{1+G_C(s)G(s)} \quad (3)$$

Where N(s) and D(s) are the numerator and denominator, respectively, of the transfer function with desired characteristics.
The control relation desired for this work is a third-order polynomial trajectory that was chosen for the control variable does not saturate in the positioning of both bases. The table coordinates should be able to follow it in a minimum time of 120 s, as well as a circular path with a radius of 25mm.
A motion trajectory is a third degree polynomial function of time defined by Equation (4) (Craig, JJ 1986).

$$P(t) = p_0 + p_1 t + p_2 t^2 + p_3 t^3 \tag{4}$$

The parameters P (t) are determined from equation (5), depending on the desired initial and final conditions in terms of displacement and velocity, namely: $P(t_0) = \ _0$, $\dot{P}(t_0) = v_0$, $\dot{P}(t_f) = v_f \quad e \quad P(t_f) = \ _f$.

$$\begin{bmatrix} 1 & t_0 & {t_0}^2 & {t_0}^3 \\ 0 & t_0 & 2t_0 & 3{t_0}^2 \\ 1 & t_f & {t_f}^2 & {t_f}^3 \\ 0 & 1 & 2t_f & 3t_f \end{bmatrix} \bullet \begin{Bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{Bmatrix} = \begin{bmatrix} _0 \\ v_0 \\ x_f \\ v_f \end{bmatrix} \tag{5}$$

For x0 = 0, v0 = 0, and x_f = 600 v_f = 0, with t0 = t_f = 0s and 130s, the polynomial P (t) is defined by equation (6).

$$P(t) = 0{,}10650887t^2 - 0{,}000546199t^3 \tag{6}$$

The path P (t) is similar to an input of a ramp as shown in Figure (3).

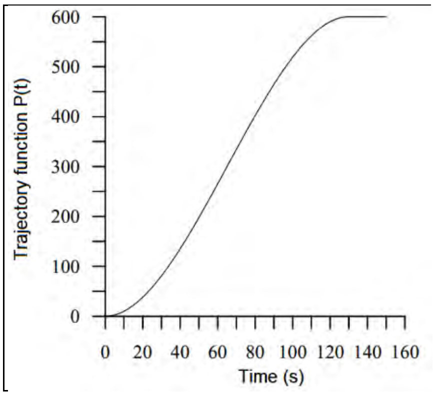


Figure 3 - Characteristic curve P(t)

In order to a transfer function having zero steady response to the error signal input of a ramp is necessary that it be a system of type 2 or greater, such as from Eq (7) (HOUPIS & D'Azzo, 1978).

$$_R(s) = \frac{f_3 {w_n}^3 s + {w_n}^4}{s^4 + f_1 w_n s^3 + f_2 {w_n}^2 s + f_3 {w_n}^3 s + {w_n}^4} \tag{7}$$

The transfer functions which represent the bases are third order and have no zeros in the numerator, which implies the need for a driver whose transfer function has at least one zero and be itself, as the function Gc (s) defined by Eq. (8).

$$_c(s) = k\frac{(s+\psi_1)}{(s+\psi_2)} \tag{8}$$

Knowing that the transfer functions of the base table coordinates has the following form:

$$(s) = \frac{k_0}{s^3 + a_1 s^2 + a_2 s} \tag{9}$$

Implies that the transfer function of the system under the action of the controller, closed loop using Eq (9) and Eq (8) is given by Eq (10).

$$T_1 = \frac{kk_0(s+\psi_1)}{s^4 + (a_1+\psi_2)s^3 + (a_2+\psi_2 a_1)s^2 + (\psi_2 a_2 + kk_0)s + kk_0\psi_1} \tag{10}$$

As T1 (s) is fourth order, there must be a reference model GR (s), also fourth order that accompanies the trajectory P (t), with zero error steady. Thus, equating equations (7 and 10) we get the following equality:

$$kk_0\psi_1 = {\omega_n}^4 \tag{11}$$

$$\psi_2 a_2 + kk_0 = f_3 {\omega_n}^3 \tag{12}$$

$$a_2 + \psi_2 a_1 = f_2 {\omega_n}^2 \tag{13}$$

$$a_1 + \psi_2 = f_1 \omega_n \tag{14}$$

$$kk_0 = f_3 {\omega_n}^3 \tag{15}$$

Replacing the equation (15) into equation (12) it follows that $\psi_2 = 0$. Thus, G_C (s) is reduced to a proportional integral controller with proportional gain $k_p = k$ and integral gain $ki = k \cdot \psi_1$, according to Equation (16).

$$_C = k_p + \frac{1}{s} k_i \tag{16}$$

The natural frequency ωn must be chosen such that G_R (s) has performance compatible with the design criteria. Thus, the parameter G_R (s), f_1 and f_2 can be determined through the equations (17 and 18):

$$f_1 = \frac{a_1}{\omega_n} \tag{17}$$

$$f_2 = \frac{a_2}{{\omega_n}^2} \tag{18}$$

Ψ_1 and f_3 parameters in equations (11 and 12) are functions of frequency proportional gain k ω_n.

The performance specifications for this project are: accommodation time of 100s, no response on signal, control variable without saturation and maximum error of 2%.

Although the method of Guillemin Truxal requires the poles and zeros desired immediately, this work seeks to accomplish a larger study, determining various functions G_R (s) that can be found by varying the parameters needed to find her. This study aims to determine the function that best meets the requirements of the project. Settling ω_n, f_3 is determined to a value of k such that G_R (s) meets these criteria. Using this value relation k and ω_n, determine ψ_1.

So, have the following script:

1. It selects a value of frequency ωn;
2. Is determined by f_1 and f_2 Equations (17 and 18);
3. Shall be chosen the desired value of the proportional gain k_p;
4. Determines the value _relation f_3 by Equation (15);
5. It is assessed if G_R (s) follows the trajectory of sinusoidal desired shape. If not, you can choose a new value for ω_n or k_p and returns to step 2. Otherwise it follows step 6;
6. Having the values determined above, are the values of ψ_1 and ψ_2;
7. How, in the project, $k_i = k_p \cdot \psi_1$, determine the integral gain and has the desired PI controller.

The function G_R (s) may initially be chosen as a criterion for performance index as the criterion Integral Absolute Error Multiplied by Time, AEMT as an initial step, which can facilitate the determination of the controller parameters easier.

3.1 Controller for the lower base

Following the script shown in previous section $\omega_n = 2$ rad/s of Eqs (17 and 18) and based on the parameters of the transfer function of the lower base, we can determine the values relation f_1 and f_2 through a computational routine created in MatLab program. The gain $k_p = 30$ was selected using as criteria the proper monitoring of the trajectory polynomial by G_R (s) and by the system under the action of the controller, because the variation of this parameter changes the correctness of it. Thus there was obtained: $f_1 = 62.65$, $916.25\ f_2 = f_3 = 291.75$. With these parameters the desired ratio of control or reference model result is shown in Equation (19).

$$_{Rinf}(s) = \frac{2334s+16}{s^4+125.3s^3+3665s^2+2334s+16} \tag{19}$$

The PI controller determined by the equations (13 and 14) has the following transfer function in Laplace

$$_{Cinf}(s) = 30 + \frac{0.20566}{s} \tag{20}$$

3.2 Controller for the upper base

Similarly the controller design for the lower base, in this case, for $\omega_n = 1$ rad/s and k = 30, it was determined: $f_1 = 64.18$, $f_2 = 863$, $f_3 = 733.2$. The function of the reference model of the upper base takes the form:

$$_{Rsup}(s) = \frac{733.2s+1}{s^4+128.36s^3+863s^2+733.2s+1} \tag{21}$$

The controller, in turn, is shown in Equation (22).

$$_{Csup}(s) = 30 + \frac{0.0409}{s} \tag{22}$$

3.3 Curves simulated and experimental trajectory polynomial

To trace the polynomial is necessary that each base moves in time following paths that composed, resulting in desired polynomial. Thus, Figure (4) shows the path followed by the polynomial lower base, and Fig (5) shows a control variable. Figure (6) shows the path followed by the base and top, Figure (7) shows that the behavior of its control variable.

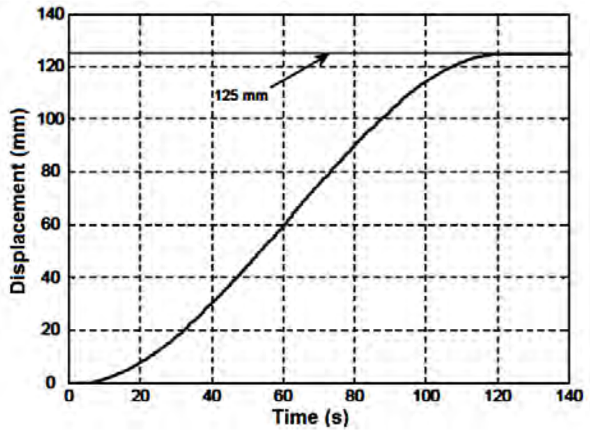


Figure 4 - Polynomial curve of third order drawn by the lower base

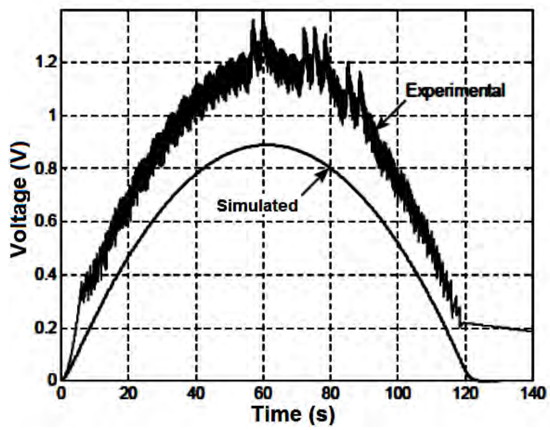


Figure 5 - Control variable for the lower base

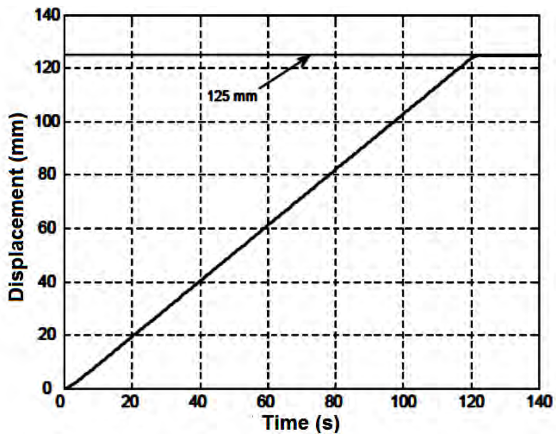


Figure 6 - Curve drawn by the upper base to follow a ramp function

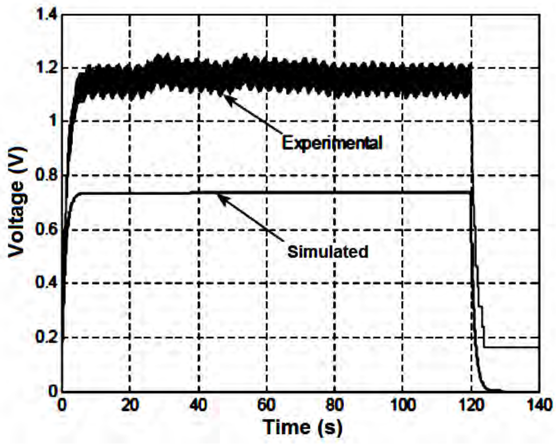


Figure 7 - Control variable for the upper base

Complementing Figure (8) shows the polynomial resulting from the movement path of the two bases.

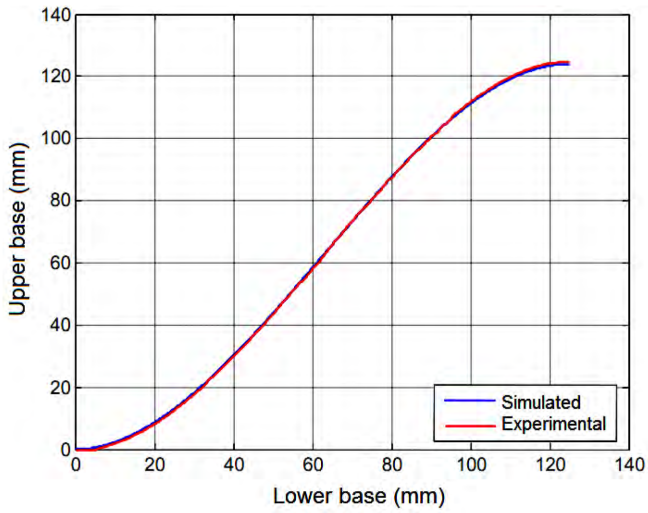


Figure 8 - Polynomial curve traced by simultaneous displacement of the two bases

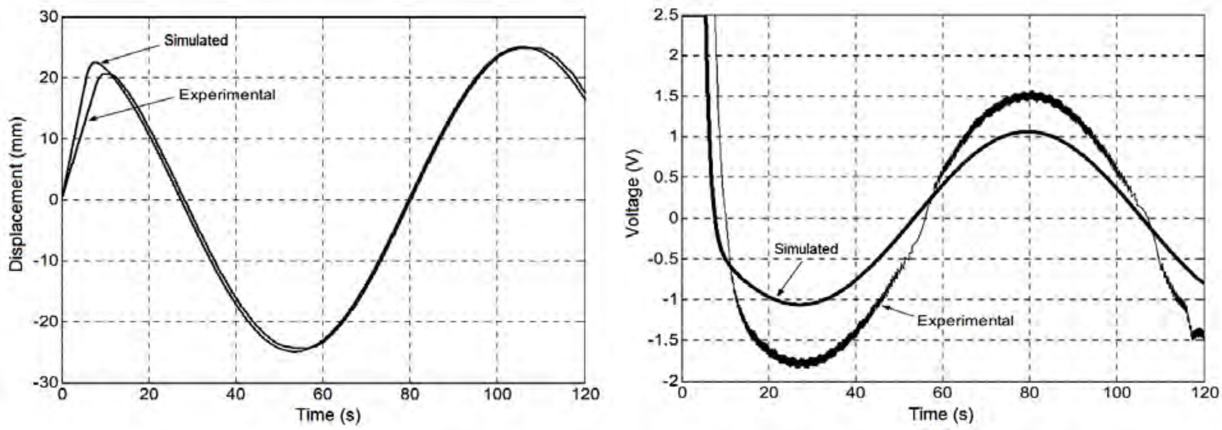
3.4 simulated and experimental curves for a circular path

The following shows the displacement curves of the upper and lower bases, resulting from the controllers influence on them to follow the explicit references in Eqs (23 and 24). Each curve shows both the simulated response and the experimental response. The composition of the paths leads to a circle of radius 25 mm and a frequency of 0.06 rad/s.

$$= 25 \cdot sin0.06t \tag{23}$$

$$y = 25 \cdot cos0.06t \tag{24}$$

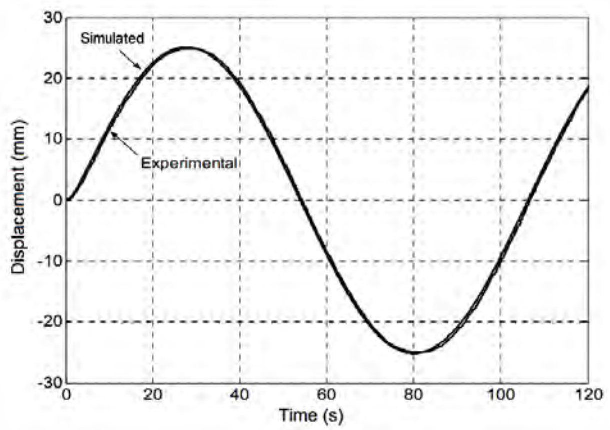
Figures (9a and 9b) show the displacement of the upper base and experimentally simulated and the behavior of the control variable for this base. It is noticed that the control variable remains without saturation (± 2.5 V), except in the small range where the trajectory is a ramp (deliberately imposed to prevent system instability, observed in experimental tests). In the comparison between the experimental and simulated response, there is a slight lag.

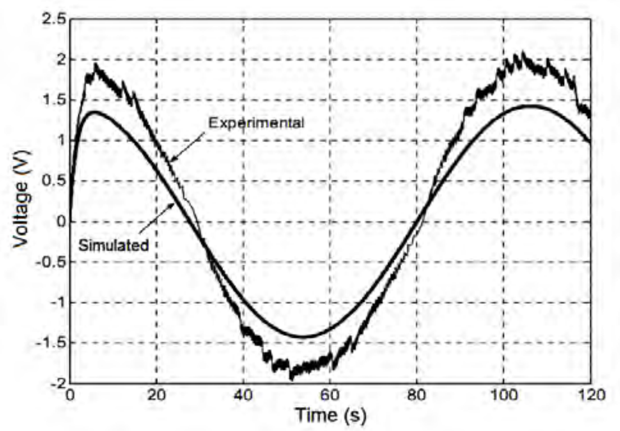


(a) (b)

Figure 9 - Curves tracking a trajectory cosine from upper base (a) and its control variable (b)

Figures (10a and 10b) respectively show the displacement of the lower base and the behavior of the control variable for this base. Again the control variable remains without saturation and the gap between the experimental and simulated response is minimal.





(a) (b)

Figure 10 - Curves tracking a sinusoidal trajectory from the lower base (a) and its control variable (b)

Figure (11) shows the resulting composition circumference of the trajectories of the upper and lower bases.

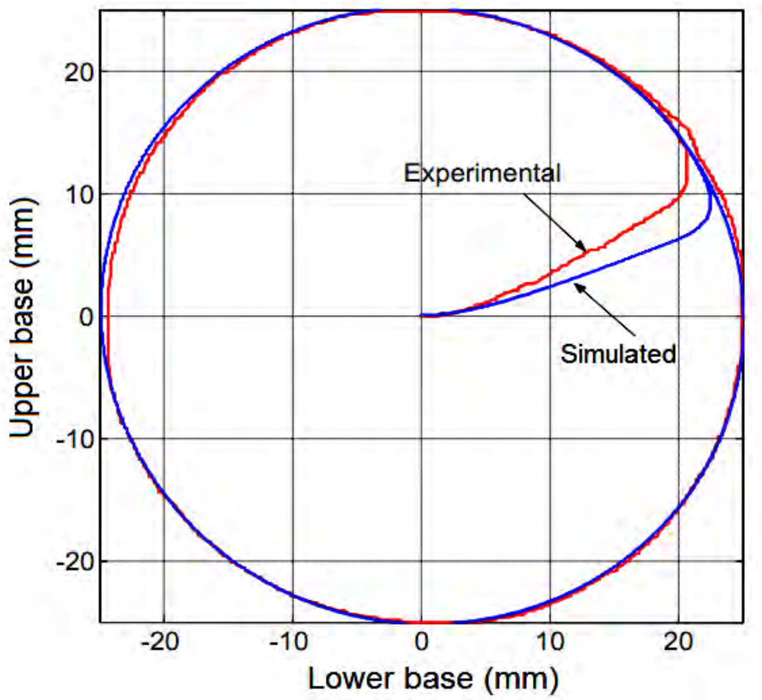


Figure 11 - Circumference simulated and experimental mapped by the composition of movements of the upper and lower bases

4. CONCLUSION

A new alternative controller design based on the procedure Guillemin - Truxal was presented in this paper. The methodology consists in designing a controller, to work on cascade with a specific system, so that the transfer function resulting in closed loop has the characteristics of a desired system. The parameters of the controllers were determined so that the system, under the action of the same controllers, accompanied a third degree polynomial trajectory from with minimum error (less than 2%), settling time of 100s and not saturation of the control variable. All of the above criteria were fulfilled according the results showed. The drivers were the type PI and also showed robustness with respect to the accompaniment of a circular path.

5. REFERENCES

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