ORIENTATION WORKSPACE OPTIMIZATION FOR A 6-RUS PARALLEL ROBOT

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Abstract: This work therefore provides a method for maximizing the workspace of the parallel robot 6 - RUS by optimizing the robot fixed joints position, evaluating the proximity of singularities and employing a hybrid approach , ie Genetic Algorithm based and derived , for optimization.
Keywords: parallel robot, optimization, hybrid optimization, PSO and Workspace

1. INTRODUCTION

According to Merlet, comparing parallel and serial robot configurations, it is observed that parallel ones are advantageousness in terms of accuracy, rigidity and ability to manipulate large loads. Therefore, parallel robots are more suitable for tasks that require such properties. However the small workspace compared to serial robots is a drawback that researchers try to minimize, optimizing the parallel robot parameters (Merlet, 2001).

The parallel robots performance is sensitive to the dimensions and geometry (Kelaiaia *et al.* (2012); Merlet (2006)), the same design may be optimized though varying these parameters. Among all kinematic measures, the workspace is one of the most important indices in design of a parallel robot (Merlet, 2001).

Zhang (2010) develops a stiffness optimization for a spatial 5-DOF parallel robot. This optmization, uses a genetic algorithm to escape from local minima. Barbosa, Pires and Lopes (Barbosa *et al.*, 2005) optimize the kinematic design a 6-dof parallel robot for maximum dexterity using a Genetic algorithm. Stan, Maties and Balan (Stan *et al.*, 2007) applied a Genetic algorithm to multicriteia optimization problem for 2-DOF micro parallel robot.

The parallel robot studied in this work is a flight simulator belonging to Udesc - Ceart in Florianopolis . This robot presents the second most common architecture for parallel robot used in flight simulator the 6 - RUS. This paper establishes a method for optimizing the robot leg positions angles (alphas) with the aim of maximizing the workspace i.e. the maximum platform rotation in a certain workspace point. To optimize this system is used a work-based index. This index measures the robot closeness to the singularities. This index is evaluated for different configurations, with respect to each motor positions on the fixed platform (base), optimizing the workspace with respect to maximum effector orientation.

A index (constraint), is based on the robot kinematics and it is used to optimize the motor joint position (optimization parameter) that maximize the orientation workspace (objective function) . The method used for optimization is a genetic Partial swam orientation (PSO) algorithm and FMINCON, the MATLAB optimization function. Initially , the genetic algorithm PSO optimizes the problem to escape from local minima and provides a set of optimized parameters in the optimum region. The FMINCON receives these optimized parameters and finds the best set that maximizes workspace. This work therefore provides a method for maximizing the workspace of the parallel robot 6 - RUS by optimizing the robot fixed joints position, evaluating the singularity proximity and employing a hybrid approach , i.e. combining genetic and gradient based algorithms.

2. PARALLEL ROBOTS

Parallel robots, also named parallel manipulators, typically consist by a moving platform connected to a fixed base by several limbs or legs (Merlet, 2001). See "Figure 1"and "Figure 2".

An n-DOF (n-degree-of-freedom) fully-parallel mechanism is composed of n independent legs connecting the mobile platform to the fixed platform. Each of these legs is a serial kinematic chain that hosts one or more motors which

actuates, directly or indirectly, the chain (Bonev, 2003). Due to distribution of external load, parallel robots present good performances in terms of accuracy, rigidity and ability to manipulate large loads (Merlet, 2001).

The 6-DoF parallel robot most studied architecture is the 6-UPS. This architecture is known as Stewart-Gough platform as presented in (Fichter, 1986). The Stewart-Gough platform presents a stiffness architecture due that the load distribution is only axial and allows the use of powerful hydraulic actuators. Motion simulators, generally, manipulate excessive loads of up to tens of tons (Bonev, 2003).

The second most common architecture is the 6-RUS kinematic chain, this chain architecture was proposed by Hunt early in 1983 (Merlet, 2001). In this architecture the actuated joint is rotational, which leads to the interchange possibility of universal and spherical joint without any change in mechanism characteristics (Bonev, 2003). This paper focus in a method to optimize the workspace the robot 6-RUS using a index of singularity closeness based on screw theory. (Ball (1900); Hunt (1983) e Campos (2001))
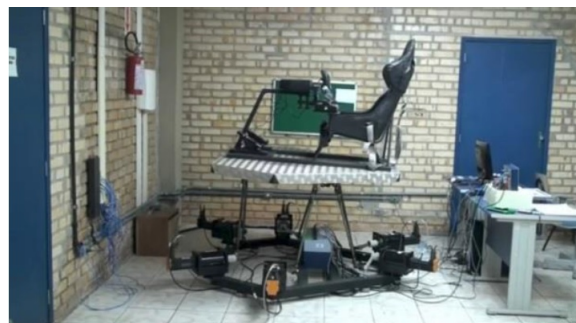

Figura 1: 6-RUS CAD Model.

Figura 2: Ceart 6-RUS Fly Simulator.

2.1 6-RUS INVERSE KINEMATIC

For inverse kinematic problem the vector describing moving platform position in cartesian coordinates system and orientation by roll, pitch and yaw angles respectively is given by $P = [P_x \ P_y \ P_z \ \varphi \ \vartheta \ \psi]$.

Few geometrical parameters are necessary to develop the inverse kinematic problem solution for 6-RUS parallel robot. Them can be obtained from the built prototype or the CAD model. See "Figure 3".

Required parameters are the fixed base radius r_b, moving platform radius r_p , half-distance between actuated joint pair d, half-distance between joints in moving platform e , crank dimension r_i which is located in active joint, rod dimension Rp , position angle of actuators pair χ_a, position angle of passive spherical joint in moving platform χ_j and the angle between the crank rotational plane and an axis parallel to y β (Campos *et al.*, 2013). These parameters are detailed in Figure 2.

Initially is needed to define the position of the actuators, which may be obtained by Eq. (1) that must be solved for all legs resulting a 6x3 vector:

$$\vec{A} = \vec{R} + \vec{m} \tag{1}$$

$$m = (-1)^{1-i}d \tag{2}$$

Being $i = 1, 2, 3, .., 6$ corresponding to each kinematic chain.

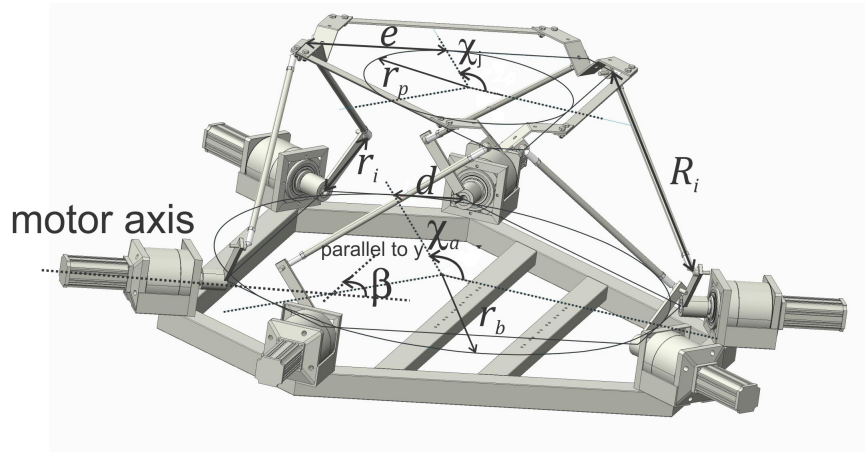$$\vec{R} = r_b \cos \chi_a \hat{i} + r_b \sin \chi_a \hat{j} \tag{3}$$

Figura 3: Parallel robot geometrical parameters (Campos *et al.*, 2013).

$$\vec{m} = -m \sin \chi_a \hat{i} + m \cos \chi_a \hat{j} + 0\hat{k} \tag{4}$$

$$\vec{A} = (r_b \cos \chi_a - m \sin \chi_a) \hat{i} + (r_b \sin \chi_a + m \cos \chi_a) \hat{j} + 0\hat{k} \tag{5}$$

In same way, all platform joints position, i.e. spherical joints attached to move platform must to be found in local coordinate system attached to moving platform:

$$\vec{C} = \vec{r} + \vec{n} \tag{6}$$

$$n = (-1)^{i-1} e \tag{7}$$

$$\vec{r} = r_p \cos \chi_j \hat{i} + r_p \sin \chi_j \hat{j} + 0\hat{k} \tag{8}$$

$$\vec{n} = -n \sin \chi_j \hat{i} + n \cos \chi_j \hat{j} + 0\hat{k} \tag{9}$$

$$^m\vec{PC} = r_p \cos \chi_j - m \sin \chi_j \hat{i} + r_p \sin \chi_j + m \cos \chi_j \hat{j} + 0\hat{k} \tag{10}$$

The rotational transformation using roll, pitch and yaw angles notation as defined in Eq.(11) is used in order to find $^m\vec{PC}$ from Tool Center Point, attached to move platform origin ^mO to spherical joints in general coordinate system attached to fixed base origin $^f\vec{O}$.

$$rot = \begin{bmatrix} c\varphi c\theta & c\varphi s\vartheta s\psi - s\varphi c\psi & c\varphi s\vartheta c\psi + s\varphi c\psi \\ s\varphi c\theta & s\varphi s\vartheta s\psi + s\varphi c\psi & s\varphi s\vartheta c\psi - c\varphi s\psi \\ -s\vartheta & c\vartheta s\psi & c\vartheta c\psi \end{bmatrix} \tag{11}$$

$$^f\vec{PC} = rot \, ^m\vec{PC} \tag{12}$$

to solve the inverse kinematic problem, which is detailed in next session.
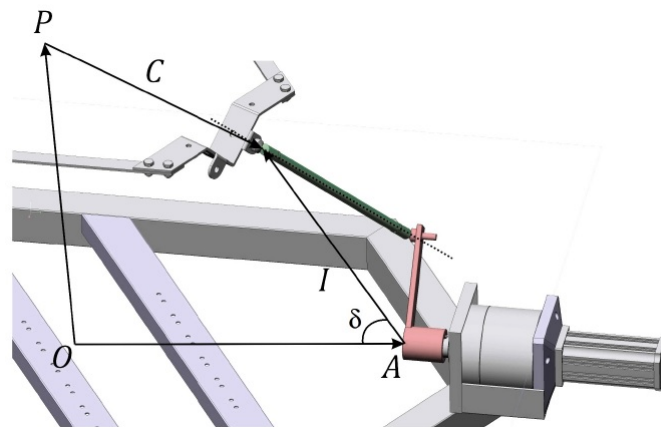

Figura 4: Vectorial chain to i-leg (Campos *et al.*, 2013).

In order to find vector from actuator to spherical joint $\vec{I}$ the vectorial equation Eq. (13) must be solved.

$$^f\vec{OA} = ^f\vec{OP} + \, ^f\vec{PC} + \, ^f\vec{I} \tag{13}$$

$$\vec{I} = {}^{f}\vec{OA} - {}^{f}\vec{OP} - {}^{f}\vec{PC} \tag{14}$$

The vector $\vec{I}$ may be decomposed in two components, one I_ω in plane ω_i, another one orthogonal to ω_i. The I_ω in ω_i may be decomposed in two components $\vec{I_z}$ and $\vec{T}$, one parallel to the z axis and is contained in plane ω_i and the other one is parallel to z axis and other one parallel to xy plane, respectively.

$$\vec{I_\omega} = \vec{I_z} + \vec{T} \tag{15}$$

The norm of vector $\vec{T}$ may be written in the components of $\vec{I}$ terms.

$$T = \left\|\vec{T}\right\| = I_x \cos\beta + I_y \sin\beta \tag{16}$$

Where I_x and I_y are $\vec{I}$ components in x and y axis, respectively.

With these definitions, it is possible to analyse crank rotational plane ω_i and define angles $\theta, \alpha, \phi, \varepsilon$ which must be found to solve inverse kinematic problem, being θ the crank angle.
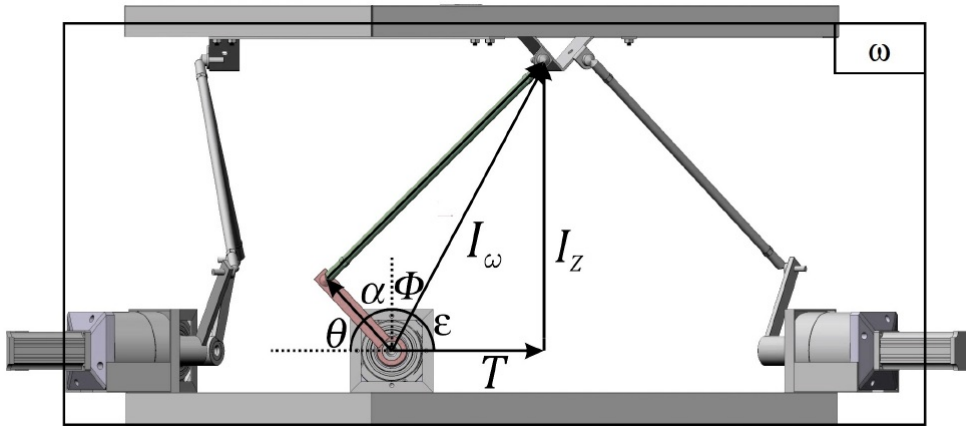

Figura 5: Plane ω_i frontal view and requeried angles (Campos *et al.*, 2013).

After some consideration can be found that θ is:

$$\theta = \frac{\pi}{2} - \arctan\frac{u}{v} - \arctan\frac{q}{w} \tag{17}$$

The parameters u, v, w and q are defined as:

$$u = 2r_i \left(I_x \cos\beta + I_y \sin\beta\right) \tag{18}$$

$$v = -2r_i I_z \tag{19}$$

$$w = R_i^2 - r_i^2 - I_x^2 - I_y^2 - I_z^2 \tag{20}$$

$$q = \sqrt{u^2 + v^2 - w^2} \tag{21}$$

With these terms, it is solved the inverse kinematic problem. Once function arctan has a dubious response, it is convenient to use $atan2$ function.

2.2 DIFFERENTIAL KINEMATIC AND INDEX BASED ON SCREW THEORY

In order to simulate the singular behaviour of parallel robots, the screw theory is used aiming at kinematic modelling. The screw is a geometric element composed by a directed line (axis) and by a scalar length parameter h called pitch (Ball, 1900). If the directed line is represented by a normalized vector, the screw is called a normalized screw (Campos *et al.*, 2011).

2.3 DIFERENTIAL KINEMTIC

The Mozzi theorem (Ceccarelli, 2000) states that the velocities of points on a rigid body with respect to an inertial reference frame $O(X, Y, Z)$ may be represented by a differential rotation ω about a given fixed axis and a simultaneous differential translation κ along the same axis. The complete movement of the rigid body, combining rotation and translation, is called screw movement or twist \$. The body "twists" around an axis instantaneously fixed with respect to the inertial reference frame. This axis is called the screw axis and the rate of the translational velocity and the angular velocity

is called the pitch of the screw $h = ||\kappa||/||\omega||$. The twist represents the differential movement of the body with respect to the inertial frame and may be expressed by a pair of vectors, in ray order, as (Hunt, 2003)

$$\$ = \begin{bmatrix} \omega \\ V_p \end{bmatrix} = [LMNP^*Q^*R^*]^T \tag{22}$$

Here ω is the angular velocity of the body with respect to the inertial frame and V_p represents the linear velocity of a point P attached to the body, which is instantaneously coincident with frame O. It is possible to express the same twist in axis order $\$ = [P^*Q^*R^*LMN]^T$. The vector V_p consists of two components: a) a velocity component parallel to the screw axis represented by $\tau = h\omega$; and b) a velocity component normal to the screw axis represented by $S_o \times \omega$, where $_o$ is the position vector of any point at the screw axis.

A twist may be decomposed into its magnitude and its corresponding normalized screw $\hat{\$}$, i.e. $\$ = \hat{\$}\Psi$ (Hunt (2003) (Campos *et al.*, 2011)).

2.4 STATIC

In the same way, the Poinsot theorem (Hunt, 1990) states that a general action, i.e. a force and a couple, upon a rigid body may be carried by a screw, called wrench $\$'$ (Ball (1900); Hunt (2003)). In this case the wrench in ray order is

$$\$' = \begin{bmatrix} f \\ C_o \end{bmatrix} = \begin{bmatrix} L' M' N' P^{*'} Q^{*'} R^{*'} \end{bmatrix}^T \tag{23}$$

where f is the resultant force and C_o is the resultant couple around O, upon the body. The wrench may be decomposed as $\$' = \$'\tau$ where τ is the wrench magnitude and the $\$' = \begin{bmatrix} L' M' N' P^{*'} Q^{*'} R^{*'} \end{bmatrix}^T$ is the normalized screw. The wrench pitch is determinated by $h' = ||C||/||f||$, being C the couple component in the screw axis direction (Campos *et al.*, 2011).

2.5 POWER

Consider a rigid body supporting a wrench $\$' = \begin{bmatrix} f^T C_o^T \end{bmatrix}^T$ while it is moving around an instantaneous twist $\$ = \begin{bmatrix} \omega^T V_p^T \end{bmatrix}^T$. Therefore, the power carried out is (Ball (1900); Hunt (2003))

$$\delta W = C_o \cdot \omega + f \cdot V_p = \$'^T\$ \tag{24}$$

where $\$'$ and $\$$ are given in axis and ray order, respectively. The screw theory is suitable to represent parallel manipulator end effector movements and actions which are used to detect singularities (Campos *et al.*, 2011).

2.6 SINGULARIT INDEX

A index based on screw theory is defined below to compute de proximity of a singular position of 6-RUS manipulator. This information is used as the constrain parameter of the optimization to maximize the workspace of this robot.

2.6.1 The Robot Wrenches

To determine the singularity index the wrenches acting in the moving platform most by found.
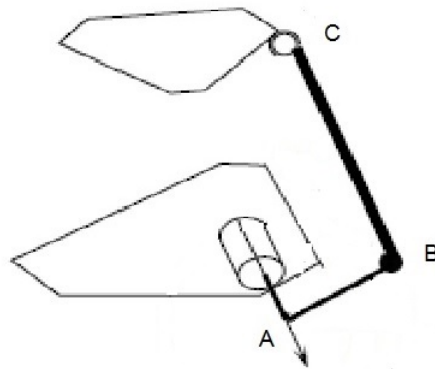

Figura 6: 6 RUS parallel robot geometrical parameters.

Being $i = 1, 2, 3, .., 6$ corresponding to each kinematic chain. The crank vector, from point A to B is:

$$\vec{b_i} = r_i \cos \chi_a \cos \theta \hat{i} + r_i \sin \chi_a \cos \theta \hat{j} - r_i \sin \theta \hat{k} \tag{25}$$

Adding Eq.(5) and Eq.(25) equations, the vector from O, origin point to B is:

$$\vec{B_i} = \vec{b_i} + \vec{A_i} \tag{26}$$

Been P_m just the moving platform position, without the orientation;

$$P_m = [P_x \ P_y \ P_z] \tag{27}$$

The vector from the base of the fixed platform to the point C_i

$$\vec{C_i} =^f \vec{PC} + P_m \tag{28}$$

The vector form point B_i to C_i is:

$$\vec{B_iC_i} = C_i - \vec{B} \tag{29}$$

The wrenches in axis-order are defined by vector multiplication of $\vec{C_i}$ and $\vec{B_iC_i}$:

$$L_i = [\vec{C_i} \times \vec{B_iC_i}, \vec{B_iC_i}]^T \tag{30}$$

Been L_i the wrench \$$_i$:

$$\$_i = L_i \tag{31}$$

The normalized screw:

$$\hat{\$}_i = \$_i/|\$_i| \tag{32}$$

2.6.2 Power Inspired Index

This technique is developed using the screw theory, specifically the power or rate of work, to determine how close the parallel manipulator is to a direct singularity (Pottmann *et al.*, 1999). The power inspired measure determines closeness to singularity through an optimization problem that results in a corresponding generalized eigenvalue problem. Using this methodology it is possible to describe the instantaneous behaviour of the end effector near singularities (Wolf and Shoham, 2003). Other measures are incorporated into a constrained optimization framework, e.g. the natural frequency measure (Voglewede, 2004).

In this approach the objective function $F(\hat{\$}, \$'_{i=1,...,6})$ to be optimized can be interpreted as the sum of the square of the power Eq.(24) of each leg upon the end effector which is constrained to move on \$$(||\omega|| = 1, h \neq \infty$, a normalized finite pitch twist; this interpretation may be done assuming unitary magnitude wrenches as normalized screws $\hat{\$'_i \simeq \$'_i}$

$$F = \sum_{t=1}^{n} \hat{\$}^T \$' \tag{33}$$

with

$$\hat{\$}' = \left[P_i^{*'} Q_i^{*'} R_i^{*'} L_i' M_i' N_i' \right]^T ; \$ = [LMNP^*Q^*R^*]^T \tag{34}$$

where $\hat{\$}'$ (dependent on x position coordinates, specifically of C_i and B_i positions) and \$ are given in axis and ray order, respectively, and n is the number of limbs. Therefore, F may be expressed as

$$F = \$^T J_x^T J_x \$ = \$^T M \$; M = \sum_{t=1}^{n} \$' \$'^T \tag{35}$$

where M is called the Graminiam matrix.

Considering that the only unconstrained movements of the end effector, in a direct singularity, are finite pitch twists (no pure translational movements are permitted) with magnitude $\Psi = 1$, the unitary twist magnitude, which is the constraint of the optimization method, is given through the invariant normalization (Voglewede, 2004).

$$||\$|| = \sqrt{\omega \cdot \omega} = \sqrt{L^2 + M^2 + N^2} = 1 \Rightarrow ||\$||^2 = \$^T D \$ = 1 \tag{36}$$

where $D = diag1, 1, 1, 0, 0, 0$. Eq.(35) under the constraint Eq. (36) may be transformed to obtain the Lagrangian L,

$$L = \$^T M \$ - \lambda(\$^T D \$ - 1) \tag{37}$$

where λ is the Lagrangian multiplier. The minimization of the Lagrangian is performed by

$$\partial L/\partial \lambda = \$^T D \$ = 1; \partial L/\partial \$ = (M - \lambda D)\$ = 0. \tag{38}$$

The matrix expression in the parenthesis, for a nontrivial solution, has to be singular, i.e.

$$det(M - \lambda D) = 0. \tag{39}$$

This is an eigenvalue problem and we can compute the eigenvalues $lambda_i$ and correspondent eigenvectors $\$_i$. Defining I as the 6×6 identity matrix and $\xi = 1/\lambda$, Eq.(39) becomes

$$det(M - \lambda D) = det(\xi I - M^{-1}D) = 0 \tag{40}$$

which has only three roots $\xi_i = 1/\lambda_i$, i.e. the eigenvalues of $[M^{-1}D]$, due to the three null elements of D diagonal. Each eigenvalue has a corresponding eigenvector $\$_i$ that satisfies Eq.(38).

Since the objective function is non-negative, given that M is a square symmetric positive semi-definite matrix (Voglewede, 2004), the end effector normalized twist which minimizes the supply power through the wrenches is the eigenvector $\$_{min}$ associated to the smallest eigenvalue λ_{min}. In this case one gets the minimum of F upon the end effector moving on $\$_{min}$, see (36), as

$$F = (\$_{min}, \$'^{\hat{}}_{i=1}, ...,_6) = \$_{min}^T M \$_{min} = \lambda_{min} \$_{min}^T D \$_{min} = \lambda_{min} \tag{41}$$

In direct singularity there is a twist $\$_{min}$ for which none of the limb wrenches can do any work and then the minimum of F, i.e. λ_{min}, goes to zero. Out of a singularity, $\$_{min}$ represents the less constrained twist and λ_{min} is a power based function measure that indicates the manipulator singularity closeness. It is worth remarking that $\delta W_{min} = \sqrt{\lambda_{min}}$ represents the minimal power. The case of two or three similar or identical minimum eigenvalues means that wrenches are in the intersection of two or three linear complexes, i.e. a linear congruence or a regulars respectively, and the end effector gains two or three degrees of freedom (Campos *et al.*, 2011).

2.7 INDEX EVALUATING

This technique is developed using the screw theory, specifically the power or rate of work, to determine how close the parallel manipulator is to a direct singularity (Lenar *et al.* (1998); Pottmann *et al.* (1999). The power inspired measure determines closeness to singularity through an optimization problem that results in a corresponding generalized eigenvalue problem (Wolf and Shoham, 2003).

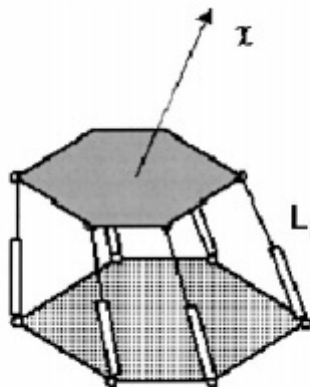Been L_i the Plucker line coordinates and x the resulting twist.

Figura 7: Gough-Stewart platform, the lines L_i and the resulting twists (Wolf and Shoham, 2003).

In the case of parallel manipulators this is interpreted as the minimization of the instantaneous work generated by a set of wrenches, given by L_i, acting on a body instantaneously moving in a twist direction given by x. If a wrench is acting on a rigid body such that it produces no work while the body is undergoing an infinitesimal twist, the two are assumed to be reciprocal to each other and their reciprocal product is zero. This is the case in a singular configuration of the robot. Consider the Gough-Stewart platform shown in "Figure 7". The rows of the Jacobian matrix of the manipulator are composed of the Plucker coordinates of the wrenches acting along its limbs L_i. Applying the method to this set of wrenches while the platform is moving instantaneously along the twist direction given by x, the smallest L_i determines the direction along which the work generated by the set of wrenches is minimal. When the reciprocal product is zero, there is no work generated by the set of wrenches when the platform moves in the twist direction given by x. We use this physical interpretation of the reciprocal product of screws in order to investigate the behavior of a given robotics structure in a given configuration (Wolf and Shoham, 2003).

2.8 OPTIMIZATION

This section have a short explanation of the optimizations algorithms applied in this work and why the use of Hybrid Optimization.

2.8.1 PSO

The particle swarm optimization (PSO) is a technique introduced by James Kennedy and Russell Eberhart in the 90s and emerged from experiments with algorithms that model the "social behaviour "observed in some species of birds (Kennedy *et al.*, 2001).

Particle Swarm (PSO) is a computational method that optimizes a problem by iteratively trying to enhance a candidate solution for a given measure of quality. PSO optimizes a problem by having a population of candidate solutions , here dubbed particles , and moving these particles around in the search space according to simple mathematical formulas on the particle position and velocity . The motion of each particle is influenced by its best known site position, but is also oriented towards the most known locations in the search space, which are updated as better positions are found by other particles . This is expected to move the swarm to the best solutions (Kennedy *et al.*, 2001).

PSO is a metaheuristic technique, as it makes little or no assumptions about the problem being optimized and can search very large spaces of candidate solutions . However , as PSO metaheuristics do not guarantee an optimal solution is always found . More specifically, the PSO does not use the gradient of the problem to be optimized (Kennedy *et al.*, 2001).

2.8.2 FMINCON

Fmincon is a gradient-based method that is designed to work on problems where the objective and constraint functions are both continuous and have continuous first derivatives. (MATLAB ®)

There is no guarantee that the Fmincom will return a global minimum, unless the global minimum is the only minimum and the function that is minimizing is continuous. As a method based on derivative may get remand to a local minimum and not find the value of the global minimum.

However, with various starting points FMINCON terminates at different v values, some of which are only local minima. Based on where FMINCON starts, it may terminate at the global minimum or at one of the local minima. The optimizer gets 'remand in the local valley' and can't escape to reach the 'global valley'.One possible way to find a global minimum is to run FMINCON with starting point in the 'global valley'. That why the use o PSO algorithm in the optimization.

2.8.3 HYBRID OPTIMIZATION

Methods for optimizing general purpose, that is, they were not designed to solve specific optimization problems, competitors must present some characteristics . Due to the possibility of local minima and the complexities of different objective functions , it is necessary to incorporate strategies to make reliable algorithm (in terms of finding the global minimum) while maintaining properties that lead to rapid convergence . One way to achieve these goals is through the coupling of two or more different optimization algorithms, which have complementary characteristics, which results in so-called hybrid algorithms. It is common to find hybrid algorithms involving an algorithm of type heuristic used to cover the entire search space and identify the region where the global minimum is found , and an algorithm with mathematical reasoning , said nonlinear programming , able to quickly reach the minimum , since the region has been identified. This type of strategy improves reliability compared to methods nonlinear programming , because it is more likely to find the global minimum , and increases efficiency compared with pure heuristic algorithms (Wang and Zheng, 2001). That the reason this work use PSO algorithm and FMINCON function to search for optimum of this problem.

3. ALGORITHM AND OPTIMIZATION RESULTS

This work developed a algorithm in MATLAB®to realize the 6-RUS robot optimization. The parameters to optimize the orientation workspace are the angles $\alpha1,\alpha2$. See "Figure 8".

The program initially receives a pair of α angles and create the robot movement simulation though inverse kinematic in all three orientation in the initial position. The screw theory is applied to establish the wrenches and twist to determined de singularity index that is the system constrain. The index can not have values below 0.0293 which determines a singular position.

The index minimum value is determined by evaluating the built robot prototype belonging to Udesc - Ceart in Florianopolis. Theoretically the index value is supposed to be zero, but due joints clearance the value of the index to be a singularity is slightly higher. See "Figure 9".

The optimization algorithms are used to adjust the alphas value that maximize the rotation positives and negatives in all three orientation axis, evaluating the index value. The program stars with an initial estimation of α angles. Then max value of these rotations is defined, for this configuration, until the movement produce the minimal index. After this loop the PSO algorithm is applied to estimate a new α pair with a better rotations values. It is the first applied optimization
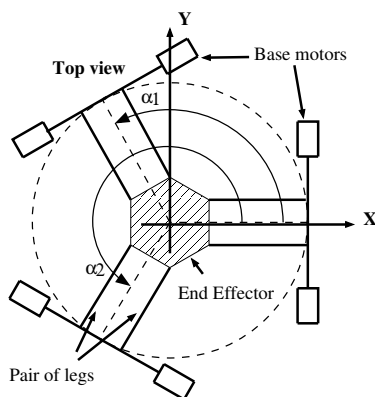
Figura 8: Base angular layout where design parameters are $\alpha 1$ and $\alpha 2$ (Campos *et al.*, 2011).
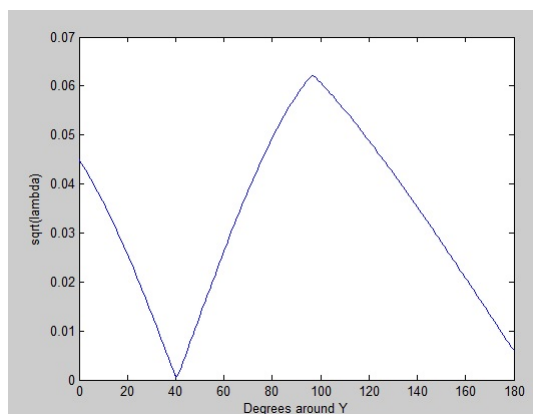

Figura 9: Singularity index for a rotation in Y axis.

algorithm to escape form local minima. PSO try many α angles until it can't find better values. Then it return a pair as the initial estimation to FMINCON. Below is the α values found by PSO:

PSO	α_1	α_2
Values	119.4951	241.1497

This pair of alphas is near the optimum, as PSO can't guarantee the optimum. Then FMINCON stars with alphas found by PSO and establish the optimum. See "Figure 10".
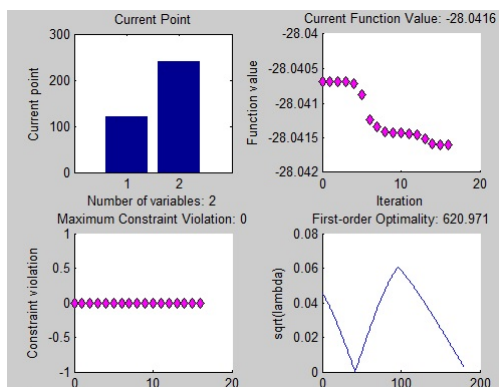

Figura 10: Graphical results from FMINCON.

Below the result α values found by FMINCON and the max orientation rotation positives and negatives of the robot:

FMINCON	X	Y	Z	α_1	α_2
Values	19.8278	18.02993	77.4263	120.3037	239.4492

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