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ATTITUDE DETERMINATION OF MULTIROTORS USING CAMERA VECTOR MEASUREMENTS

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Abstract. *The employment of camera in low-cost navigation and guidance of multirotor unmanned aerial vehicles (UAV) has recently been the focus of many investigations. Nevertheless, in the previous works, camera measurements was adopted either to aid in the position/velocity estimation or to directly provide feedback for guidance, but not specifically for assisting in the attitude determination process. This work is concerned with the attitude determination of multirotor UAVs using vector measurements taken from a camera. The vehicle is assumed to be equipped with an altimeter, a triad of rate-gyros, and a downward-facing strapdown camera. It is assumed to fly in an indoor environment containing various landmarks placed in known positions on the floor. The quantity and positions of the landmarks are chosen in such a way that at least two of them always remain in the camera field of view. Therefore, at each time instant, two noncollinear unit vectors directed from the camera to the center of area of the landmarks can be computed. In order to carry out attitude determination, two quaternion estimation methods are adopted: the multiplicative extended Kalman filter (MEKF) and the quaternion extended Kalman filter (QEKF). The proposed multirotor attitude determination scheme is evaluated by computational simulations.*

Keywords: *aerial robotics, attitude determination, Kalman filtering, computer vision*

1. INTRODUCTION

The attitude determination (AD) is a fundamental part of any control system for unmanned aerial vehicles (UAV). In general, it is concerned with the estimation of the vehicle's attitude and angular velocity with respect to a given reference coordinate system. The estimates computed by the AD function is then used to provide the attitude control laws with feedback information.

The literature on AD is very extensive and has mainly been developed in the aerospace field (Wertz, 1978), (Yang, 2012). The AD methods stems from the Wabba Problem (Markley, 1988), which defined a framework to estimate attitude from vector measurements. (Cheng *et al.*, 2008) uses the extended Kalman filter (EKF) to estimate pitch and roll angles of a Micro Aerial Vehicle (MAV). The third column of the Direction Cosine Matrix (DCM) and the rate gyro bias are used as state variables. Gravity is used as the observation vector in the measurement model. The yaw angle is obtained from geomagnetic field vector. Gebre-Egziabher and Elkaim (2008) use both gravity and geomagnetic field as observation vectors in two different approaches to estimate the attitude quaternion. The first approach is an iterated least-square estimator (LSE) and the second is an EKF. The LSE executes a global search of the attitude at each time step. On the contrary, the EKF algorithm accounts for *a priori* information, resulting in a better performance. The above two methods were designed to be gyro-free and GPS assisted. (Bar-Itzhack and Oshman, 1985) proposes a quaternion extended Kalman filter (QEKF), which, to ensure estimates with unit norm, realizes an Euclidian normalization step after each measurement update. (Idan, 1996) proposes a minimum-variance filter to estimate attitude parameterized by Rodrigues parameters. Due to simpler algebraic expressions, this approach has a relative computational advantage over the quaternion estimators. (Markley and Crassidis, 1996) presents a multiplicative extended Kalman filter (MEKF) that estimates an attitude error in MRP and updates the total attitude represented by quaternion by means of quaternion multiplication.

In satellite AD methods, vector measurements are typically taken from solar sensors (Sun direction), magnetometers (local geomagnetic field vector), horizon sensors (direction of nadir), star sensors (direction of stars) (Wertz, 1978). On the other hand, the multirotor UAV literature usually relies only on two vector measurements taken, respectively, from accelerometers (local vertical) and magnetometers.

This work presents a multirotor UAV attitude determination method using vector measurements taken from images. It is assumed that the vehicle is equipped with three strapdown sensors: a downward-facing camera, a triad of rate-gyros and an altimeter. The vehicle is assumed to fly indoors over a flat ground with various landmarks. Both vehicle and landmarks have known positions with respect to the adopted reference coordinate system. The landmarks are disposed, in quantity and positions, in such a way that at least two of them always remain in the camera field of view (FOV). Using measurements taken from the camera and the altimeter, two noncollinear vector measurements pointing from the camera to the landmarks' centers can be computed. In order to obtain a scheme for attitude determination of multirotor UAVs, these vector measurements as well as rate-gyro data are considered in two attitude estimation methods: the QEKF (Bar-Itzhack and Oshman, 1985) and the MEKF (Markley and Crassidis, 1996). The proposed scheme is evaluated by computational simulation. The remaining text is organized in the following manner. Section II defines the paper problem. Section III reformulates the attitude estimation methods. Section IV presents some simulation results. Finally, Section V presents the paper's conclusions.

Notation. $\mathbf{I}_{\mathrm{N}}$ is the $\mathrm{N}\times\mathrm{N}$ identity matrix, $[\bullet\times]$ denotes the cross product matrix and $\bullet'$ defines the matrix transpose.

2. PROBLEM STATEMENT

Consider the multirotor helicopter and the three Cartesian coordinate systems (CCS) illustrated in Fig. 1. The body CCS $S_{\mathrm{B}}=\{X_{\mathrm{B}},Y_{\mathrm{B}},Z_{\mathrm{B}}\}$ is attached to the vehicle at its center of mass (CM). The ground CCS $S_{\mathrm{G}}=\{X_{\mathrm{G}},Y_{\mathrm{G}},Z_{\mathrm{G}}\}$ is fixed on the ground at point O. The reference CCS $S_{\mathrm{R}}=\{X_{\mathrm{R}},Y_{\mathrm{R}},Z_{\mathrm{R}}\}$ is parallel to S_{G} but is centered at CM.

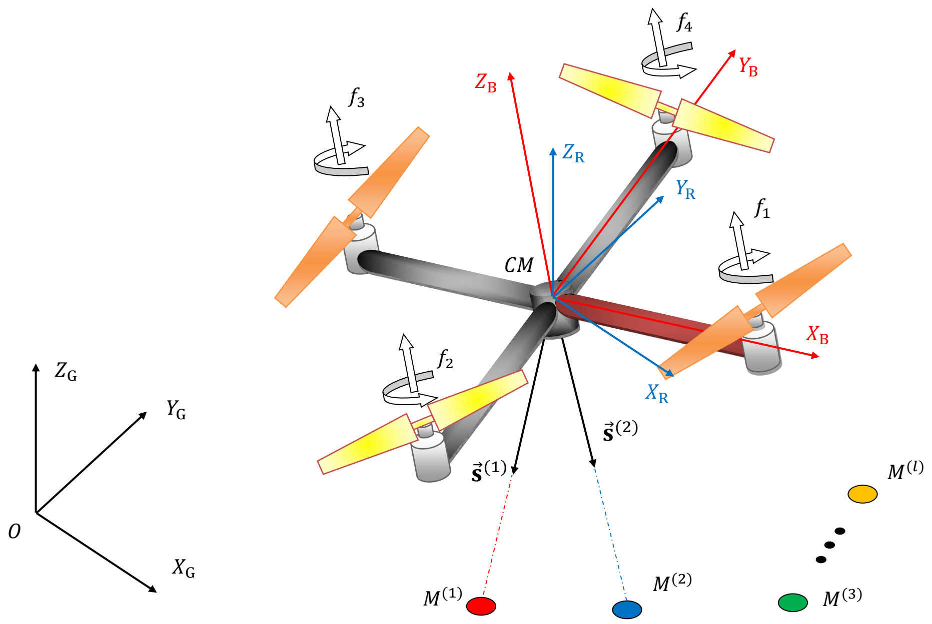


Figure 1: The Cartesian coordinate systems and the flight environment.

Assume that the camera is positioned at the CM and the triad of rate-gyros is aligned with S_{B}. Define the set of landmark indexes to be $\mathcal{I}\triangleq\{1,2,...,l\}$. Denote the center of the i-th landmark by $M^{(i)}$. Define $\vec{\mathbf{s}}^{(i)}$ to be the unit geometric vector pointing from CM to $M^{(i)}$. Denote the representations of $\vec{\mathbf{s}}^{(i)}$ in S_{B} and S_{R} by $\mathbf{b}^{(i)}\in\mathbb{R}^3$ and $\mathbf{r}^{(i)}\in\mathbb{R}^3$, respectively. The representations $\mathbf{b}^{(i)}$ and $\mathbf{r}^{(i)}$ are interrelated by $\mathbf{b}^{(i)}=\mathbf{D}\mathbf{r}^{(i)}$, where $\mathbf{D}\in SO(3)$ is the attitude matrix of S_{B} with respect to S_{R}. In order to measure two noncollinear pairs $(\mathbf{b}^{(i)},\mathbf{r}^{(i)})$, one assumes that both CM and landmarks have known positions and, moreover, at least two landmarks are measured by the camera at each sample instant. This yields the following two pairs of vector measurements:

$$\mathcal{V}_k\triangleq\left\{\left(\hat{\mathbf{b}}_k^{(i_1)},\hat{\mathbf{r}}_k^{(i_1)}\right),\left(\hat{\mathbf{b}}_k^{(i_2)},\hat{\mathbf{r}}_k^{(i_2)}\right)\right\},i_1\in\mathcal{I},i_2\in\mathcal{I},i_1\neq i_2, \tag{1}$$

where k denotes the discrete-time instant and, for $i=i_1,i_2$,

$$\hat{\mathbf{b}}_k^{(i)}=\mathbf{D}(\mathbf{a}_k)\mathbf{r}_k^{(i)}+\delta\mathbf{b}_k^{(i)}, \tag{2}$$

$$\mathbf{r}_k^{(i)}=\hat{\mathbf{r}}_k^{(i)}+\delta\mathbf{r}_k^{(i)}, \tag{3}$$

where $\hat{\mathbf{r}}_k^{(i)}$ is a sample of $\mathbf{r}^{(i)}$ at instant k, $\delta\mathbf{b}_k^{(i)}$ and $\delta\mathbf{r}_k^{(i)}$ are zero-mean Gaussian white sequences with covariances $\mathbf{R}_{b,k}^{(i)}$

and $\mathbf{R}_{r,k}^{(i)}$, respectively, and $\mathbf{a}_k \in \mathbb{R}^n$ is a discrete-time attitude representation vector which parameterizes the attitude matrix $\mathbf{D}(\mathbf{a}_k)$.

Let the attitude kinematics be modeled by the differential equation (Wertz, 1978)

$$\dot{\mathbf{a}}(t) = \mathbf{f}(\mathbf{a}(t), \boldsymbol{\omega}(t)), \tag{4}$$

where $\mathbf{a}(t)$ is a continuous-time version of $\mathbf{a}_k$, $\boldsymbol{\omega}(t) \in \mathbb{R}^3$ is the true angular velocity. Since the rate-gyros are not perfect, the true angular velocity is given by the following stochastic model:

$$\boldsymbol{\omega}(t) = \hat{\boldsymbol{\omega}}(t) + \delta\boldsymbol{\omega}(t), \tag{5}$$

where $\hat{\boldsymbol{\omega}}(t) \in \mathbb{R}^3$ is the measured angular velocity and $\delta\boldsymbol{\omega}(t) \in \mathbb{R}^3$ is the rate-gyro measurement noise, which is assumed to be a zero-mean Gaussian white sequence with covariance $\mathbf{Q}$. A discrete-time version of Eq.(5) is given by $\boldsymbol{\omega}_k = \hat{\boldsymbol{\omega}}_k + \delta\boldsymbol{\omega}_k$, where $\delta\boldsymbol{\omega}_k$ has the same characteristics of $\delta\boldsymbol{\omega}(t)$.

The main problem of the paper is to recursively compute the minimum-variance (MV) estimate $\hat{\mathbf{a}}_{k|k}$ of the true attitude vector $\mathbf{a}_k$ using the dynamic equation (4), the sequence of angular velocity measurements $\hat{\boldsymbol{\omega}}_{1:k}$, and sequence of vector measurements $\mathcal{V}_{1:k}$.

3. PROBLEM SOLUTION

This section presents two estimation methods to face the afore-defined problem: The Quaternion Extended Kalman Filter (QEKF) (Bar-Itzhack and Oshman, 1985) and the Multiplicative Extended Kalman Filter (MEKF) (Markley and Crassidis, 1996).

3.1 Quaternion extended Kalman filter - QEKF

Bar-Itzhack and Oshman (1985) proposed a discrete-time extended Kalman filter to estimate the attitude quaternion. This method is described in the sequel. Let the vector $\mathbf{a}(t)$ assumes the form of the attitude quaternion

$$\mathbf{q}(t) \triangleq \begin{bmatrix} q_{1,t} \\ \mathbf{e}_t \end{bmatrix}, \tag{6}$$

subject to the unit norm constrain

$$\|\mathbf{q}(t)\| = q_{1,t}^2 + \|\mathbf{e}_t\| = 1, \tag{7}$$

where $q_{1,t}$ and $\mathbf{e}_t$ are, respectively, the scalar and the complex part of the attitude quaternion. This gives rise to the following attitude kinematic equation (Wertz, 1978):

$$\dot{\mathbf{q}}(t) = \boldsymbol{\Omega}(t)\mathbf{q}(t), \tag{8}$$

where

$$\boldsymbol{\Omega}(t) = \frac{1}{2}\begin{bmatrix} 0 & -\boldsymbol{\omega}(t)' \\ \boldsymbol{\omega}(t) & -[\boldsymbol{\omega}(t)\times] \end{bmatrix}. \tag{9}$$

Integrating Eq.(9) from t_k to t_{k+1}, yields

$$\mathbf{q}_{k+1} = \boldsymbol{\Phi}(t_{k+1}, t_k)\mathbf{q}_k, \tag{10}$$

where $\boldsymbol{\Phi}(t_{k+1}, t_k) \in \mathbb{R}^{4\times 4}$ is the state transition matrix. Define $T_s \triangleq (t_{k+1} - t_k)$ as the sampling time. Assuming constant angular velocity $\boldsymbol{\omega}(t)$ during the interval T_s, the state transition matrix can be written as

$$\boldsymbol{\Phi}(t_{k+1}, t_k) = \mathrm{e}^{\boldsymbol{\Omega}_k T_s}, \tag{11}$$

where $\boldsymbol{\Omega}_k$ has the same form of Eq.(9), however it is computed using $\boldsymbol{\omega}_k$. Rewriting Eq.(11) by using the discrete-time version of Eq.(5), results

$$\mathbf{\Phi}(t_{k+1}, t_k) = \mathrm{e}^{\hat{\mathbf{\Omega}}_k T_s} \mathrm{e}^{\delta \mathbf{\Omega}_k T_s}, \tag{12}$$

where $\hat{\mathbf{\Omega}}_k$ and $\delta\mathbf{\Omega}_k$ are given by Eq.(9), but computed using $\hat{\boldsymbol{\omega}}_k$ and $\delta\boldsymbol{\omega}_k$, respectively. The second factor at the right-hand side of Eq.(12) is expanded in power series, yielding

$$\mathbf{\Phi}(t_{k+1}, t_k) = \mathrm{e}^{\hat{\mathbf{\Omega}}_k T_s} (\mathbf{I}_4 + \delta\mathbf{\Omega}_k T_s + ...). \tag{13}$$

By truncating the series in Eq.(13) after the first order term, it is possible to approximate Eq.(10) by

$$\mathbf{q}_{k+1} \approx \mathrm{e}^{\hat{\mathbf{\Omega}}_k T_s} \mathbf{q}_k + \mathrm{e}^{\hat{\mathbf{\Omega}}_k T_s} \delta\mathbf{\Omega}_k T_s \mathbf{q}_k. \tag{14}$$

Manipulating the second term in the right-hand side of Eq.(14), which is the state noise, one can obtain the discrete-time state model as follows:

$$\mathbf{q}_{k+1} = \mathrm{e}^{\hat{\mathbf{\Omega}}_k T_s} \mathbf{q}_k + \frac{T_s}{2} \mathrm{e}^{\hat{\mathbf{\Omega}}_k T_s} \mathbf{\Xi}_k \delta\boldsymbol{\omega}_k, \tag{15}$$

where

$$\mathbf{\Xi}_k \triangleq \begin{bmatrix} -\mathbf{e}_k' \\ [\mathbf{e}_k \times] + q_{1,k}\mathbf{I}_3 \end{bmatrix}. \tag{16}$$

Let $\mathbf{\Gamma}_k$ be defined by

$$\mathbf{\Gamma}_k = \frac{T_s}{2} \mathrm{e}^{\hat{\mathbf{\Omega}}_k T_s} \hat{\mathbf{\Xi}}_k, \tag{17}$$

where $\hat{\mathbf{\Xi}}_k$ is given by Eq.(16), but computed using $\hat{\mathbf{q}}_{k|k}$. The state noise covariance is approximated as follows:

$$\mathbf{Q}_k^q = \mathbf{\Gamma}_k \mathbf{Q} \mathbf{\Gamma}_k'. \tag{18}$$

The discrete-time nonlinear measurement model is now described in quaternion as follows:

$$\hat{\mathbf{b}}_k^{(i)} = \mathbf{D}(\mathbf{q}_k)\mathbf{r}_k^{(i)} + \delta\mathbf{b}_k^{(i)}, \tag{19}$$

where

$$\mathbf{D}(\mathbf{q}_k) = (q_{1,k}^2 - |\mathbf{e}_k|^2)\mathbf{I}_3 + 2\mathbf{e}_k\mathbf{e}_k' - 2q_{1,k}[\mathbf{e}_k \times]. \tag{20}$$

The QEKF requires the Jacobian matrix of the nonlinear measurement model of Eq.(20), which is defined as

$$\mathbf{H}_{\mathbf{q},k+1}^{(i)} \triangleq \frac{\partial \mathbf{D}(\mathbf{q})\mathbf{r}_{k+1}^{(i)}}{\partial \mathbf{q}}|_{\mathbf{q}=\hat{\mathbf{q}}_{k+1|k}} = \begin{bmatrix} \frac{\partial \mathbf{D}(\mathbf{q})}{\partial q_1}\mathbf{r}_{k+1}^{(i)} & \frac{\partial \mathbf{D}(\mathbf{q})}{\partial q_2}\mathbf{r}_{k+1}^{(i)} & \frac{\partial \mathbf{D}(\mathbf{q})}{\partial q_3}\mathbf{r}_{k+1}^{(i)} & \frac{\partial \mathbf{D}(\mathbf{q})}{\partial q_4}\mathbf{r}_{k+1}^{(i)} \end{bmatrix}_{\mathbf{q}=\hat{\mathbf{q}}_{k+1|k}}, \tag{21}$$

where the partial derivatives are given by

$$\frac{\partial \mathbf{D}(\mathbf{q})}{\partial q_1}|_{\mathbf{q}=\hat{\mathbf{q}}_{k+1|k}} = 2 \begin{bmatrix} \hat{q}_1 & \hat{q}_4 & -\hat{q}_3 \\ -\hat{q}_4 & \hat{q}_1 & \hat{q}_2 \\ \hat{q}_3 & -\hat{q}_2 & \hat{q}_1 \end{bmatrix}_{k+1|k}, \tag{22}$$

$$\frac{\partial \mathbf{D}(\mathbf{q})}{\partial q_2}|_{\mathbf{q}=\hat{\mathbf{q}}_{k+1|k}} = 2 \begin{bmatrix} \hat{q}_2 & \hat{q}_3 & \hat{q}_4 \\ \hat{q}_3 & -\hat{q}_2 & \hat{q}_1 \\ \hat{q}_4 & -\hat{q}_1 & -\hat{q}_2 \end{bmatrix}_{k+1|k}, \tag{23}$$

$$\frac{\partial \mathbf{D}(\mathbf{q})}{\partial q_3}|_{\mathbf{q}=\hat{\mathbf{q}}_{k+1|k}} = 2\begin{bmatrix} -\hat{q}_3 & \hat{q}_2 & -\hat{q}_1 \\ \hat{q}_2 & \hat{q}_3 & \hat{q}_4 \\ \hat{q}_1 & \hat{q}_4 & -\hat{q}_3 \end{bmatrix}_{k+1|k}, \tag{24}$$

$$\frac{\partial \mathbf{D}(\mathbf{q})}{\partial q_4}|_{\mathbf{q}=\hat{\mathbf{q}}_{k+1|k}} = 2\begin{bmatrix} -\hat{q}_4 & \hat{q}_1 & \hat{q}_2 \\ -\hat{q}_1 & -\hat{q}_4 & \hat{q}_3 \\ \hat{q}_2 & \hat{q}_3 & \hat{q}_4 \end{bmatrix}_{k+1|k}. \tag{25}$$

The QEKF consists of a discrete-time formulation of the extended Kalman filter (Bar-Shalom and Li, 1993) applied to the system modeled by the state equation (15) and the measurement equation (19). In order to force an unit norm property, an Euclidean normalization is carried out at each filter iteration, after the measurement update. For simplicity, the error covariance of the normalized estimate is approximated by $\mathbf{P}^*_{k+1|k+1} = \mathbf{P}_{k+1|k+1}$.

The QEKF algorithm is summarized as follows:

Initial conditions

$\hat{\mathbf{q}}^*_{0|0} = \hat{\mathbf{q}}_0$
$\mathbf{P}^*_{0|0} = \mathbf{P}^q_0$

State propagation

$\hat{\mathbf{q}}_{k+1|k} = \mathrm{e}^{\hat{\mathbf{\Omega}}_k T_s}\hat{\mathbf{q}}^*_{k|k}$
$\mathbf{P}_{k+1|k} = (\mathrm{e}^{\hat{\mathbf{\Omega}}_k T_s})\mathbf{P}^*_{k|k}(\mathrm{e}^{\hat{\mathbf{\Omega}}_k T_s})' + \mathbf{Q}^q_k$

Measurement prediction

$\begin{bmatrix} (\hat{\mathbf{b}}^{(i_1)}_{k+1|k})' & (\hat{\mathbf{b}}^{(i_2)}_{k+1|k})' \end{bmatrix}' = \begin{bmatrix} \mathbf{D}(\hat{\mathbf{q}}_{k+1|k})(\mathbf{r}^{(i_1)}_{k+1})' & \mathbf{D}(\hat{\mathbf{q}}_{k+1|k})(\mathbf{r}^{(i_2)}_{k+1})' \end{bmatrix}'$
$\mathbf{P}^b_{k+1|k} = \mathbf{H}_{\mathbf{q},k+1}\mathbf{P}_{k+1|k}\mathbf{H}'_{\mathbf{q},k+1} + \mathbf{R}_{k+1}$

Update

$\mathbf{K}_{k+1} = \mathbf{P}_{k+1|k}\mathbf{H}'_{\mathbf{q},k+1}(\mathbf{P}^b_{k+1|k})^{-1}$
$\hat{\mathbf{q}}_{k+1|k+1} = \hat{\mathbf{q}}_{k+1|k} + \mathbf{K}_{k+1}\left(\begin{bmatrix} (\mathbf{b}^{(i_1)}_{k+1})' & (\mathbf{b}^{(i_2)}_{k+1})' \end{bmatrix}' - \begin{bmatrix} (\hat{\mathbf{b}}^{(i_1)}_{k+1|k})' & (\hat{\mathbf{b}}^{(i_2)}_{k+1|k})' \end{bmatrix}'\right)$
$\mathbf{P}_{k+1|k+1} = \mathbf{P}_{k+1|k} - \mathbf{K}_{k+1}\mathbf{P}^b_{k+1|k}\mathbf{K}'_{k+1}$

Normalization
$\hat{\mathbf{q}}^*_{k+1|k+1} = \frac{\hat{\mathbf{q}}_{k+1|k+1}}{||\hat{\mathbf{q}}_{k+1|k+1}||}$
$\mathbf{P}^*_{k+1|k+1} = \mathbf{P}_{k+1|k+1}$

Note that

$$\mathbf{H}_{\mathbf{q},k+1} = \begin{bmatrix} \mathbf{H}^{(i_1)}_{\mathbf{q},k+1} & \mathbf{H}^{(i_2)}_{\mathbf{q},k+1} \end{bmatrix}', \tag{26}$$

and

$$\mathbf{R}_{k+1} = \begin{bmatrix} \mathbf{R}^{(i_1)}_{b,k+1} & \mathbf{0}_{3\times 3} \\ \mathbf{0}_{3\times 3} & \mathbf{R}^{(i_2)}_{b,k+1} \end{bmatrix}. \tag{27}$$

The estate transition matrix is solved by [(Wertz, 1978), pp.567]

$$e^{\hat{\mathbf{\Omega}}_k T_s} = \cos(\|\hat{\boldsymbol{\omega}}_k\|\frac{T_s}{2})\mathbf{I}_4 + \frac{1}{\|\hat{\boldsymbol{\omega}}_k\|}\sin(\|\hat{\boldsymbol{\omega}}_k\|\frac{T_s}{2})\hat{\mathbf{\Omega}}_k. \tag{28}$$

3.2 Multiplicative extended Kalman filter - MEKF

Markley and Crassidis (1996) proposed a continuous/discrete-time filter which represents the true attitude quaternion by

$$\mathbf{q}(t) = \delta\mathbf{q}(\mathbf{p}(t)) \otimes \hat{\mathbf{q}}(t), \tag{29}$$

where $\hat{\mathbf{q}}(t)$ is a reference quaternion, $\delta\mathbf{q}(\mathbf{p}(t))$ is the multiplicative error quaternion parameterized by modified Rodrigues parameters $\mathbf{p}(t)$, and $\otimes$ denotes the quaternion product (Shuster, 1993). The reference quaternion $\hat{\mathbf{q}}(t)$ is considered the best estimate of the true quaternion $\mathbf{q}(t)$ between the interval $[t_k, t_{k+1})$. Thus, the MRP assumes $\mathbf{p}(t) = 0$ for $t \in [t_k, t_{k+1})$, which eliminates the redundancy of two paramerizations use.

Let Eq.(4) be redefined by the MRP $\mathbf{p}(t) \triangleq \begin{bmatrix} p_{1,t} & p_{2,t} & p_{3,t} \end{bmatrix}'$ as follows:

$$\dot{\mathbf{p}}(t) = \mathbf{f}(\mathbf{p}(t), \boldsymbol{\omega}(t)). \tag{30}$$

The nonlinear function $\mathbf{f}(\mathbf{p}(t), \boldsymbol{\omega}(t))$ is defined by

$$\mathbf{f}(\mathbf{p}(t), \boldsymbol{\omega}(t)) = \mathbf{G}(\mathbf{p}(t))\boldsymbol{\omega}(t), \tag{31}$$

where (Schaub, 1998)

$$\mathbf{G}(\mathbf{p}(t)) = \frac{1}{4}\left\{(1 - ||\mathbf{p}(t)||^2)\mathbf{I}_3 + 2[\mathbf{p}(t)\times] + 2\mathbf{p}(t)\mathbf{p}(t)'\right\}. \tag{32}$$

Applying Eq.(5) in Eq.(31), and the result in Eq.(30), yields in the state model as follows:

$$\dot{\mathbf{p}}(t) = \mathbf{G}(\mathbf{p}(t))\hat{\boldsymbol{\omega}}(t) + \mathbf{G}(\mathbf{p}(t))\delta\boldsymbol{\omega}(t), \tag{33}$$

where the second term in the right-hand side of Eq.(33) is the state noise. Its covariance is approximated as:

$$\mathbf{Q}^{\mathbf{p}}(t) = \boldsymbol{\Gamma}(t)\mathbf{Q}\boldsymbol{\Gamma}(t)', \tag{34}$$

where $\boldsymbol{\Gamma}(t) = \mathbf{G}(\hat{\mathbf{p}}(t)), \forall t \in [t_k, t_{k+1})$.

The MEKF requires the Jacobian matrix of Eq.(33), as follows:

$$\mathbf{F}(\hat{\mathbf{p}}(t), \hat{\boldsymbol{\omega}}(t)) \triangleq \frac{\partial \mathbf{G}(\mathbf{p}(t))\boldsymbol{\omega}(t)}{\partial \mathbf{p}}|_{\mathbf{p}=\hat{\mathbf{p}}(t)}. \tag{35}$$

Assuming null MRP for $[t_k, t_{k+1})$, Eq.(35) results in

$$\mathbf{F}(\hat{\mathbf{p}}(t), \hat{\boldsymbol{\omega}}(t)) = \frac{1}{2}(-[\hat{\boldsymbol{\omega}}\times]). \tag{36}$$

Let discrete-time measurement model be defined in MRP by

$$\hat{\mathbf{b}}_k^{(i)} = \mathbf{D}(\mathbf{p}_k)\mathbf{r}_k^{(i)} + \delta\mathbf{b}_k^{(i)}, \tag{37}$$

where (Shuster, 1993)

$$\mathbf{D}(\mathbf{p}_k) = \mathbf{I}_3 + \frac{8[\mathbf{p}_k\times]^2 - 4(1 - \|\mathbf{p}_k\|^2)[\mathbf{p}_k\times]}{(1 + \|\mathbf{p}_k\|^2)^2}, \tag{38}$$

is the attitude matrix in MRP. In order to obtain a linear model of Eq.(37), first order Taylor expansion is applied. The Jacobian of this operation is given by

$$\mathbf{H}^{(i)}_{\mathbf{p},k+1} \triangleq \frac{\partial \mathbf{D}(\mathbf{p})\mathbf{r}^{(i)}_{k+1}}{\partial \mathbf{p}}|_{\mathbf{p}=\hat{\mathbf{p}}_{k+1|k}} = \begin{bmatrix} \frac{\partial \mathbf{D}(\mathbf{p})}{\partial p_1}\mathbf{r}^{(i)}_{k+1} & \frac{\partial \mathbf{D}(\mathbf{p})}{\partial p_2}\mathbf{r}^{(i)}_{k+1} & \frac{\partial \mathbf{D}(\mathbf{p})}{\partial p_3}\mathbf{r}^{(i)}_{k+1} \end{bmatrix}_{\mathbf{p}=\hat{\mathbf{p}}_{k+1|k}}, \tag{39}$$

where the partial derivatives, assuming null MRP $\forall t \in [t_k, t_{k+1})$, yield

$$\frac{\partial \mathbf{D}(\mathbf{p})}{\partial p_1}|_{\mathbf{p}=\hat{\mathbf{p}}_{k+1|k}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & -4 & 0 \end{bmatrix}, \tag{40}$$

$$\frac{\partial \mathbf{D}(\mathbf{p})}{\partial p_2}|_{\mathbf{p}=\hat{\mathbf{p}}_{k+1|k}} = \begin{bmatrix} 0 & 0 & -4 \\ 0 & 0 & 0 \\ 4 & 0 & 0 \end{bmatrix}, \tag{41}$$

$$\frac{\partial \mathbf{D}(\mathbf{p})}{\partial p_3}|_{\mathbf{p}=\hat{\mathbf{p}}_{k+1|k}} = \begin{bmatrix} 0 & 4 & 0 \\ -4 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{42}$$

By means of the continuous-discrete EKF, both state model given by Eq.(33) and measurement model given by Eq.(37) are fused in order to estimate the attitude error in MRP. The global nonsingular attitude is propagated in quaternion by Eq.(15) in the interval $[t_k, t_{k+1})$. The update of the global attitude is given by the discrete-time version of Eq.(29), where

$$\delta\mathbf{q}(\mathbf{p}_{k+1|k+1}) = \begin{bmatrix} \frac{1-||\mathbf{p}_{k+1|k+1}||^2}{1+||\mathbf{p}_{k+1|k+1}||^2} \\ \frac{2p_{1,k+1|k+1}}{1+||\mathbf{p}_{k+1|k+1}||^2} \\ \frac{2p_{2,k+1|k+1}}{1+||\mathbf{p}_{k+1|k+1}||^2} \\ \frac{2p_{3,k+1|k+1}}{1+||\mathbf{p}_{k+1|k+1}||^2} \end{bmatrix}. \tag{43}$$

The MEKF algorithm is summarized as follows:

Initial conditions

$\hat{\mathbf{q}}_{0|0} = \hat{\mathbf{q}}_0$
$\mathbf{P}^{\mathbf{p}}_{0|0} = \mathbf{P}^{\mathbf{p}}_0$
$\hat{\mathbf{m}}_{0|0} = 0$

State propagation

$\hat{\mathbf{p}}(t) = 0, t \in [t_k, t_{k+1})$
$\dot{\mathbf{P}}^{\mathbf{p}}(t) = \mathbf{F}(\hat{\mathbf{p}}(t), \hat{\boldsymbol{\omega}}(t))\mathbf{P}^{\mathbf{p}}(t) + \mathbf{P}^{\mathbf{p}}(t)\mathbf{F}(\hat{\mathbf{p}}(t), \hat{\boldsymbol{\omega}}(t))' + \mathbf{Q}^{\mathbf{p}}(t)$
$\hat{\mathbf{q}}_{k+1|k} = \mathrm{e}^{\hat{\boldsymbol{\Omega}}_k T_s}\hat{\mathbf{q}}^*_{k|k}$

Measurement prediction

$\begin{bmatrix} (\hat{\mathbf{b}}^{(i_1)}_{k+1|k})' & (\hat{\mathbf{b}}^{(i_2)}_{k+1|k})' \end{bmatrix}' = \begin{bmatrix} \mathbf{D}(\hat{\mathbf{q}}_{k+1|k})(\mathbf{r}^{(i_1)}_{k+1})' & \mathbf{D}(\hat{\mathbf{q}}_{k+1|k})(\mathbf{r}^{(i_2)}_{k+1})' \end{bmatrix}'$
$\mathbf{P}^b_{k+1|k} = \mathbf{H}_{\mathbf{p},k+1}\mathbf{P}^{\mathbf{p}}_{k+1|k}\mathbf{H}'_{\mathbf{p},k+1} + \mathbf{R}_{k+1}$

Update

$\mathbf{K}_{k+1} = \mathbf{P}^{\mathbf{p}}_{k+1|k}\mathbf{H}'_{\mathbf{p},k+1}(\mathbf{P}^b_{k+1|k})^{-1}$
$\hat{\mathbf{p}}_{k+1|k+1} = \mathbf{K}_{k+1}\left(\begin{bmatrix} (\mathbf{b}^{(i_1)}_{k+1})' & (\mathbf{b}^{(i_2)}_{k+1})' \end{bmatrix}' - \begin{bmatrix} (\hat{\mathbf{b}}^{(i_1)}_{k+1|k})' & (\hat{\mathbf{b}}^{(i_2)}_{k+1|k})' \end{bmatrix}'\right)$
$\mathbf{P}^{\mathbf{p}}_{k+1|k+1} = \mathbf{P}^{\mathbf{p}}_{k+1|k} - \mathbf{K}_{k+1}\mathbf{P}^b_{k+1|k}\mathbf{K}'_{k+1}$

$\hat{\mathbf{q}}_{k+1|k+1} = \delta\mathbf{q}(\mathbf{p}_{k+1|k+1}) \otimes \hat{\mathbf{q}}_{k+1|k}$

Note that

$$\mathbf{H}_{\mathbf{q},k+1} = \begin{bmatrix} \mathbf{H}^{(i_1)}_{\mathbf{q},k+1} & \mathbf{H}^{(i_2)}_{\mathbf{q},k+1} \end{bmatrix}', \tag{44}$$

and

$$\mathbf{R}_{k+1} = \begin{bmatrix} \mathbf{R}^{(i_1)}_{b,k+1} & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{R}^{(i_2)}_{b,k+1} \end{bmatrix}. \tag{45}$$

4. SIMULATION AND RESULTS

The performance of both presented estimators will be compared using simulated data. The multirotor true attitude $\mathbf{q}_k$ is propagated by Eq.(15) with the following angular velocity:

$$\boldsymbol{\omega}_k = \begin{bmatrix} 0.1\sin(kT_s) \\ 0.1\cos(kT_s) \\ -0.1\sin(kT_s)\cos(kT_s) \end{bmatrix}, \tag{46}$$

where $T_s = 0.1s$. The camera vector measurements were generated using Eq.(2), where

$$\mathbf{r}^{(1)}_k = \begin{bmatrix} 0 \\ \frac{5}{13} \\ -\frac{12}{13} \end{bmatrix}, \tag{47}$$

$$\mathbf{r}^{(2)}_k = \begin{bmatrix} 0 \\ -\frac{5}{13} \\ -\frac{12}{13} \end{bmatrix}. \tag{48}$$

Both rate-gyro and camera noise covariances were tuned in order to not diverge the filter estimate. Table 1 shows the assumed measurement noise covariances.

Table 1: Measurement noise covariances.

Sensor	Covariance
Rate-gyro	$\mathbf{Q}_k = (0.005)^2\mathbf{I}_3\ (rad/s)^2$
Camera	$\mathbf{R}^{(i)}_{b,k} = (0.01)^2\mathbf{I}_3$

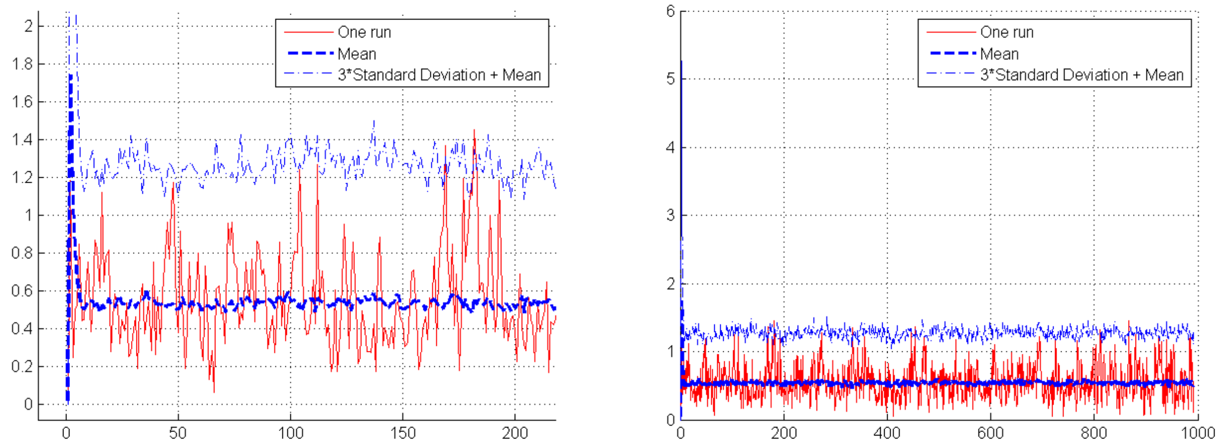
Using the simulated measurements, both QEKF and MEKF were submitted to one hundred Monte-Carlo runs with $1000s$ of duration each. The integration for MEKF is given by fourth order Runge-Kutta. The initial conditions assumed are shown in Tab.2.

Table 2: Initial conditions.

Parameter	QEKF	MEKF
True attitude	$\mathbf{q}_0 \sim \mathcal{N}\left(\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}', \mathbf{P}_{0\|0}\right)$	$\mathbf{q}_0 \sim \mathcal{N}\left(\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}', \mathbf{P}_{0\|0}\right)$
State	$\hat{\mathbf{q}}_{0\|0} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}'$	$\hat{\mathbf{q}}_{0\|0} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}', \hat{\mathbf{p}}_{0\|0} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}'$
Covariance	$\mathbf{P}_{0\|0} = 0.01\mathbf{I}_4$	$\mathbf{P}^{\mathbf{p}}_{0\|0} = 0.01\mathbf{I}_3$

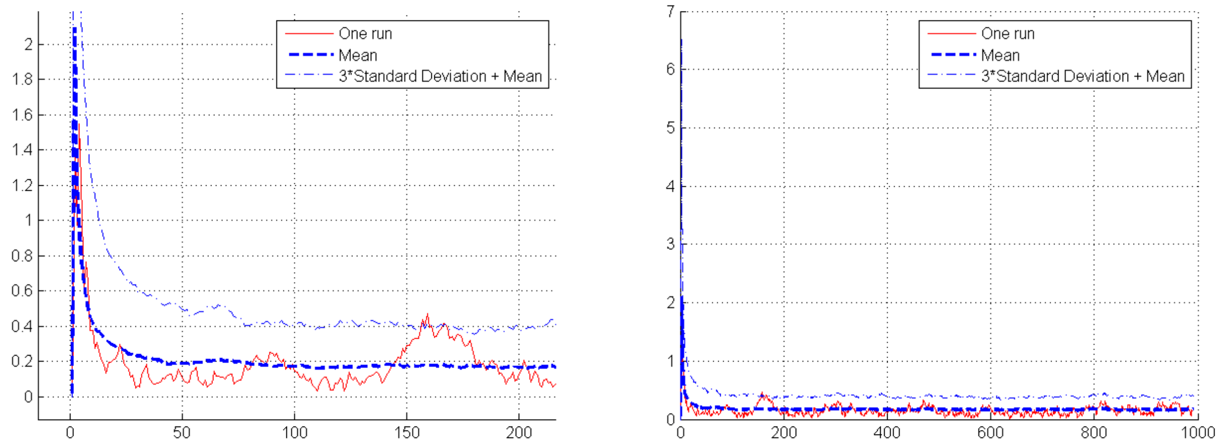
Accuracy, orthogonality and relative computational burden are the parameters to be examined. The accuracy is measured as follows (Wertz, 1978):

$$I_k = |acos\left(\frac{1}{2}\left[tr\left(\mathbf{D}(\hat{\mathbf{q}}_{k|k})'\mathbf{D}(\mathbf{q}_k)\right) - 1\right]\right)|, \tag{49}$$



(a) Zoom in 0-200 seconds (b) 0-1000 seconds

Figure 2: MEKF angular error in degree



(a) Zoom in 0-200 seconds (b) 0-1000 seconds

Figure 3: QEKF angular error in degree

where the index I_k corresponds to the error angle between the true and the estimate attitudes in the Euler principal angle notation. The orthogonality is given by

$$J_k = tr\left\{\left[\mathbf{D}(\hat{\mathbf{q}}_{k|k})'\mathbf{D}(\hat{\mathbf{q}}_{k|k}) - \mathbf{I}_3\right]'\left[\mathbf{D}(\hat{\mathbf{q}}_{k|k})'\mathbf{D}(\hat{\mathbf{q}}_{k|k}) - \mathbf{I}_3\right]\right\} \tag{50}$$

where the index J_k describes how close the estimate attitude matrix is to the orthogonal matrix, as it gets closer to zero.

Since the CPU performs tasks parallel to the simulation, it is not possible to use the cycle time for measure an absolute computational burden of each filter. Rather, the cycle time is used to measure how fast is one algorithm relative to the other.

Defined the simulation conditions, the mean and the standard deviation values of both indexes I_k and J_k are calculated. Figure (2) shows the MEKF mean accuracy index between 0.4 and 0.6 degrees, while Fig.(3) presents same index for QEKF approximately equal to 0.2 degrees. For this simulation conditions, the QEKF shows better accuracy than the MEKF. From Figs. (5) and (4), one can conclude that the QEKF attitude matrix is closer to the orthogonal matrix than the MEKF one. The QEKF spent an average of $0.115ms$ per cycle while the MEKF spent $0.152ms$, resulting in 24.34% more time consumption for the MEKF over the QEKF.

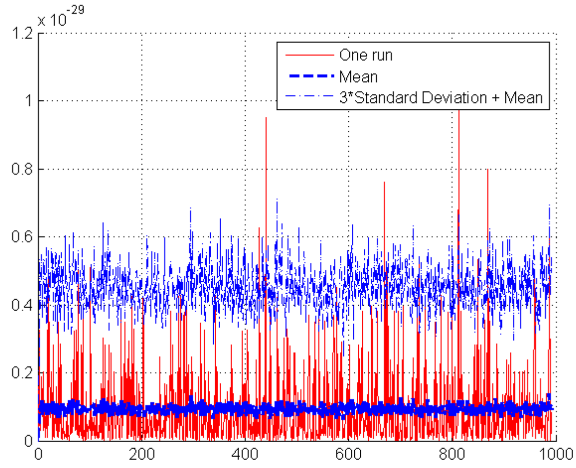


Figure 4: QEKF orthogonality Index

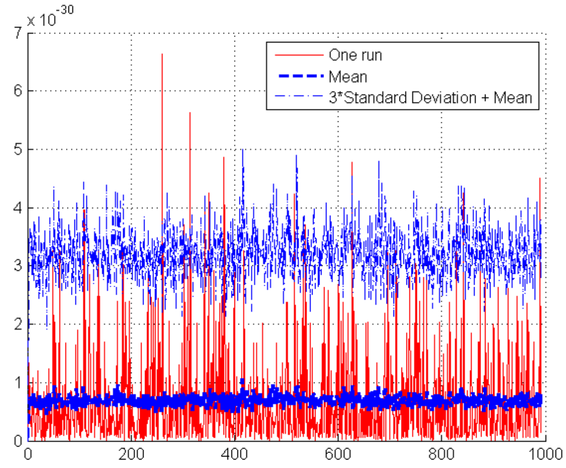


Figure 5: MEKF orthogonality Index

5. CONCLUSION

Two attitude determination methods based on camera vector measurements were presented. The quaternion extended Kalman filter performed better than the multiplicative extended Kalman filter for the proposed simulation scheme. However, MEKF does not need a normalization step after the state update. These methods are suitable for indoor environments, since they do not use GPS. An alternative upgrade for outdoor flight is to use gravity direction and geomagnetic field vector along with camera vector measurements. An experimental flight test is being prepared in order to evaluate the embedded computational burden.

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