ANALYTICAL EQUATION FOR MOTION CONSTRAINTS IN CONFINED ENVIRONMENTS FOR A $P6R$ REDUNDANT ROBOT

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Abstract. *Redundant robots have additional mobilities that allows applications beyond the conventional robot. The additional mobilities of robots enable to include extras tasks, depending on the degree of redundancy. Several references present techniques and methodologies to solve the kinematic redundancy to robots with open or closed kinematics chains. In the cases of operation of robots in confined environments, the classical methods like Denavit-Hartenberg, for example, makes more difficult the precise analysis of the limits and constraints of robot's joints movements. Recent works have shown that the use of screw theory, and its tools, turn the robots analysis easier, even for complex differential and kinematic model, when compared with the classical methods. The screw theory allows to introduce new solutions on study of singularity and motion constraint in confined environments. Again, researches have also shown ways to evaluate the movement constraints derived from a differential model of redundant robots, based on analytical expressions. These studies are limited to computational experiments in a planar redundant robot model. This paper presents theoretical aspects of the differential model and the mathematical strategies to obtain the expression of the constrained movement for a $P6R$ redundant robot operating in confined environments including the avoiding collision as a secondary task. It is presented theoretical issues and the development of the position and differential kinematics model, based on screw theory. As result, it is explored the mathematical model to identify the $P6R$ singular postures including the influences of the collision avoidance task*

Keywords: *Redundant robots, kinematic constraints, collision avoidance, screw theory, analytical singularities*

1. INTRODUCTION

Robotic operations in confined environments imply in the imminent possibility of collision of the robot with some part of its workspace. The avoidance and treatment of collision possibilities present themselves as an additional restriction to be planned resulting in a secondary task, since the primary task, the robot will perform with its end-effector. The collision avoidance, as secondary task in robotic systems, requires additional movements of the robot, beyond those required for performing the primary task, to reposition its kinematic chain away from to the collision points in the environment around the task. In this sense, the robot must have some degree of redundancy.

The robot redundancy r is computed by the difference between robot DOF, in other words, the number of joints n, and the DOF necessary to perform the task m, expressed by: $r = n - m$ (Siciliano *et al.*, 2009). The degree of redundancy determines the number of constraints, or the DOF, for a second task.

The use of the redundancy suggests new solutions for direct and inverse kinematics, since traditional methods may turn difficult to get adequate results (Chang, 1996)(Piaggio, 1999)(Muller, 2004)(Soucy and Payeur, 2005). The main methods are based on nullspace of the Jacobian matrix, like Pseudoinverse (Siciliano *et al.*, 2009) and task priority (Chiaverini, 1997)(Antonelli and Chiaverini, 1998). Recent works presented new methods for solution of redundancy based on screw theory.

The screw theory is based that any spatial movement can be represented as a combination of a linear movement with a rotational movement (Hunt, 2000). From the screw theory, several tools and mathematical methods have been developed for modelling the kinematic of mechanisms and robots. Highlighting among these tools, the Davies' method (Davies, 1981), and Assur virtual chains (Simas *et al.*, 2009), have been extensively used lately in several studies (Campos *et al.*, 2009) (Nokleby and Podhorodeski, 2001) (Dai and Jones, 2001)(Simas *et al.*, 2011), and among these, are taken as the basis for developing the solution for collision avoidance of a redundant robot operating tasks in confined environment (Simas *et al.*, 2004)(Simas *et al.*, 2009).

The potential of application of the screw theory in differential kinematic modeling and solution for redundant robots has been studied by Campos (Campos *et al.*, 2009) and Simas (Simas *et al.*, 2009) and specifically for analysis of kinematic singularities for a planar redundant robots by Simas (Simas *et al.*, 2011). The paper goal is to present an advanced study of singularities for spatial redundant robots operating in confined environment and subjected to kinematic constraints imposed derived from virtual kinematic chains, used to task of collision avoidance. The study results allows to write mathematical formulas for the evaluation of kinematic singularity considering the restriction and limitation of movement in terms of the secondary task, or the collision avoidance task. It avoided with this result, work and control with algorithmic singularities, quite common in other methods previously presented in the literature (Chiaverini, 1997) (Muller, 2004)(Soucy and Payeur, 2005).

Firstly, it is presented the theoretical aspects of the screw theory and their respective methods and mathematical tools used in the study developed. Following the kinematic model is developed for a $P6R$ redundant robot, operating in an environment delimited by a plane, where an Assur virtual chain is used to perform the collision avoidance. Finally, are presented the partial equations of the model, and the final mathematical equation for the kinematic singularities control of the complete robotic system is obtained, including the Assur virtual chain, as a result of development. The mathematical equations for the kinematic singularities control are analised and discussed through images that represent the final results.

2. TOOLS FROM SCREW THEORY

The approach, here proposed in this paper is based on the Davies' method, Assur virtual chains, direct graph notation and extended Jacobian from kinematic restrictions, where the screw displacement are successively applied. Those topics are extensively explored in literature and briefly presented in following sections.

2.1 The description of movements through Screws

The general spatial differential movement of a rigid body consists of a differential rotation about an axis, and a differential translation along the same axis named the instantaneous screw axis. The complete movement of the rigid body, can be described as a combining rotation (θ displacement) and translation (t displacement) called screw movement or twist, here denoted by \$. The ratio of the linear velocity to the angular velocity is called pitch of the screw denoted as h (Tsai, 1999).

The twist may be expressed by a pair of vectors $\$ = \left[\omega^T; V_p^T\right]^T$, where ω represents the angular velocity of the body with respect to the inertial frame (reference frame or link r) and V_p represents the linear velocity of a point P attached to the body which is instantaneously coincident with the origin O of the reference frame.

So, a twist may be decomposed into its magnitude and its corresponding normalized screw. The twist magnitude $\dot{q}$ is either the magnitude of the angular velocity of the body, $\|\omega\|$, if the kinematic pair is rotative ($h = 0$) or helical, or the magnitude of the linear velocity, $\|V_p\|$, if the kinematic pair is prismatic ($h \rightarrow \infty$) (Hunt, 2000). The normalized screw $\hat{\$}$ is a twist of unitary magnitude, i.e.

$$\$ = \hat{\$}\dot{q} \tag{1}$$

The normalized screw coordinates $\hat{\$}$ is written as:

$$\hat{\$} = \begin{bmatrix} s_i \\ s_{oi} \times s_i \end{bmatrix} \text{ for rotative pairs and } \quad \hat{\$} = \begin{bmatrix} 0 \\ s_i \end{bmatrix} \text{ for prismatic pairs (Tsai, 1999),} \tag{2}$$

where $s_i = \left[s_{i_x}, s_{i_y}, s_{i_z}\right]$ denotes an unit vector along the direction of the screw axis, and vector s_{oi} represents the position vector of a point lying on the screw axis.

Thus, the twist in Eq.(2) expresses the general spatial differential movement (velocity) of a rigid body relative to an inertial reference frame $O - xyz$.

If the kinematic pair is rotative, s_i points in the direction of rotative axis, and, if the kinematic pair is prismatic, s_i points in the direction of kinematic pair displacement. It is important to note that $s_i^T s_{oi} = 0$, ie, they are perpendicular (Tsai, 1999) (Hunt, 2000). Figures 1 and 2 depict the location of vector s_i and s_{oi} for a rotative and a prismatic kinematic pair respectively .

The twist can also represent the movement between two adjacent links of a kinematic chain, the sucessive screw displacement (Tsai, 1999). In this case, s_i and s_{oi} represents the movement of link i relative to link $(i - 1)$ (see Fig. 1 and Fig. 2) and a homogeneous transformation is obtained (Tsai, 1999), as Eq.(3).

$$A_i = \begin{bmatrix} a_{11} & a_{12} & a_{13} & ts_x - s_{ox}(a_{11} - 1) - s_{oy}a_{12} - s_{oz}a_{13} \\ a_{21} & a_{22} & a_{23} & ts_y - s_{ox}a_{21} - s_{oy}(a_{22} - 1) - s_{oz}a_{23} \\ a_{31} & a_{32} & a_{33} & ts_z - s_{ox}a_{31} - s_{oy}a_{32} - s_{oz}(a_{33} - 1) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

where:

$$\begin{aligned} a_{11} &= (s_x^2 - 1)(1 - c_\theta) + 1; & a_{12} &= s_x s_y(1 - c_\theta) - s_z s_\theta; & a_{13} &= s_x s_z(1 - c_\theta) + s_y s_\theta; \\ a_{21} &= s_y s_x(1 - c_\theta) - s_z s_\theta; & a_{22} &= (s_y^2 - 1)(1 - c_\theta) + 1; & a_{23} &= s_y s_z(1 - c_\theta) - s_x s_\theta; \\ a_{31} &= s_z s_x(1 - c_\theta) - s_y s_\theta; & a_{32} &= s_z s_y(1 - c_\theta) + s_x s_\theta; & a_{33} &= (s_z^2 - 1)(1 - c_\theta) + 1. \end{aligned} \tag{4}$$

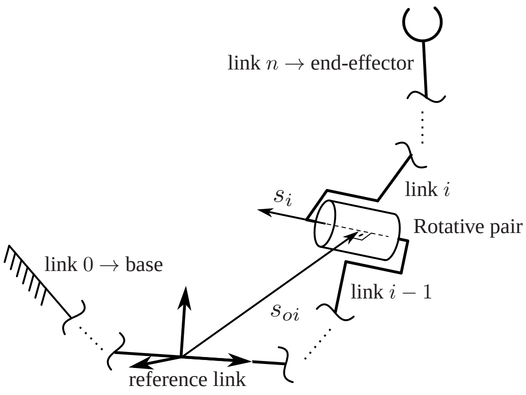

Figure 1. Location of the vectors s_i and s_{oi} for a rotative kinematic pair

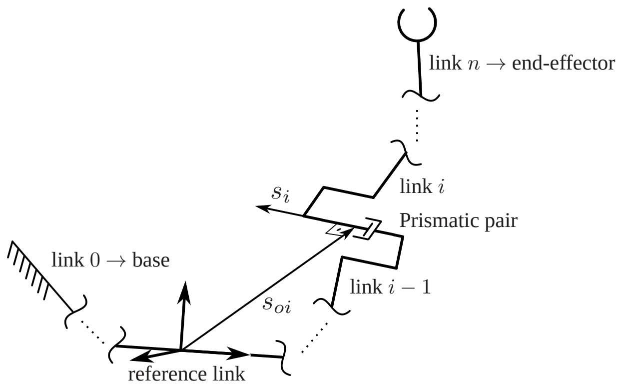

Figure 2. Location of the vectors s_i and s_{oi} for a prismatic kinematic pair

The vector s_i and s_{io} are function of θ_i (for rotative kinematic pair) and t_i (for prismatic kinematic pair) associated with the i kinematic pair. Then the s_i and s_{oi} are computed by the relation presented on Eq.(5) and Eq.(6) .

$$s_i^r = R_i^r \boldsymbol{s_i} \tag{5}$$

where, s_i^r is the vector s_i as function of the kinematic displacements between the link i and the reference link r, $\boldsymbol{s_i}$ is the coordinates of the vetor s_i in the initial kinematic posture and R_i^r, extracted from homogeneous transformation on Eq.(3), is the rotation matrix of the projections of the axis of the frame of the link i on the coordinates of the reference frame, on the link r.

$$\begin{bmatrix} s_{oi}^r \\ \cdots \\ 1 \end{bmatrix} = A_i^r \begin{bmatrix} \boldsymbol{s_{oi}} \\ \cdots \\ 1 \end{bmatrix} \tag{6}$$

where, s_{oi}^r is the vector s_{oi} as function of the kinematic displacements between the link i and the reference link r; $\boldsymbol{s_{oi}}$ is the coordinates of the vetor s_{oi} in the initial kinematic posture and A_i^r is the homogeneuos transformation matrix between the frame of the link i on the coordinates of the reference frame, on the link r computed using Eq.(3).

The choice of a link as reference (link r) aims to simplify the final expressions for the screw representations. In general, it is necessary to transform the coordinates of a screw represented in the link r to a new reference on link j. In this case, it used the coordinate screw transformation matrix T_r^j that has its structure presented in Eq.(7) (Tsai, 1999)

$$T_r^j = \left[\begin{array}{c:c} R_r^j & \mathbf{0}_{3\times 3} \\ \hdashline W_r^j & R_r^j \end{array}\right] \tag{7}$$

where R_r^j is the rotation matrix of the reference frame on link r in relation to the frame on link j; W_r^j is a 3×3 skew-symmetric matrix representing the vector from the origin O_j of the frame j to the origin O_r, on frame r, expressed in the jth frame.

More details of the screw theory and its applications can be found in works composed chronologically of Davies (1981),Tsai (1999) and Hunt (2000).

2.2 Davies method

Davies method is a systematic way to relate the joint velocities in closed kinematic chains. Davies derived a solution to the differential kinematics of closed kinematic chains from Kirchhoff circulation law for electrical circuits. The resulting Kirchhoff-Davies circulation law states that "The algebraic sum of relative velocities of kinematic pairs along any closed kinematic chain is zero" (Campos *et al.*, 2009). This method is used to obtain the relationship between the velocities of a closed kinematic chain. Since the velocity of a link with respect to itself is null, the circulation law can be expressed as:

$$\sum_{0}^{n} \hat{\$}_i \dot{q}_i = 0 \tag{8}$$

where $\hat{\$}_i$ (expressed on the coordinates of the frame reference link r), $\dot{q}_i$ represents respectively the normalized screw and the magnitude of twist $\$_i$ and n is the number of joints.

Equation (8) is the constraint equation which, in general can be written as

$$N\dot{q} = 0 \tag{9}$$

where $N = [\hat{\$}_1 \quad \hat{\$}_2 \quad \cdots \quad \hat{\$}_n]$ is the network matrix containing the normalized screws, with the signs of the screws depend on the definition of the circuit orientation (as will be presented later) (Campos *et al.*, 2009), and $\dot{q} = [\dot{q}_1 \quad \dot{q}_2 \quad \cdots \quad \dot{q}_n]$ is the magnitude vector of the velocities of each joint.

A closed kinematic chain has actuated joints, here assigned as primary joints, and passive joints, assigned as secondary joints. The constraint equation, Eq.(9), allows the computation of the secondary joint velocities as functions of the primary joint velocities. To achieve this, the constraint equation is rearranged highlighting the primary and secondary joint velocities and Eq.(9) is rewritten as follows:

$$\begin{bmatrix} N_p & \vdots & N_s \end{bmatrix} \begin{bmatrix} \dot{q}_p \\ \cdots \\ \dot{q}_s \end{bmatrix} = 0 \tag{10}$$

where N_p and N_s are the primary and secondary network matrices, respectively, and $\dot{q}_p$ and $\dot{q}_p$ are the corresponding primary and secondary magnitude vectors, respectively.

So, Eq.(10) can be rewritten as

$$N_p\dot{q}_p + N_s\dot{q}_s = 0 \tag{11}$$

The secondary joint velocities can be computed by Eq.(12) as follows:

$$\dot{q}_s = -N_s^{-1}N_p\dot{q}_p \tag{12}$$

The secondary joint position can be computed by numerical method, as a screw-based integration method proposed by (Simas *et al.*, 2009)

2.3 Assur virtual chains

The concept of Assur virtual kinematic chain, or just virtual chain, is essentially a tool to get information on the movement of a kinematic chain or to impose movements on a kinematic chain (Campos *et al.*, 2009).

This concept was first introduced by (Campos *et al.*, 2009), which defines the virtual chain as a kinematic chain composed of links (virtual links) and joints (virtual joints) which possesses three properties: a) the virtual chain is open; b) it has joints whose normalized screws are linearly independent; c) it does not change the mobility of the real kinematic chain.

From the third property, the virtual chain proposed by (Campos *et al.*, 2009) is in fact an Assur group, i.e. a kinematic subchain with null mobility such that, when connected to another kinematic chain preserves its mobility (Campos *et al.*, 2009).

2.4 Direct graph notation

Consider a kinematic pair composed of two links E_i and E_{i+1}. This kinematic pair has its relative velocity defined by a screw ${}^R\$_j$ (joint j) relative to a reference frame R. Joint j represents the relative movement of the link E_i with respect to the link E_{i+1}. This relation can be represented by a graph (Campos *et al.*, 2009), where the vertices represent links and the arcs represent joints.

Now, studying a simple graph, where joint j is part of two closed chains. For each closed chain the circuit direction is chosen (Campos *et al.*, 2009). In a direct mechanism graph, if the joint has the same direction as the circuit, the twist associated with the joint has a positive sign in the circuit equation (constraint equation on Eq.(8)), and a negative sign if the joint has the opposite direction to the circuit.

2.5 Extended Jacobian from kinematic constraints

The method of extended Jacobian proposes a solution to solve the redundancy of robots creating kinematic constraints in differential space. These constraints when added to the Jacobian matrix, produce a non-redundant kinematic system and making the Jacobian matrix invertible.

A method to compute additional constraints has been proposed by Simas *et al.* (2011) based on reciprocal screws (Dai and Jones, 2001)(Nokleby and Podhorodeski, 2001). The extended Jacobian based on reciprocal screws arises from the fact that N_s matrix, must be inverted as can be seen on Eq (12), contains screws from virtual kinematic pairs. So, to simplify the inversion of the matrix N_s, it is necessary to eliminate its screws of the virtual chains.

The elimination of secondary virtual screws can be performed through reciprocal screws. The reciprocal screws are arranged in a matrix defined as annihilating matrix (Campos *et al.*, 2009).

To eliminate these screws (columns) from secondary matrix (Eq.(11)), another partition is performed, as follows in Eq.(13).

$$N_s \dot{q}_s = N_{sa} \dot{q}_{sa} + N_{sp} \dot{q}_{sp} \tag{13}$$

where N_{sa} corresponds to the screws of the joints of interest (here called active) and N_{sp} corresponds to the screws of the joints which there is no interest (here called passive).

The passive joints are eliminated using an annihilate matrix $\mathcal{K}$ which has the following structure on Eq.(14) (Campos *et al.*, 2009).

$$\mathcal{K} = \left[\begin{array}{c|c} I_{m \times m} & 0 \\ \hline 0 & {}^{ref}W_{N_{sp}(n-m) \times d} \end{array} \right] \tag{14}$$

where ${}^{ref}W_{N_{sp}}$, whose dimension is $(n - m) \times d$, is a set of reciprocal screws from secondary passive matrix N_{sp}. (Campos *et al.*, 2009)(Martins, 2002).

The reciprocal screws represent a set of external forces and torques that do not generate movements on secondary passive joints. Therefore pre-multiplying N_{sp} by $\mathcal{K}$, produces:

$$\mathcal{K} N_{sp} = 0 \tag{15}$$

To maintain equality, it is necessary that the Eq.(11) is rewritten, considering the Eq.(13), as follows in Eq.(16).

$$\mathcal{K} N_p \dot{q}_p + \mathcal{K} N_{sa} \dot{q}_{sa} + \mathcal{K} N_{sp} \dot{q}_{sp} = 0 \tag{16}$$

Using equality in Eq.(15) the following result is obtained in Eq.(17).

$$\mathcal{K} N_p \dot{q}_p + \mathcal{K} N_{sa} \dot{q}_{sa} = 0 \tag{17}$$

The velocities of the secondary joints are then obtained by Eq.(18).

$$\dot{q}_{sa} = -(\mathcal{K} N_{sa})^{-1} \mathcal{K} N_p \dot{q}_p \tag{18}$$

So using the usual definition of the Jacobian, the following result is obtained in Eq.(19).

$$J = -(\mathcal{K} N_p)^{-1} \mathcal{K} N_{sa} \tag{19}$$

The Jacobian expressed by the Eq.(19) is a desired extended Jacobian matrix (Simas *et al.*, 2011) (Campos *et al.*, 2009).

In order to evaluate the singularities postures on the whole kinematic chain, including real and virtual kinematic pairs on extended Jacobian, it is necessary to compute the determinant of $\mathcal{K} N_{sa}$. The next step is to invert the Jacobian matrix as shown in Eq.(18) with objetive to compute the velocities on secondary kinematic pairs ($\dot{q}_{sa}$), and its respectively position.

3. MODELING OF A $P6R$ REDUNDANT ROBOT OPERATING IN A CONFINED ENVIRONMENT

This section presents the $P6R$ model, including the virtual chains to generate the trajectories, as primary task, and to the collision avoidance, considered as secondary task. The resultant extended Jacobian is used to obtain an analytical formula that expresses mathematically, and allows the control of the postures, that the $P6R$ robot assumes their singular conditions, together with the collision avoidance strategy.

Figure 3 depicts the $P6R$ redundant robot including a collision plane inside its workspace.

The $P6R$ robot presented in Fig. 3 is composed of seven joints, where the first joint is a prismatic with displacement L_1, and the next six rotative joints, with displacements θ_2, θ_3, θ_4, θ_5, θ_6 and θ_7, respectively described through screws $\$_i$ (for $i = 1, \ldots, 7$) pointing its directions. The robot has three links, enumerated by 2, with length a_2 , 3, with length a_3 and 4, with length a_4 and d_4, and has its end-effector attached in a spherical wrist (screws $\$_5$, $\$_6$ and $\$_7$). The collision

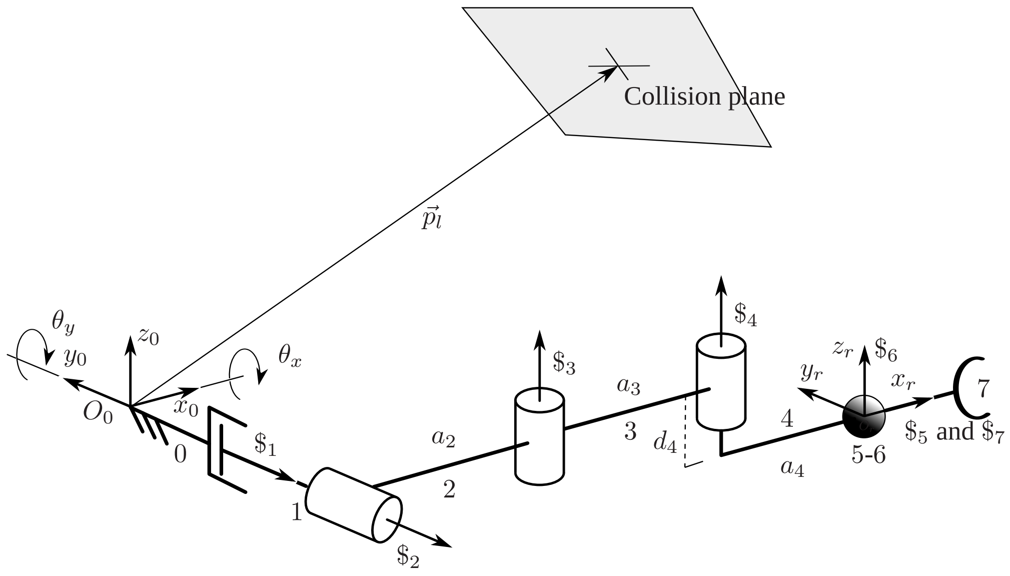

Figure 3. $P6R$ redundant robot.

Table 1. Coordinates of $\boldsymbol{s_i}$ and $\boldsymbol{s_{oi}}$ for $P6R$ robot

Joint i	$\boldsymbol{s_i}$	$\boldsymbol{s_{oi}}$	θ_i	t_i
1	$0, -1, 0$	$-(a_2 + a_3 + a_4), 0, d_4$	0	L_1
2	$0, -1, 0$	$-(a_2 + a_3 + a_4), 0, d_4$	θ_2	0
3	$0, 0, 1$	$-(a_3 + a_4), 0, d_4$	θ_3	0
4	$0, 0, 1$	$-a_4, 0, d_4$	θ_4	0
5	$1, 0, 0$	$0, 0, 0$	θ_5	0
6	$0, 0, 1$	$0, 0, 0$	θ_6	0
7	$1, 0, 0$	$0, 0, 0$	θ_7	0

plane is defined through a point with coordinates of the vector $\vec{p_l}$ and orientation defined by the angles θ_x and θ_y measured in relation to the inertial frame $O_0 - x_0y_0z_0$.

Considering the link 4 as reference link, and that its frame $O_r - x_ry_rz_r$ (or $O_4 - x_4y_4z_4$) located in the center of the spherical wrist, it can be obtained the coordinates of $\boldsymbol{s_i}$ and $\boldsymbol{s_{oi}}$, as function of the joint variables, as presented on the Tab. 1.

Computing the screws for $P6R$ using Eq.(6) and the initial posture from Tab. 1, the matrix of normalized screws are obtained as: $J_{P6R} = [\hat{\$}_1, \hat{\$}_2, \hat{\$}_3, \hat{\$}_4, \hat{\$}_5, \hat{\$}_6, \hat{\$}_7]$ and given by the Eq.(20)

$$J_{P6R} = \begin{bmatrix} 0 & -s_{34} & 0 & 0 & 1 & 0 & c_6 \\ 0 & -c_{34} & 0 & 0 & 0 & -s_5 & c_5s_6 \\ 0 & 0 & 1 & 1 & 0 & c_5 & s_5s_6 \\ -s_{34} & d_4c_{34} & a_3s_4 & 0 & 0 & 0 & 0 \\ -c_{34} & -d_4s_{34} & a_4 + a_3c_4 & a_4 & 0 & 0 & 0 \\ 0 & a_2 + a_3c_3 + a_4c_{34} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{20}$$

where J_{P6R} is the matrix of the normalized screw described in function of the joint displacements and referenced on coordinates of the frame of the link 4, the reference frame, $s_3 = sin(\theta_3)$, $c_3 = cos(\theta_3)$, $s_4 = sin(\theta_4)$, $c_4 = cos(\theta_4)$, $s_5 = sin(\theta_5)$, $c_5 = cos(\theta_5)$, $s_6 = sin(\theta_6)$, $c_6 = cos(\theta_6)$, $s_{34} = sin(\theta_3 + \theta_4)$ and $c_{34} = cos(\theta_3 + \theta_4)$.

The model has as inertial reference the $P6R$ base or the frame $O_0 - x_0y_0z_0$ (see Fig. 3), so the respective screw transformation matrix necessary to represent the velocities of the end-effector on the reference frame ($P6R$ base) is given in Eq.(21)

$$T_{P6R} = \begin{bmatrix} c_2c_{34} & -c_2s_{34} & -s_2 & 0 & 0 & 0 \\ s_{34} & c_{34} & 0 & 0 & 0 & 0 \\ s_2c_{34} & -s_2s_{34} & c_2 & 0 & 0 & 0 \\ d_4c_2s_{34} - s_2k_1 & d_4c_2c_{34} - s_2k_2 & c_2(a_3s_3 + a_4s_{34} - L_1) & c_2c_{34} & -c_2s_{34} & -s_2 \\ -d_4c_{34} & d_4s_{34} & -a_2 - a_3c_3 - a_4c_{34} & s_{34} & c_{34} & 0 \\ d_4s_2c_{34} + c_2k_1 & d_4s_2c_{34} + c_2k_2 & s_2(a_3s_3 + a_4s_{34} - L_1) & s_2c_{34} & -s_2s_{34} & c_2 \end{bmatrix} \quad (21)$$

where T_{P6R} is the screw transformation matrix responsible for transforming the coordinates of the $P6R$ screw expressed on the frame r on the coordinates of the frame 0, $k_1 = L_1c_{34} + a_2s_{34} + a_3s_4$ and $k_2 = -L_1s_{34} + a_2c_{34} + a_3c_4 + a_4$

The next step is to define the screws of the collision avoidance virtual chain. The most common virtual chain used to collision avoidance tasks is the $3P3R$ virtual chain (Campos *et al.*, 2009)(Simas *et al.*, 2009)(Simas *et al.*, 2011).

The $3P3R$ virtual chain is composed by three prismatic joints, perpendicular to each other, and three rotative joints composing a spherical wrist. The first two prismatic joints, x_c and y_c, are defined as tangents in relation to the collision plane and have displacements p_{x_c} and p_{y_c} respectively, as consequence the third prismatic joint z_c is normal to the collision plane and has displacements p_{z_c}. The last three rotative joint are definied as rx_c, ry_c and rz_c pointed on the same directions of x_c, y_c and z_c respectively, with displacements θ_{l_x}, θ_{l_y} and θ_{l_z}. The base of the $3P3R$ virtual chain is located on the coordinates of the $\vec{p}_l = [p_{x_l}, p_{y_l}, p_{z_l}]$ expressed on inertial frame $O_0 - x_0y_0z_0$. The last link (end-effector) of the $3P3R$ collision virtual chain is attached along the link 3 in the intersection with the line of the screw $\$_4$ (see Fig. 3).

In order to simplify the model and obtain tractable results, some considerations were adopted to compute screws of the $3P3R$ virtual kinematic chains, as follows:

- The screws of the $3P3R$ virtual chains are obtained using the $P6R$ base as reference, or the inertial frame $O_0 - x_0y_0z_0$, and will not be required compute the screw transformation matrix;
- The $3P3R$ is located on the collision plane such that the displacements $p_{x_c} = 0$ and $p_{y_c} = 0$;
- The collision plane is oriented in function of two angles θ_x and θ_y (see Fig. 3);
- are obtained using the $P6R$ base as reference.

Considering the simplifications presented above, the screws are obtained for the $3P3R$ virtual chains and disposed in a matrix $J_{pla} = [\hat{\$}_{x_c}, \hat{\$}_{y_c}, \hat{\$}_{z_c}, \hat{\$}_{rx_c}, \hat{\$}_{ry_c}, \hat{\$}_{rz_c}]$

$$J_{pla} = \begin{bmatrix} 0 & 0 & 0 & c_{l_y} & 0 & s_{l_y} \\ 0 & 0 & 0 & s_{l_x}s_{l_y} & c_{l_x} & s_{l_x}c_{l_y} \\ 0 & 0 & 0 & -c_{l_x}s_{l_y} & s_{l_x} & -c_{l_x}c_{l_y} \\ c_{l_y} & 0 & s_{l_y} & -s_{l_y}(p_{y_l}c_{l_x} + p_{z_l}s_{l_x}) & -p_{z_l}c_{l_y} - p_{c_z}c_{l_y} + p_{y_l}s_{l_x} & c_{l_y}(p_{y_l}c_{l_x} + p_{z_l}s_{l_x}) \\ s_{l_x}s_{l_y} & c_{l_x} & -s_{l_x}c_{l_y} & p_{z_l}c_{l_y} + c_{l_x}(p_{c_z} + p_{x_l}s_{l_y}) & -s_{l_x}(p_{x_l} + p_{c_z}s_{l_y}) & -p_{x_l}c_{l_x}c_{l_y} + p_{z_l}s_{l_y} \\ -c_{l_x}s_{l_y} & s_{l_x} & c_{l_x}c_{l_y} & -p_{y_l}c_{l_y} + s_{l_x}(p_{c_z} + p_{x_l}s_{l_y}) & c_{l_x}(p_{x_l} + p_{c_z}s_{l_y}) & -p_{x_l}s_{l_x}c_{l_y} - p_{y_l}s_{l_y} \end{bmatrix} \quad (22)$$

where $s_{l_x} = sin(\theta_{l_x})$, $c_{l_x} = cos(\theta_{l_x})$, $s_{l_y} = sin(\theta_{l_y})$, $c_{l_y} = cos(\theta_{l_y})$, $s_{l_z} = sin(\theta_{l_z})$, $c_{l_z} = cos(\theta_{l_z})$ and $\vec{p}_l$ can be expressed geometrically in function of the $P6R$ parameters, as following expressions obtained through geometric inspection:

- $p_{x_l} = c_2(a_2 + a_3c_3) - p_{c_z}s_{l_y}$;
- $p_{y_l} = -L_1 + a_3s_3 + p_{c_z}s_{l_x}c_{l_y}$;
- $p_{z_l} = s_2(a_2 + a_3c_3) - p_{c_z}c_{l_x}c_{l_y}$.

With the purpose to generate the trajectory for the $P6R$ redundant robot, another $3P3R$ virtual chain is used. The trajectory virtual chain has as base the inertial frame $O_0 - x_0y_0z_0$ and its end-effector attached on the end-effector of the $P6R$ redundant robot. The three prismatic joint are defined as x_d, y_d and z_d and their screws are pointed on direction of x_0, y_0 and z_0 with displacements p_{d_x}, p_{d_y} and p_{d_z}, respectively. The three rotative joints are defined as rx_d, ry_d and rz_d and their screws are pointed, also, on direction of x_0, y_0 and z_0 with displacements θ_{d_x}, θ_{d_y} and

θ_{d_z} respectively. Equation (23) presents the normalized screws of the trajectory virtual chain disposed in the matrix $J_{traj} = [\hat{\$}_{x_d}, \hat{\$}_{y_d}, \hat{\$}_{z_d}, \hat{\$}_{rx_d}, \hat{\$}_{ry_d}, \hat{\$}_{rz_d}]$

$$J_{traj} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & s_{d_y} \\ 0 & 0 & 0 & 0 & c_{d_x} & -s_{d_x}c_{d_y} \\ 0 & 0 & 0 & 0 & s_{d_x} & c_{d_x}c_{d_y} \\ 1 & 0 & 0 & 0 & p_{d_y}s_{d_x} - p_{d_z}c_{d_x} & c_{d_y}(p_{d_y}c_{d_x} + p_{d_z}s_{d_x}) \\ 0 & 1 & 0 & p_{d_z} & -p_{d_x}s_{d_x} & -p_{d_x}c_{d_x}c_{d_y} - p_{d_z}s_{d_y} \\ 0 & 0 & 1 & -p_{d_y} & p_{d_x}c_{d_x} & -p_{d_x}s_{d_x}c_{d_y} - p_{d_y}s_{d_y} \end{bmatrix} \quad (23)$$

where $s_{d_x} = sin(\theta_{d_x})$, $c_{d_x} = cos(\theta_{d_x})$, $s_{d_y} = sin(\theta_{d_y})$, $c_{d_y} = cos(\theta_{d_y})$, $s_{d_z} = sin(\theta_{d_z})$, $c_{d_z} = cos(\theta_{d_z})$

Since all normalized screw, of all joints, are represented in the inertial reference frame and the constraint equation (Eq.(9)) can be constructed, and presented in Eq.(24), considering that the complete kinematic have two circuits, in agree with Davies method and graph notation (Davies, 1981)(Campos *et al.*, 2009)(Simas *et al.*, 2009).

$$N\dot{q} = \begin{bmatrix} \hat{\$}_1 & \hat{\$}_2 & \hat{\$}_3 & \hat{\$}_4 & \hat{\$}_5 & \hat{\$}_6 & \hat{\$}_7 & 0 & 0 & 0 & 0 & 0 & 0 & -\hat{\$}_{x_d} & -\hat{\$}_{y_d} & -\hat{\$}_{z_d} & -\hat{\$}_{rx_d} & -\hat{\$}_{ry_d} & -\hat{\$}_{rz_d} \\ \hat{\$}_1 & \hat{\$}_2 & \hat{\$}_3 & 0 & 0 & 0 & 0 & -\hat{\$}_{x_c} & -\hat{\$}_{y_c} & -\hat{\$}_{z_c} & -\hat{\$}_{rx_c} & -\hat{\$}_{ry_c} & -\hat{\$}_{rz_c} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \dot{q} \quad (24)$$

where $\dot{q} = [\dot{L}_1, \dot{\theta}_2, \dot{\theta}_3, \dot{\theta}_4, \dot{\theta}_5, \dot{\theta}_6, \dot{\theta}_7, \dot{p}_{x_c}, \dot{p}_{y_c}, \dot{p}_{z_c}, \dot{r}x_c, \dot{r}y_c, \dot{r}z_c, \dot{p}_{x_d}, \dot{p}_{y_d}, \dot{p}_{z_d}, \dot{r}x_d, \dot{r}y_d, \dot{r}z_d]^T$.

The collision avoidance is accomplished through activation of the z_c joint of the collision avoidance virtual chain, because of that, the joint z_c is primary, together with all six joint of the trajectory virtual chain (Simas *et al.*, 2011). The definition of the primary joints, determines a first partition on matrix N in N_p, with normalized screws of the primary joints, and N_s, with normalized screws of the secondary joints as shown in Eq.(25) and Eq.(26).

$$N_p\dot{q}_p = \begin{bmatrix} 0 & -\hat{\$}_{x_d} & -\hat{\$}_{y_d} & -\hat{\$}_{z_d} & -\hat{\$}_{rx_d} & -\hat{\$}_{ry_d} & -\hat{\$}_{rz_d} \\ -\hat{\$}_{rz_c} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \dot{q}_p \quad (25)$$

where $\dot{q}_p = [\dot{r}z_c, \dot{p}_{x_d}, \dot{p}_{y_d}, \dot{p}_{z_d}, \dot{r}x_d, \dot{r}y_d, \dot{r}z_d]^T$.

$$N_s\dot{q}_s = \begin{bmatrix} \hat{\$}_1 & \hat{\$}_2 & \hat{\$}_3 & \hat{\$}_4 & \hat{\$}_5 & \hat{\$}_6 & \hat{\$}_7 & 0 & 0 & 0 & 0 & 0 \\ \hat{\$}_1 & \hat{\$}_2 & \hat{\$}_3 & 0 & 0 & 0 & 0 & -\hat{\$}_{x_c} & -\hat{\$}_{y_c} & -\hat{\$}_{z_c} & -\hat{\$}_{rx_c} & -\hat{\$}_{ry_c} \end{bmatrix} \dot{q}_s \quad (26)$$

where $\dot{q}_s = [\dot{L}_1, \dot{\theta}_2, \dot{\theta}_3, \dot{\theta}_4, \dot{\theta}_5, \dot{\theta}_6, \dot{\theta}_7, \dot{p}_{x_c}, \dot{p}_{y_c}, \dot{p}_{z_c}, \dot{r}x_c, \dot{r}y_c]^T$.

In the Eq.(26), N_s has screws of active and passive joints. In according to Eq.(13) a second partition can be performed on the N_s as presented in Eq.(27)

$$N_s = \left[\begin{array}{ccccccc|ccccc} \hat{\$}_1 & \hat{\$}_2 & \hat{\$}_3 & \hat{\$}_4 & \hat{\$}_5 & \hat{\$}_6 & \hat{\$}_7 & 0 & 0 & 0 & 0 & 0 \\ \hat{\$}_1 & \hat{\$}_2 & \hat{\$}_3 & 0 & 0 & 0 & 0 & -\hat{\$}_{x_c} & -\hat{\$}_{y_c} & -\hat{\$}_{z_c} & -\hat{\$}_{rx_c} & -\hat{\$}_{ry_c} \end{array}\right] = [\; N_{sa_{(12\times7)}} \mid N_{sp_{(12\times5)}} \;] \quad (27)$$

Computing the matrix ${}^{ref}W_{N_{sp}(n-m)\times d}$, with $ref = 4$, $n = 6$, $m = 5$ and $d = 6$, that corresponds to the null space of N_{sp}, we have the Eq.(28).

$${}^{4}W_{N_{sp}(1\times6)} = [\; w_1 \quad w_2 \quad w_3 \quad w_4 \quad w_5 \quad w_6 \;]_{1\times6} \quad (28)$$

where:

- $w_1 = -L_1 + a_3s_3 + (a_2 + a_3c_3)s_2\frac{s_{lx}}{c_{lx}}$;
- $w_2 = -(a_2 + a_3c_3)(c_2 - \frac{s_2}{c_{lx}}\frac{s_{ly}}{c_{ly}})$
- $w_3 = -c_2(a_2 + a_3c_3)\frac{s_{lx}}{c_{lx}} + \frac{1}{c_{lx}}(L_1 - a_3s_3)\frac{s_{ly}}{c_{ly}}$;
- $w_4 = \frac{1}{c_{lx}}\frac{s_{ly}}{c_{ly}}$;
- $w_5 = \frac{s_{lx}}{c_{lx}}$;
- $w_6 = 1$

The annihilating matrix has the following structure in Eq.(29)

$$\mathcal{K} = \left[\begin{array}{cccccc|cccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & w_1 & w_2 & w_3 & w_4 & w_5 & w_6 \end{array}\right] \tag{29}$$

Using the annihilating matrix $\mathcal{K}$, the matrix $\mathcal{K}N_{sp}$ will be a null matrix with dimension 7×5, and the matrix $\mathcal{K}N_{sa}$ has as result the Eq.(30)

$$\mathcal{K}N_{sa} = \begin{bmatrix} \hat{\$}_1 & \hat{\$}_2 & \hat{\$}_3 & \hat{\$}_4 & \hat{\$}_5 & \hat{\$}_6 & \hat{\$}_7 \\ \frac{s_{l_x}}{c_{l_x}} & (a_2 + a_3 c_3)\left(c_2 - \frac{s_2}{c_{l_x} c_{l_y}}\right) & -a_3\left(\frac{c_3 s_{l_x}}{c_{l_x}}\left(s_2 + \frac{c_2 c_{l_y}}{c_{l_x} c_{l_y}}\right)\right) & 0 & 0 & 0 & 0 \end{bmatrix}_{7\times 7} \tag{30}$$

where the respective $\dot{q}_{sa} = [\dot{L}_1, \dot{\theta}_2, \dot{\theta}_3, \dot{\theta}_4, \dot{\theta}_5, \dot{\theta}_6, \dot{\theta}_7]$

It is important to emphasize that the screws presented on Eq.(30) are represented on the inertial frame $O_0 - x_0y_0z_0$. In following, it is analized the singularities of the kinematic system composed by $P6R$ redundant robot, $3P3R$ collision avoidance virtual chain and $3P3R$ trajectory virtual chain.

4. ANALYSIS OF SINGULARITIES

Equation (18) shows how to compute the velocities of the active secondary joints. The computation is feasible if the matrix $\mathcal{K}N_{sa}$ can be inverted. The determinant of $\mathcal{K}N_{sa}$ may be used to evaluate the conditions under which this matrix is not invertible, and respectively, the singular postures that the kinematic system can assume. So, computing the determinant of $\mathcal{K}N_{sa}$, defined as $D_{\mathcal{K}N_{sa}}$ we have the Eq.(31).

$$D_{\mathcal{K}N_{sa}} = -a_3 a_4 (a_2 + a_3 c_3 + a_4 c_{34}) s_3 s_6 (s_{l_y} c_2 s_{34} + s_{l_x} c_{l_y} c_{34} + c_{l_x} c_{l_y} s_2 s_{34}) \tag{31}$$

Analyzing the expression of the determinant in Eq.(31), the following singularities can be observed:

1. $s_3 = 0 \rightarrow$ Corresponds to the alignment of links 2 e 3, when $\theta_3 = k\pi$, for $k = 0, 1, \cdots$

2. $s_6 = 0 \rightarrow$ Corresponds to the alignment of joint 6 and 7, when $\theta_6 = k\pi$, for $k = 0, 1, \cdots$ - note in Fig. 3, that in the initial posture of the $P6R$, the screw axes $\$_5$ and $\$_7$ are parallel.

3. $(a_2 + a_3 c_3 + a_4 c_{34}) = 0 \rightarrow$ According with the structure of the robot, it can be observed that the links 2 and 3, with dimensions a_2 and a_3 respectively, are contained in a plane, defined as p_a (when $\theta_3 \neq 0$). A second plane p_b parallel to p_a can be defined, distanced d_4 from the plane p_a and containing the link 4 along to the its length a_4. Making a geometric analisys, the singular posture of this expression shows that the distance between the center of the spherical wrist and the line along of the axis of the joint 2 ($\$_2$ axis, see Fig. 3) can not be equal to d_4. If this distance is d_4, the robot has restricted their movements imposed in the normal direction of the plane p_b. Using another form of interpretation, it can be say that, in this singular posture, the velocities imposed by screws directed to the normal of p_b will be reciprocal to the screw axis of the joint 2. Figure 4 shown the $P6R$ posture corresponding to the singular posture described.

4. $(s_{l_y} c_2 s_{34} + s_{l_x} c_{l_y} c_{34} + c_{l_x} c_{l_y} s_2 s_{34}) = 0 \rightarrow$ This expression can be rewritten as a dot product of the vectors $\vec{u}$ and $\vec{v}$ as show the Eq.(32)

$$\begin{bmatrix} s_{l_y} \\ -s_{l_x} c_{l_y} \\ c_{l_x} c_{l_y} \end{bmatrix} \cdot \begin{bmatrix} c_2 s_{34} \\ -c_{34} \\ s_2 c_{34} \end{bmatrix} = \vec{u} \cdot \vec{v} = (s_{l_y} c_2 s_{34} + s_{l_x} c_{l_y} c_{34} + c_{l_x} c_{l_y} s_2 s_{34}) \tag{32}$$

Studying vectors $\vec{u}$ and $\vec{v}$, it can be identified that these vectors correspond to direction of the joint z_c, on $3P3R$ collision avoidance virtual chain and the vector y_4 of the frame of the link 4, respectively. In this sense, the singularity occurs when these vectors are perpendicular. Figure 5 presents the vectors z_c and y_4, and the angle ϑ, which should be different from $\pi/2$.

The analysis developed here, shown the potential of using virtual kinematic chains in obtaining an extended Jacobian for spatial robots. The extended Jacobian allowed to evaluate the singularities belonging to the robot kinematic structure only, without the appearance of algorithmic singularities, as in the classical methods.

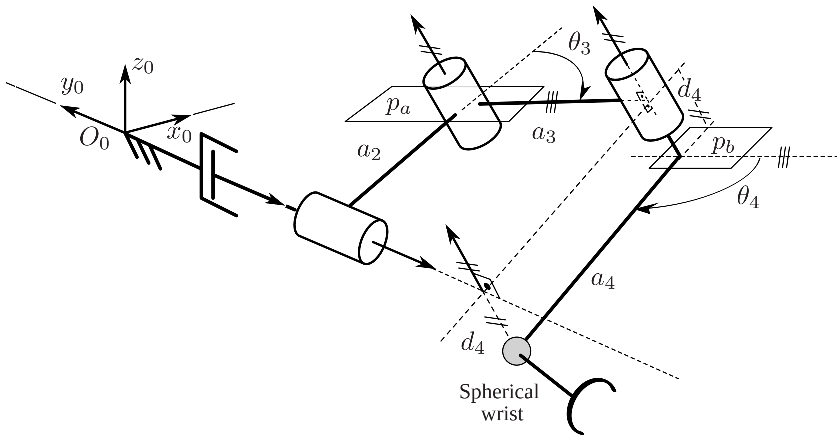

Figure 4. Singular posture for $P6R$ redundant robot corresponding to expression: $(a_2 + a_3c_3 + a_4c_{34}) = 0$

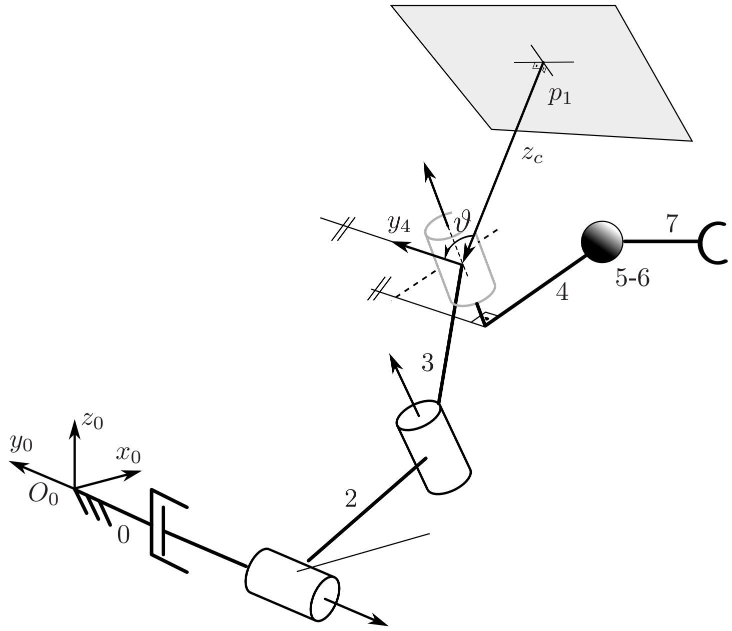

Figure 5. Singular condition for $P6R$ redundant robot from collision avoidance virtual chain.

5. CONCLUSION

Many manipulators has been designed to operate in confined environments. As shown in these paper, operations in confined environments require methods to evaluate and to prevent that any collision may come along during the task performing.

Therefore, the Simas approach solution may be apply to avoid collisions, studying a $P6R$ redundant robot operating in confined enviroments (Simas *et al.*, 2009). It is based on screw theory and in the use of virtual chains to determine its trajectories and to avoid collision. However, the paper do not present a study of the motion limits and the singular postures of the $P6R$ including the collision avoidance virtual chain. Another work, (Simas *et al.*, 2011) presents an analitycal study of singularities for a planar redundant robot, based on reciprocal screws using virtual chains and annihilate matrix.

This paper presented a study of singularities for a $P6R$ redundant robot operating in a confined environment. Considering the possibility of collisions, Assur virtual chains were used in order to reposition the $P6R$ robot away of some imminent collision with any part inside its workspace.

The use Assur virtual chains for collision avoidance has became complex the differential kinematic model, obtained by the Davies method. A simplified differential kinematic model was then obtained using the concept of annihilating matrix, resulting in an extended Jacobian, similar to others proposed in references, but without algorithmic singularities that do not belong to kinematic chain.

Partial results were presented and a final expression for the determinant of the differential kinematics, $D_{\mathcal{K}N_{sa}}$, identified all singular postures that the kinematic system, formed by $P6R$ redundant robot and virtual chains for trajectory generation and collision avoidance, can assume.

Results showed the viability of using the annihilating matrix as a way to simplify the differential kinematic modeling for spatial kinematic systems. Furthermore, the development showed that the obtained Jacobian is an extended Jacobian, invertible, whose the determinant represents only singularities belonging to the kinematic system.

Agreement with the second task developed as example in this paper, the result is useful in generation trajectory systems, since singular conditions should be monitored, and most importantly, includes the singular postures arising from collision avoidance strategy.

Future works can be developed aiming to minimize the need for geometric inspection, as shown in Eq.(22), and to further simplify the screws obtained, since they have its formulation dependent on the choice of a link reference.

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