

A theoretical poro-elasto-visco-plastic model for mechanical and chemo-mechanical deformation in sedimentary basins

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Abstract

Intergranular pressure-solution (IPS) represents a major mechanism of deformation in sedimentary basins during diagenesis. The aim of the present contribution is to provide a comprehensive 3D framework for modeling both mechanical and chemo-mechanical compaction in sedimentary basins. Extending concepts, previously proposed for the modeling of purely mechanical compaction in finite poroplasticity, deformation by IPS is addressed by means of additional viscoplastic terms in the state equations of the porous material. Resorting to both micromechanical reasonings and phenomenological arguments, the primary focus is on the analysis of the effects of large irreversible porosity changes on the poromechanical properties of the sediment material. The last part of the paper is intended to illustrate, through the analysis of a simplified compaction model, the main features of the proposed theoretical constitutive model.

Keywords: sedimentary basin, chemo-mechanical compaction, Intergranular Pressure-Solution, poroplasticity, poroviscoplasticity.

1 Introduction

Simulation of sedimentary basins is a complex multidisciplinary problem involving geological, chemical and mechanical aspects. Due to potential applications in the field of geoscience, which include petroleum exploration, reserve assessment and production, reconstructing the stress and deformation history of a sedimentary basin still remain a challenging and important problem in geoscience. In this context, compaction behavior of sediments material is one of the key elements controlling the sedimentary basin deformation, such as diagenetic densification, poromechanical properties evolution over geological time and its implications on the reservoir formation to hydrocarbon production.

Actually, the mechanical response of a sedimentary basin is the consequence of complex processes involving mechanical, geochemical, geophysical and geological aspects. As far as the mechanical analysis at the macroscopic scale is concerned, two principal deformation mechanisms contribute to the compaction of sediments: (1) purely-mechanical compaction which originates mainly from rearrangement

of the solid particles during burial, and (2) chemo-mechanical compaction resulting from intergranular dissolution-precipitation mechanisms, generally induced by stress and referred to as pression-solution [1–3]. Purely mechanical phenomena prevail in the upper layers, whereas chemical compaction dominates for deeper burial as stress and temperature increase (e.g. Schmidt and McDonald [4]).

The paper deals with the theoretical modeling of both mechanical and chemo-mechanical compaction in sedimentary basins.

As regards mechanical compaction, the basic models are still based on phenomenological relationships relating porosity to effective vertical stress. At this respect, one may refer to Bernaud *et al.* [5] where a review of the state of art on this topic is given. Furthermore, a 3D constitutive model was formulated in the framework of finite poroplasticity, extending previous ideas to a more comprehensive description of the mechanics involved in basin simulation.

Evidencies from both field and laboratory studies have confirmed that intergranular pressure-solution (IPS) represents an important mechanism of bulk deformation and porosity change. It is a key aspect of deformation in sedimentary basins during diagenesis. From a schematic point of view, IPS mechanism may be described as follows. Dissolution and diffusion occur at the grain-to-grain contacts subject to elevated stress, precipitation occurs at the free pore walls. The net result is compaction of the grains and porosity reduction, which in turn implies a decrease in permeability. Dissolution and precipitation processes are expected to produce time-dependent compaction creep and play therefore a crucial role as far as compaction of sediments over geological time periods is concerned.

Creep deformation resulting from IPS has widely been investigated since it has been identified as an important mechanism of rock deformation in the upper crust. The driving force for this deformation is the difference in effective stress between grain contacts and free pore walls. Two models have been basically proposed for IPS mechanisms in fluid saturated media, namely the “thin water film” model and the “island & channel” model (see for instance the review article by Hellmann *et al.* [6]). They both assume that the grain-to-grain contacts contain a trapped fluid which cannot be squeezed out by applied stress [7]. Both of these models are based on three material transfer processes occurring in series: dissolution of solid material at stressed grain-to-grain contacts, diffusion of dissolved material through the intergranular fluid, and precipitation on pore walls. The slowest of the three serial processes controls the overall deformation. It is primarily the physical nature of the intergranular regions that differentiates these two models. The “thin water film” is based on the presence of a continuous thin film of viscous fluid at grain-to-grain contacts [7, 8]. The “island & channel” model posits an intergranular region consisting of solid-solid contacts at islands that support the imposed stresses. These islands are surrounded by a network of interconnected channels containing fluid (e.g., Raj [9]; Lehner [10], Schutjens and Spiers [11]).

At the macroscopic scale, the intergranular pressure-solution phenomena are classically addressed in the constitutive equations by means of viscous terms. This kind of approach will be adopted in this work.

The paper is organized as follows. Section 2 describes the micromechanics-based constitutive model for the sedimentary model within the finite poro-visco-plasticity framework. Special attention is devoted to the influence of microstructure modifications on the poromechanical properties of the sediment material. The principle of finite element implementation is then briefly presented. In section

3, the main features of the proposed constitutive equations are illustrated through the analysis of simplified compaction model which is intended to represent the local response of the sediment particle under oedometric conditions of compaction.

2 Theoretical aspects

During compaction process, the sediment material is subjected to large volumetric strains resulting in a significant porosity reduction, exceeding in some basins 50% as burial goes [12]. In turns, this porosity change affects the physical and mechanical properties of the sediment material. In addition to the constitutive non-linearity, a comprehensive modeling of the basin response should thus also take into account both the geometric non-linearities induced by large strains. In this context, a 3D constitutive model for sediment materials has been formulated within the framework of finite poroplasticity in a recent paper by Bernaud *et al.* [5]. Disregarding chemical aspects, this constitutive modeling was dedicated to purely mechanical compaction in sedimentary basins. At many aspects, this contribution represented an appreciable improvement with respect to predecessor works which are mainly formulated within a 1D setting, basically through phenomenological relationships relating porosity to effective vertical stress. The purpose of this section is to provide a mechanical framework aiming to extend the model proposed in Bernaud *et al.* [5] in order to account for both mechanical and chemo-mechanical compaction.

2.1 Constitutive equations

At the macroscopic scale, the intergranular pressure-solution phenomena are classically addressed in the constitutive equations by means of viscous terms (see for example Laubsher [13], Fletcher [14], Schneider *et al.* [15, 16], Schneider and Hay [17] or Lehner and Leroy [18], to cite a few). The fundamental idea consists in relating measured strain rates resulting from dissolution-precipitation mechanisms induced by stress to porosity change. Assuming a phenomenological relationship (proportionality under certain conditions) between the latter and the vertical effective stress, a viscous law type is therefore derived for chemo-mechanical compaction.

The sedimentary rock is modelled as a fully saturated poro-elasto-visco-plastic material undergoing large strains. The anisotropy induced by compaction on the mechanical properties of the sediment material is disregarded. In addition, the elastic part of the deformation gradient of the skeleton particles is assumed to remain infinitesimal. Large strains involved during compaction process are only of irreversible nature.

The constitutive behavior formulated in Bernaud *et al.* [5] was based on the theoretical framework of finite poroplasticity proposed in Dormieux and Maghous [19, 20] and Bernaud *et al.* [21]. The above mentioned formulation is extended herein to finite poro-visco-plasticity. Denoting by $\underline{\underline{\sigma}}$ and p respectively the Cauchy stress tensor and the pore pressure, the first state equation relates the stress rate $\underline{\underline{\dot{\sigma}}}$ and pore pressure rate \dot{p} to the strain rate tensor $\underline{\underline{d}}$. The proposed form for the first state equation writes

$$\frac{D_J \underline{\underline{\sigma}}^{e}}{Dt} = \underline{\underline{\dot{\sigma}}}^{e} + \underline{\underline{\sigma}}^{e} \cdot \underline{\underline{\Omega}} - \underline{\underline{\Omega}} \cdot \underline{\underline{\sigma}}^{e} = \underline{\underline{c}} : (\underline{\underline{d}} - \underline{\underline{d}}^{ir}) + \underline{\underline{c}}^{-1} : \underline{\underline{\sigma}}^{e} \quad (1)$$

where $\underline{\underline{d}}^{ir}$ stands for the irreversible strain rate

$$\underline{\underline{d}}^{ir} = \underline{\underline{d}}^p + \underline{\underline{d}}^{vp} \quad (2)$$

$\underline{\underline{d}}^p$ (resp. $\underline{\underline{d}}^{vp}$) is the plastic (resp. viscoplastic) strain rate, while $\underline{\underline{\Omega}}$ is the rotation (spin) rate tensor which aims at taking the large rotation of the elementary volume into account. This equation involves a rotational time derivative D_J/Dt of the Biot effective stress tensor $\underline{\underline{\sigma}}^{le} = \underline{\underline{\sigma}} + b p \underline{\underline{1}}$, where b is the Biot coefficient. It also includes a term related to the particulate derivative $\dot{\underline{\underline{c}}}$ of the tensor of drained elastic moduli $\underline{\underline{c}}$, which aims at capturing the evolution of the elastic properties with the microstructural changes due to large irreversible strains.

Relationship (1) represents a generalization of the rate form formulation provided in Bernaud *et al.* [5].

In view of the derivation of the second state equation which relates the pore volume change to the rate \dot{p} of the pore pressure and to the strain rate $\underline{\underline{d}}$, it is convenient to introduce the jacobian J of the transformation of the elementary volume, defined as the ratio between its volume in the current configuration and its initial one. Similarly, the irreversible part J^{ir} of the jacobian is defined as the jacobian in the unloaded configuration of the elementary volume. It may be split into its plastic part J^p and viscoplastic one J^{vp} . The rates of these jacobians are given by

$$\dot{J} = J \text{tr} \underline{\underline{d}} \quad ; \quad \dot{J}^\alpha = J^\alpha \text{tr} \underline{\underline{d}}^\alpha \quad \text{with} \quad \alpha \in \{ir, p, vp\} \quad (3)$$

Let us denote by ϕ the lagrangian porosity in the current configuration of the elementary volume and by ϕ^{ir} the lagrangian porosity in the corresponding unloaded configuration. Starting from the rate form derived in finite poroplasticity by Bernaud *et al.* [21], the second state equation is generalized as

$$\dot{p} = M \left(-b \text{tr}(\underline{\underline{d}} - \underline{\underline{d}}^{ir}) + \frac{\dot{\phi} - \dot{\phi}^{ir}}{J^{ir}} \right) + \frac{\dot{M}}{M} p - M \dot{b} \text{tr}(\underline{\underline{c}}^{-1} : \underline{\underline{\sigma}}^{le}) \quad (4)$$

M is the Biot modulus. The terms involving \dot{M} and \dot{b} in (4) are related to the influence of large irreversible strains on the poroelastic properties.

The complementary equations prescribe the flow rule during the irreversible transformation of the elementary volume. Within the present framework, the latter deals with the irreversible strain rate $\underline{\underline{d}}^{ir}$ and with the rate of the irreversible part of the lagrangian porosity $\dot{\phi}^{ir}$. As regards the plastic strain rate, we introduce a plastic potential $g^p(\underline{\underline{\sigma}}')$ depending on $\underline{\underline{\sigma}}$ and p through the so-called Terzaghi effective stress $\underline{\underline{\sigma}}' = \underline{\underline{\sigma}} + p \underline{\underline{1}}$:

$$\underline{\underline{d}}^p = \dot{\chi} \frac{\partial g^p}{\partial \underline{\underline{\sigma}}'} \quad (5)$$

where $\dot{\chi}$ is a non-negative plastic multiplier.

The time-dependent (viscoplastic) component of the strain rate is formulated adopting the following

generalized viscoplasticity model [22]:

$$\underline{\underline{d}}^{vp} = \frac{1}{\eta} \left\langle f^{vp}(\underline{\underline{\sigma}}', \zeta) \right\rangle^n \frac{\partial g^{vp}}{\partial \underline{\underline{\sigma}}'} \quad (6)$$

where η is the viscosity coefficient, n the viscosity exponent, f^{vp} the viscoplastic yield function, $g^{vp}(\underline{\underline{\sigma}}')$ the viscoplastic potential and ζ the set of hardening variables. It should be recalled that the pressure-solution deformation described by the viscoplastic component $\underline{\underline{d}}^{vp}$ of the strain is generally addressed in basin engineering by means of explicit relationships of the form (see for instance Raj [9]; de Meer and Spiers, [23, 24] or Zhang and Spiers [25])

$$\underline{\underline{d}}^{vp} = \mathcal{F}(\underline{\underline{\sigma}}') \quad (7)$$

where expression of \mathcal{F} depends on the rate-limiting process during intergranular pressure-solution and incorporate microstructural data of the sedimentary material as well as temperature and physical properties of the solute in the intergranular regions. Actually, these models attempt to account in a simplified way for available measures obtained from laboratory tests which are generally performed in a 1D setting. The ingredients of the theoretical model described by (6), should therefore be consistent with the above mentioned experiment-based models.

It should be emphasized that the theoretical descriptions of the process of intergranular pressure solution in terms of creep laws can explain features such as grain-size dependence of creep rates. However as pointed out in Lehner [26], a significant shortcoming of the creep laws proposed in the literature is their limited applicability to macroscopically closed systems, with no solution transfer over long range transport in the pore fluid phase, as may be expected during diagenesis or metamorphism [27].

Since the irreversible part of the strain rate is given by (2), the conjunction of the plastic and viscoplastic flow rules (5) and (6) achieve the prescription of the flow rule (direction) for $\underline{\underline{d}}^{ir}$.

To complete the formulation of the flow rule, we need to precise the flow rule for the rate of irreversible porosity $\dot{\phi}^{ir}$. A simple way to achieve this formulation relies upon the assumption of incompressibility of the solid phase during the irreversible transformation of the elementary volume. This implies that the irreversible part of the pore volume change is equal to the total volume change and yields identity $\dot{\phi}^{ir} = \dot{J}^{ir}$. Owing to (3), it is further obtained that

$$\dot{\phi}^{ir} = J^{ir} \text{tr} \underline{\underline{d}}^{ir} \quad (8)$$

which indicates that the flow rule for $\dot{\phi}^{ir}$ stems directly from the flow rules (5) and (6). It is worth noting that because the irreversible strains in a sedimentary basin reduce to plastic strains in the upper layers, incompressibility of the solid phase during the irreversible transformation of the sediment particles means either plastic or viscoplastic incompressibility.

The above model requires to specify:

- the dependence of the elastic moduli $\hat{\zeta}$ as well as the poroelastic coefficients M and b on large irreversible strains;

- the influence of large irreversible strains on the plastic and viscoplastic properties of the sedimentary material.

These issues are addressed in the next section within the similar framework adopted in Bernaud *et al.* [5].

2.2 Poromechanics properties versus microstructural changes

The question of the influence of large plastic strains on the poro-elastic and plastic properties of the sediment material has been amply investigated in Dormieux and Maghous [20], Bernaud *et al.* [21] and Deudé *et al.* [28].

Since incompressibility is assumed for the solid matrix in the irreversible range, large irreversible strain are expected to induce significant significant porosity and pore shape changes. A micromechanics-based reasoning is used in order to capture the influence of the plastic strains on the poroelastic properties. In the subsequent analysis, the anisotropy induced during compaction process is disregarded. This means that the pore space is entirely characterized by its volume fraction in the current configuration of the representative elementary volume, namely the eulerian or classical porosity $\varphi = \phi/J$. The Hashin-Shtrikman upper bounds which are adopted as estimates for the elastic properties of the isotropic porous media. Accordingly, the bulk K and shear μ moduli of the porous medium appear as functions of the porosity as well as of the elastic properties of the solid phase:

$$K(\varphi) = \frac{4k^s \mu^s (1 - \varphi)}{3k^s \varphi + 4\mu^s} \quad ; \quad \mu(\varphi) = \frac{\mu^s (1 - \varphi)(9k^s + 8\mu^s)}{k^s(9 + 6\varphi) + \mu^s(8 + 12\varphi)} \quad (9)$$

where k^s and μ^s are bulk and shear moduli of the solid phase. It is recalled that the Biot coefficient and modulus are connected to K through

$$b(\varphi) = 1 - \frac{K(\varphi)}{k^s} \quad \text{and} \quad \frac{1}{M(\varphi)} = \frac{b(\varphi) - \varphi}{k^s} \quad (10)$$

In a second step, the porosity change will be connected to the irreversible strain undergone by the porous material. It can be first observed that the condition of incompressibility of the solid constituent during the irreversible transformation of the elementary volume reads

$$J^{ir} - \phi^{ir} = 1 - \varphi_o \quad (11)$$

where φ_o denotes the initial value of the porosity. In the framework of infinitesimal elastic strains, it is possible to neglect the variation of the pore volume and that of the total volume between the loaded and the unloaded configurations of the elementary volume, since these variations are reversible by definition. This justifies the following approximations

$$J^{ir} \approx J \quad ; \quad \phi^{ir} \approx \phi \quad (12)$$

Introducing (12) into (11) yields :

$$\varphi \approx 1 - \frac{1 - \varphi_o}{J} \approx 1 - \frac{1 - \varphi_o}{J^{ir}} \quad (13)$$

In view of (13), Eqs. (9) show that the macroscopic stiffness tensor $\underline{\underline{c}}$ is a function of the (total or irreversible) jacobian : $\underline{\underline{c}} = \underline{\underline{c}}(J^{ir})$. The same conclusion holds for the poroelastic coefficients b and M . Neglecting the induced anisotropy therefore amounts to consider that the elastic-plastic coupling is only governed by the plastic volumetric strains.

We deal now with the evolution of plastic properties of the sediment material. As regards the plastic yield surface f corresponds to the standard modified Cam-Clay (e.g., Muir Wood [29]):

$$f(\underline{\underline{\sigma}}' = \underline{\underline{\sigma}} + p\underline{\underline{1}}, p_c) = \frac{3}{2}\underline{\underline{s}} : \underline{\underline{s}} + M_{cs}^2 p'(p' + p_c) \quad (14)$$

where $\underline{\underline{s}} = \underline{\underline{\sigma}} - \frac{1}{3} \text{tr} \underline{\underline{\sigma}} \underline{\underline{1}}$ is the deviatoric stress tensor, $p' = \frac{1}{3} \text{tr} \underline{\underline{\sigma}}'$ is the mean effective stress. p_c is the consolidation pressure and represents the hardening parameter of the model. The constant M_{cs} represents the slope of the critical state line. The plastic flow rule is associated, i.e. $g = f$.

It is assumed in the sequel that the shape of the yield locus is not affected by large irreversible strains. In contrast, the hardening law, that is the influence of large irreversible strains on the consolidation pressure, is a crucial feature of the model. In the framework of finite poroplasticity, a micromechanics-based model for the hardening law has been proposed by Deudé *et al.* [28], and implemented later by Bernaud *et al.* [5]. Unlike the classical Cam-Clay hardening law, the micromechanics-based one avoids the development of negative porosities under high isotropic compression. We adopt in the sequel the heuristic approach which consists first, in considering that only the plastic part of the irreversible strains affect the parameter p_c , and then, that the same law can reasonably model the hardening law within the present framework, which is characterized by both plastic and viscoplastic strains. Accordingly

$$p_c(J^p) = \frac{p_{co}}{\ln \varphi_o} \ln \left(1 - \frac{1 - \varphi_o}{J^p} \right) \quad (15)$$

As regards the viscoplastic properties, the simple way to proceed is to adopt or extend the phenomenological models which are schematically described by (7). Indeed, these models already account in some extent for the morphology of the sediment microstructure through the value of porosity. Clearly enough, there is still a need for a comprehensive micromechanical approach specifically devoted to the formulation of a viscous law for deformation by pressure-solution. In this context, works had already been developed in the objective to relate, for specific microstructures, the macroscopic viscous behavior to the chemical phenomena occurring at a smaller scale (see for instance Lehner [26]; Renard *et al.* [30]; Ghoussoub and Leroy [31]).

3 A simplified compaction model

This section is intended to illustrate the main features of the theoretical model proposed in the previous sections, as well as its capabilities to simulate the stress and deformation of the sediment material during burial. A simplified framework is adopted for this purpose.

3.1 Statement of the problem and basic assumptions

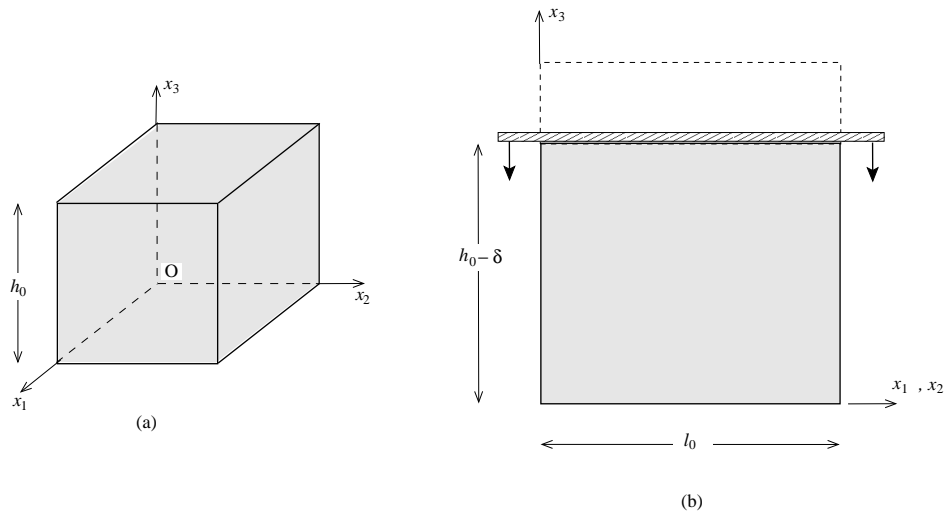


Figure 1: Geometry and loading of the simplified model

The geometry of the representative elementary volume (r.e.v) of the sedimentary material is depicted in Fig. 1a. In its initial configuration, the r.e.v is a parallelepipedic domain Ω_0 of high h_0 and horizontal sides transversal l_0 . The r.e.v refers herein to the macroscopic particle of the sediment material which is undergoing compaction during burial. The loading of the r.e.v consists in one-dimensional compressive solicitation, i.e., oedometric compression, as indicated in Fig. 1b. The vertical faces parallel to direction \underline{e}_3 are kept in contact with smooth and fixed walls, while the lower and upper horizontal are placed between two rigid and smooth rams. The lower one is kept fixed whereas the upper one has a downwards vertical displacement of magnitude $\delta > 0$. In terms of displacement, the boundary conditions read as follows:

$$\left\{ \begin{array}{ll} \xi_i = 0 & \text{on the vertical faces } x_i = 0, l_0 \\ \xi_3 = 0 & \text{on the lower face} \\ \xi_3 = -\delta & \text{on the upper face} \end{array} \right. \quad (16)$$

The value of the displacement δ imposed to the r.e.v accounts in some extent for the depth level of the particle within the sedimentary basin. Indeed, the quantity $\Lambda = 1 - \delta/h_0$ represents the vertical stretch of the particle, which is obviously related to the depth within the basin. In other words, the response of the r.e.v for increasing values of δ is supposed to simulate the stress and deformation of particles located at increasing depths along the basin.

The problem is treated in drained conditions (no overpressure). The material assumptions will be described in the sequel. The constitutive material of the r.e.v is homogeneous with isotropic elastic, plastic and viscous mechanical properties. Besides, effects of large strains on these properties, such as stiffness increase and hardening induced by large plastic volumetric strains, are disregarded [20, 28]. Accordingly, the elastic Lamé coefficients λ and μ are in particular considered as constant.

Plasticity is modeled by a simplified cap model with associated flow rule, where the straight-line corresponding to positive hardening is defined by

$$f^p(\underline{\sigma}, p_c) = -\frac{1}{3}\text{tr}\underline{\sigma} - p_c \tag{17}$$

p_c is identical to the consolidation pressure of the Cam-Clay model (e.g. Muir Wood [29]; Charlez [32]). In addition, the hardening modulus $m = -dp_c/dJ^p$ is taken constant (i.e., linear variations of p_c with respect to the plastic volumetric strains):

$$p_c(J^p) = m(1 - J^p) + p_{co} \tag{18}$$

p_{co} is the initial value of p_c . If plastic yielding occurs, the plastic part of the strain rate writes

$$\underline{d}^p = \dot{\chi} \frac{\partial f^p}{\partial \underline{\sigma}} = -\frac{\dot{\chi}}{3} \underline{1} \tag{19}$$

As regards the viscous behavior, which macroscopically models the effects of pressure-solution phenomena, a simplified viscoplastic criterion similar to that controlling plasticity is adopted:

$$f^{vp}(\underline{\sigma}) = -\frac{1}{3}\text{tr}\underline{\sigma} - p_{vp} \tag{20}$$

Unlike p_c , the viscoplastic threshold p_{vp} is assumed as constant. In addition, its value complies with condition

$$p_{vp} > p_{co} \tag{21}$$

in order to ensure that only plastic compaction occurs for moderate values of the loading δ (i.e., in the upper layers of the sedimentary basin). The corresponding viscoplastic strain rate is derived from the “thin water film” model [33]. If diffusion is the rate-limiting process, the strain rate resulting from intergranular pressure-solution is classically taken as [25, 34]

$$\underline{d}^{vp} = \frac{3}{\eta} \langle f^{vp}(\underline{\sigma}) \rangle \frac{\partial f^{vp}}{\partial \underline{\sigma}} = \frac{1}{\eta} \left(\frac{1}{3}\text{tr}\underline{\sigma} + p_{vp} \right) \underline{1} \quad \text{if } f^{vp}(\underline{\sigma}) \geq 0 \tag{22}$$

where $\langle X \rangle = \frac{X+|X|}{2}$ is the positive value of the scalar X , and η is the viscosity coefficient of the sediment. It incorporates the different fundamental parameters governing the creep law for pressure-solution (see for instance Rutter [7, 8]; Durney [35]), such as the characteristic grain size, the diffusivity of the solute in the grain contact or the equilibrium concentration of the dissolved solid in the pore fluid. This macroscopic viscosity is taken as constant in the present analysis. It should however be observed that a more realistic modeling should in particular account for the effects of temperature

[1, 16] and of the microstructural changes due to large irreversible strains.

This simplified framework obviously corresponds to a highly academic situation. It should be recalled that the purpose herein is only to provide a qualitative insight on the relative contribution of both the mechanical and mechano-chemical compaction on the local evolution of the stresses and strains.

3.2 Stress and strain in the r.e.v: elasto-visco-plastic analysis

Starting from $\delta = 0$ at time $t = 0$, the loading process of the r.e.v consists in prescribing the continuously increasing function $t \rightarrow \delta(t)$ (i.e., $\dot{\delta}(t) > 0$). The initial state of stress in the r.e.v is natural: $\underline{\underline{\sigma}}(t = 0) = 0$. The mechanical response of the r.e.v is determined in the sequel at any stage of the compaction level δ/h_0 . Under oedometric loading, the strain rate is homogeneous in the r.e.v and reads

$$\underline{\underline{d}} = d \underline{e}_3 \otimes \underline{e}_3 \quad \text{with} \quad d = -\frac{\dot{\delta}/h_0}{1 - \delta/h_0} \quad (23)$$

and the jacobian of the associated transformation of the r.e.v, which characterizes the volume change of the r.e.v is $J = 1 - \delta/h_0$. The rate form of the constitutive equation reduces to

$$\underline{\underline{\dot{\sigma}}} = 2\mu (\underline{\underline{d}} - \underline{\underline{d}}^{ir}) + \lambda \text{tr}(\underline{\underline{d}} - \underline{\underline{d}}^{ir}) \underline{\underline{1}} \quad (24)$$

A) Infinitesimal elastic response: $0 \leq \delta \leq \delta^e$ ($0 \leq t \leq t^e$)

Because of the elastic strains are assumed to remain infinitesimal, this phase corresponds to a small range of the loading $\delta \leq \delta^e \ll h_0$, where the expression of the elastic limit δ^e will be specified below. In basin engineering practice, purely elastic compaction prevails along a very thin upper layer, and should not therefore affect the global response of the whole basin.

In the elastic range $\underline{\underline{d}}^{ir} = 0$, the stress reads

$$\underline{\underline{\sigma}} = \sigma \left[\lambda (\underline{e}_1 \otimes \underline{e}_1 + \underline{e}_2 \otimes \underline{e}_2) + (\lambda + 2\mu) \underline{e}_3 \otimes \underline{e}_3 \right] \quad \text{with} \quad \sigma = \ln(1 - \delta/h_0) \simeq -\delta/h_0 \quad (25)$$

The conjunction of (17), (20) and condition (21) implies that plasticity occurs at the end of the elastic phase. This means that the elastic phase holds for interval $t \in [0, t^e]$ such that

$$\delta(t^e) = \delta^e \quad \text{with} \quad \frac{\delta^e}{h_0} = \left(1 - \exp \frac{-p_{co}}{K} \right) \simeq \frac{p_{co}}{K} \quad (26)$$

where $K = \lambda + 2\mu/3$ is the elastic bulk modulus. At the end of this elastic phase, the stress is

$$\underline{\underline{\sigma}}(t^e) = \underline{\underline{\sigma}}^e = -\frac{p_{co}}{K} \left[\lambda (\underline{e}_1 \otimes \underline{e}_1 + \underline{e}_2 \otimes \underline{e}_2) + (\lambda + 2\mu) \underline{e}_3 \otimes \underline{e}_3 \right] \quad (27)$$

Time $t = t^e$ characterizes the end of the elastic phase and the activation of plasticity.

B) Elasto-plastic response: $\delta^e \leq \delta \leq \delta^p$ ($t^e \leq t \leq t^p$)

In this situation $t \geq t^e$, $\underline{d}^{ir} = \underline{d}^p$ is given by (19). It comes from (25) and the consistency condition $\dot{f}^p = 0$ the following expression for the plastic multiplier

$$\dot{\chi} = \frac{\dot{\delta}/h_0}{(1 - \delta/h_0)(1 + mJ^p/K)} \geq 0 \quad (28)$$

Owing to (28), relation $\dot{J}^p = J^p \text{tr} \underline{d}^p$ leads a differential equation governing the evolution of J^p

$$\frac{\dot{J}^p}{J^p} + \frac{m}{K} j^p + \frac{\dot{\delta}/h_0}{1 - \delta/h_0} = 0 \quad (29)$$

which solution is

$$J^p(\delta) = \frac{K}{m} W \left(\frac{m}{K} \frac{1 - \delta/h_0}{1 - \delta^e/h_0} \exp \frac{m}{K} \right) = \frac{K}{m} W \left(\frac{m}{K} (1 - \delta/h_0) \exp \frac{m + p_{co}}{K} \right) \quad (30)$$

where $W(x)$ is the LambertW function defined by $W(x) \exp W(x) = x$. It is recalled that this function satisfies $W(x \exp x) = x$.

The stress field within the r.e.v results from the integration of the constitutive equation (24)

$$\underline{\sigma} = \underline{\sigma}^e + m (J^p - 1) \underline{1} + 2\mu \left(\ln(1 - \delta/h_0) + \frac{p_{co}}{K} \right) \left[\underline{e}_3 \otimes \underline{e}_3 - \frac{1}{3} \underline{1} \right] \quad (31)$$

$J^p = J^p(\delta)$ being given by (30). This elasto-plastic phase prevails as long as $f^p(\underline{\sigma}, p_c) \leq f^{vp}(\underline{\sigma})$. This condition may be expressed by

$$p_c = p_{co} - m (J^p - 1) \leq p_{vp} \quad (32)$$

whic by virtue of (30) leads to

$$\delta(t) \leq \delta(t^p) = \delta^p \quad \text{with} \quad \frac{\delta^p}{h_0} = 1 - \left(1 - \frac{p_{vp} - p_{co}}{m} \right) \exp \frac{-p_{vp}}{K} \quad (33)$$

The end of the phase is characterized by the following strain and stress related quantity values:

$$J^p(t^p) = J^p(\delta^p) = 1 - \frac{p_{vp} - p_{co}}{m} \quad ; \quad p_c(\delta^p) = p_{vp} \quad (34)$$

$$\underline{\sigma}(t^p) = \underline{\sigma}(\delta^p) = \underline{\sigma}^e - (p_{vp} - p_{co}) \underline{1} + 2\mu \left(\ln(1 - \frac{p_{vp} - p_{co}}{m}) - \frac{p_{vp} - p_{co}}{K} \right) \left[\underline{e}_3 \otimes \underline{e}_3 - \frac{1}{3} \underline{1} \right] \quad (35)$$

Time $t = t^p$ corresponds to characterizes the end of the elasto-plastic elastic phase and appearance of viscous strains.

C) Elasto-visco-plastic phase: $\delta^p \leq \delta \leq \delta^{vp}$ ($t^p \leq t \leq t^{vp}$)

If the loading is continued beyond δ^p , the v.e.r will *a priori* undergo viscoplastic strains due to pressure-solution phenomenon. In this case, $\underline{d}^{ir} = \underline{d}^p + \underline{d}^{vp}$. The plastic and viscoplastic parts of the irreversible strain rate are given respectively by (19) and (22). Introducing (19), (22) and (23) into the constitutive equation (24) leads to

$$\text{tr } \underline{\dot{\sigma}}/3 = K \left(-\frac{\dot{\delta}/h_0}{1 - \delta/h_0} + \dot{\chi} - \frac{3}{\eta} (\text{tr } \underline{\sigma}/3 + p_{vp}) \right) \quad (36)$$

On the other hand, the plastic condition $f^p = 0$ implies that $\text{tr } \underline{\sigma} = -3p_c$ while the consistency condition $\dot{f}^p = 0$ reads

$$\text{tr } \underline{\dot{\sigma}}/3 = -\dot{p}_c = m \dot{J}^p = m J^p \text{tr } \underline{d}^p = -m J^p \dot{\chi} \quad (37)$$

The combination of (36) and (37) yields

$$\dot{\chi} = \frac{1}{1 + mJ^p/K} \left(\frac{\dot{\delta}/h_0}{1 - \delta/h_0} - \frac{3}{\eta} (p_c - p_{vp}) \right) \quad (38)$$

Observing that the beginning of the phase corresponds to $p_c(t^p) - p_{vp} = 0$, (38) indicates that the plastic flow rule $\dot{\chi} \geq 0$ is satisfied at $t = t^p$. Consequently, provided that the compaction rate $\dot{\delta}/h_0$ remains positive, there will exist a non-empty time interval $t^p \leq t \leq t^{vp}$ such that the plastic flow rule shall be fulfilled. Clearly enough, the quantity $p_c(t) - p_{vp}$ is an increasing function during this time interval.

Eq. (38) and the identity $\dot{\chi} = -\dot{J}^p/J^p$, which can readily be deduced from (37), lead to

$$\frac{\dot{J}^p}{J^p} + \frac{m}{K} \dot{J}^p - \frac{3}{\eta} (p_c - p_{vp}) + \frac{\dot{\delta}/h_0}{1 - \delta/h_0} = 0 \quad (39)$$

By substituting expression (18) of $p_c = p_c(J^p)$ in the above equation, one obtains the differential equation governing the evolution in time of the plastic jacobian

$$\frac{\dot{J}^p}{J^p} + \frac{m}{K} \dot{J}^p + \frac{3m}{\eta} J^p - \frac{3}{\eta} (p_{co} + m - p_{vp}) + \frac{\dot{\delta}/h_0}{1 - \delta/h_0} = 0 \quad (40)$$

It is recalled that $t \rightarrow \delta(t)$ which defines the loading of the v.e.r is prescribed. Except for very particular situations, no closed-form solution of (40) can be derived and a numerical procedure is therefore necessary to solve this differential equation. This may be achieved by making use, for instance, of the software Maple.

Interestingly, a direct integration of (40) between $t = t^p$ (starting of the elasto-visco-plastic regime) and the current time t yields

$$\begin{aligned} \frac{3m}{\eta} \int_{t^p}^t J^p(\tau) d\tau = & \frac{3(p_{co} + m - p_{vp})}{\eta} (t - t^p) + \ln \frac{1 - \delta(t)/h_0}{1 - \delta(t^p)/h_0} \\ & - \ln \frac{J^p(t)}{J^p(t^p)} - \frac{m}{K} (J^p(t) - J^p(t^p)) \end{aligned} \quad (41)$$

with $J^p(t^p)$ and $\delta(t^p)$ given respectively by (33) and (34). The interest of (41) lies in fact that it provides an alternative way to compute numerically the unknown $t \rightarrow J^p(t)$. This can simply be achieved through the discretization of interval $[t^p, t]$ and evaluating the term integral in (41) by means of a numerical integration. If, for instance, the trapezoidal rule is adopted together with the time discretization $t_0 = t^p, \dots, t_i = t_{i-1} + \Delta t$ (the time increment Δt is chosen constant), it can readily be shown that

$$J^p(t_i) = \frac{1}{\frac{m}{K} + \frac{3m}{2\eta}\Delta t} W \left(\left[\frac{m}{K} + \frac{3m}{2\eta}\Delta t \right] \exp(\mathcal{F}_i) \right) \quad (42)$$

where

$$\begin{aligned} \mathcal{F}_i = & \frac{3(p_{co} + m - p_{vp})}{\eta} i \Delta t + \ln \frac{1 - \delta(t_i)/h_0}{1 - \delta(t_0)/h_0} + \ln J^p(t_0) + \left(\frac{m}{K} - \frac{3m}{2\eta}\Delta t \right) J^p(t_0) \\ & - \frac{3m}{\eta}\Delta t \sum_{k=1}^{i-1} J^p(t_k) \end{aligned} \quad (43)$$

Whatever the option chosen to handle with (40), $J^p(t)$ and consequently $p_c(t)$ by virtue of (18), can theoretically be considered as known at the end of the resolution procedure. The plastic multiplier $\dot{\chi}$ can thus be calculated from (38) as well as the plastic strain rate $\underline{\underline{d}}^p = -\dot{\chi}/3\underline{\underline{1}}$. Finally, the expression of the viscous strain rate $\underline{\underline{d}}^{vp} = \frac{1}{\eta} \left(\frac{1}{3}\text{tr}\underline{\underline{\sigma}} + p_{vp} \right) \underline{\underline{1}}$ is deduced from condition $f^p = 0$ which implies that $\text{tr}\underline{\underline{\sigma}}/3 = -p_c$.

To complete the analysis, the stress field should also be evaluated during this regime. It comes from the constitutive equation (24) that

$$\underline{\underline{\dot{\sigma}}} = d \left(2\mu \underline{\underline{e}}_3 \otimes \underline{\underline{e}}_3 + \lambda \underline{\underline{1}} \right) + K \left(\dot{\chi} + \frac{3}{\eta} (p_c - p_{vp}) \right) \quad (44)$$

where (19), (22) and (23) have been used. The final expression for the stress rate is derived by combining the above equation with (37) and (38)

$$\underline{\underline{\dot{\sigma}}} = m J^p \underline{\underline{1}} + 2\mu d \left[\underline{\underline{e}}_3 \otimes \underline{\underline{e}}_3 - \frac{1}{3} \underline{\underline{1}} \right] \quad \text{with} \quad d = -\frac{\dot{\delta}/h_0}{1 - \delta/h_0} \quad (45)$$

Integration of the above equation between t^p and the current time t yields

$$\underline{\underline{\sigma}}(t) = \underline{\underline{\sigma}}(t^p) + m (J^p(t) - J^p(t^p)) \underline{\underline{1}} + 2\mu \ln \left(\frac{1 - \delta(t)/h_0}{1 - \delta(t^p)/h_0} \right) \left[\underline{\underline{e}}_3 \otimes \underline{\underline{e}}_3 - \frac{1}{3} \underline{\underline{1}} \right] \quad (46)$$

in which $\delta(t^p)$ is given by (33), $J^p(t^p)$ by (34) and $\underline{\underline{\sigma}}(t^p)$ by (35).

This phase holds as long as the coexistence of plastic $\underline{\underline{d}}^p \neq 0$ and viscoplastic $\underline{\underline{d}}^{vp} \neq 0$ strain rates is compatible with the plastic flow rule, that is, with the positivity of the plastic multiplier. Consequently, the end of this regime coincides with time t^{vp} where condition $\dot{\chi} \geq 0$ ceases to be valid. Accordingly,

$$\dot{\chi}(t^{vp}) = 0 \implies p_c(t^{vp}) = p_{vp} + \frac{\eta}{3} \frac{\dot{\delta}(t^{vp})/h_0}{1 - \delta(t^{vp})/h_0} \quad (47)$$

As $t \rightarrow \delta(t)$ is prescribed and function $p_c(t)$ can be computed once $J^p(t)$ is determined, Eq. (47) can theoretically be solved with respect to t^{vp} . Mathematically, the existence of an effective solution t^{vp} will obviously depend on both material and loading characteristics.

At time $t = t^{vp}$, the v.e.r switches from the elaso-visco-plastic regime ($\underline{\underline{d}}^{ir} = \underline{\underline{d}}^p + \underline{\underline{d}}^{vp}$) to the elasto-viscoplastic regime ($\underline{\underline{d}}^{ir} = \underline{\underline{d}}^{vp}$). It should be observed from (18) and (47) that

$$J^p(t^{vp}) = J^p(\delta^{vp}) = 1 - \frac{p_{vp} - p_{co}}{m} - \frac{\eta}{3m} \frac{\dot{\delta}(t^{vp})/h_0}{1 - \delta(t^{vp})/h_0} \quad (48)$$

where $\delta^{vp} = \delta(t^{vp})$. Clearly enough, one should check whether the following restrictions are fulfilled: $J^p(t^{vp})$ remains positive and lower than $J^p(t^p)$ characterizing the initial value of the plastic jacobian during the current phase. Owing to (34) and (48), one obtains from (46)

$$\underline{\underline{\sigma}}(t^{vp}) = \underline{\underline{\sigma}}(\delta^{vp}) = \underline{\underline{\sigma}}(t^p) - \frac{\eta}{3} \frac{\dot{\delta}(t^{vp})/h_0}{1 - \delta(t^{vp})/h_0} \underline{\underline{1}} + 2\mu \ln \left(\frac{1 - \delta(t^{vp})/h_0}{1 - \delta(t^p)/h_0} \right) \left[\underline{\underline{e}}_3 \otimes \underline{\underline{e}}_3 - \frac{1}{3} \underline{\underline{1}} \right] \quad (49)$$

D) Elasto-viscoplastic phase: $\delta^{vp} \leq \delta(t^{vp} \leq t)$

Provided that $J^p(t^{vp}) \in]0, J^p(t^p)[$, this situation occurs if the loading δ is pursued beyond δ^{vp} . The v.e.r undergoes purely elavisoplastic strains ($\underline{\underline{d}}^{ir} = \underline{\underline{d}}^{vp}$). In other terms, no additional plastic strain appears in the v.e.r after $t = t^{vp}$ (i.e., $\underline{\underline{d}}^p = 0$). This means in particular that $\dot{J}^p(t) = 0$ for $t^{vp} \leq t$, which in turn implies identity $\dot{p}_c(t) = 0$. In reason of the particular form (17) of the plasticity criterion, the situation of plastic unloading (i.e., $\dot{f}^p < 0$) would imply condition $\text{tr} \underline{\underline{\dot{\sigma}}} > 0$, which is physically

irrelevant in the perspective of sedimentary basins modeling. We therefore adopt the assumption that the v.e.r does not experience any unloading process during this phase. Consequently,

$$\dot{f}^p = -\frac{1}{3}\text{tr}\underline{\dot{\sigma}} - \dot{p}_c = 0 \implies \text{tr}\underline{\dot{\sigma}} = 0 \quad \forall t \geq t^{vp} \quad (50)$$

During this phase, each one of the quantities J^p , p_c and $\text{tr}\underline{\sigma}$ keeps a constant value, equal to that reached at $t = t^{vp}$. Hence,

$$\text{tr}\underline{\sigma}(t) = -3p_c(t^{vp}) = -3p_{vp} - \eta \frac{\dot{\delta}(t^{vp})/h_0}{1 - \delta(t^{vp})/h_0} \quad (51)$$

and

$$\underline{d}^{vp}(t) = \underbrace{-\frac{1}{3} \frac{\dot{\delta}(t^{vp})/h_0}{1 - \delta(t^{vp})/h_0}}_{\frac{1}{3} d(t^{vp}) \underline{\underline{1}}} \underline{\underline{1}} \quad (52)$$

The constitutive equation (24) reduces to

$$\underline{\dot{\sigma}} = d(t) \left(2\mu \underline{e}_3 \otimes \underline{e}_3 + \lambda \underline{\underline{1}} \right) - K d(t^{vp}) \underline{\underline{1}} \quad (53)$$

The consistency with condition $\text{tr}\underline{\dot{\sigma}} = 0$ leads to the following restriction on the loading

$$d(t) = d(t^{vp}) \quad \text{or} \quad \frac{\dot{\delta}(t)/h_0}{1 - \delta(t)/h_0} = \frac{\dot{\delta}(t^{vp})/h_0}{1 - \delta(t^{vp})/h_0} \quad \forall t \geq t^{vp} \quad (54)$$

This restriction clearly emphasizes one of the shortcomings of the simplified model analyzed herein.

Integration of (53) between t^{vp} and t yields

$$\underline{\sigma}(t) = \underline{\sigma}(t^{vp}) - 2\mu \frac{\dot{\delta}(t^{vp})/h_0}{1 - \delta(t^{vp})/h_0} (t - t^{vp}) \left[\underline{e}_3 \otimes \underline{e}_3 - \frac{1}{3} \underline{\underline{1}} \right] \quad (55)$$

3.3 Comments

The purpose herein is to summarize the previous results by analyzing a numerical example. The model parameter are taken as: Young modulus $E = 10^3$ MPa, Poisson's ratio $\nu = 0.33$, $p_{c0} = 1$ MPa, hardening modulus $m = 100$ MPa, $p_{vp} = 10$ MPa, viscosity $\eta = 10$ MPa×My, where My stands for million years. The loading rate is constant and equal to $\dot{\delta} = 10$ m/My. Figure 2 displays the evolution in time of the jacobian J of the material transformation as well as its plastic part J^p . It is worth noting that the ratio J/J^p represents within the range $t \geq t^p \simeq 0.010$ My, a measure of the viscoplastic volumetric strains undergone by the material and which refers to the compaction induced by pressure-solution.

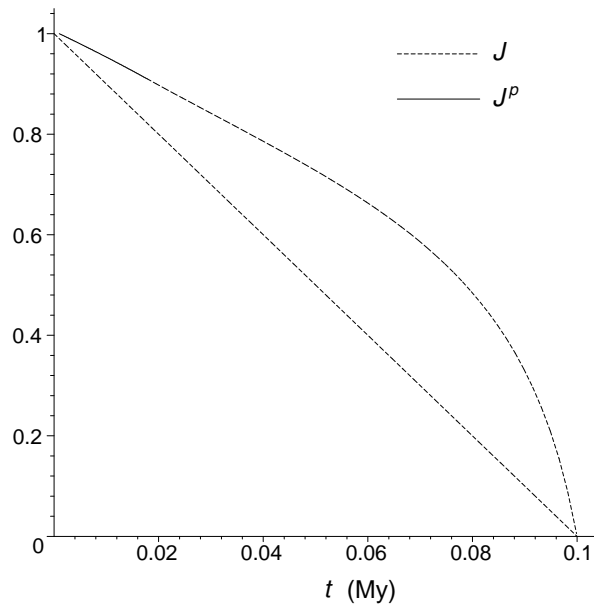


Figure 2: Transformation of the r.e.v.: time evolution of the total jacobian and its plastic part

As regards the stresses within the r.e.v, the variations of the vertical stress $\sigma_V = \sigma_{33}$ and of the horizontal stress $\sigma_H = \sigma_{11} = \sigma_{22}$ during the different phases of the macroscopic behavior are reported in Figure 3.

4 Conclusion

A 3D theoretical constitutive model has been formulated for sedimentary materials. It incorporates some of the fundamental coupled phenomena involved in the process of basin compaction by resorting to either micromechanical reasonings or phenomenological approaches. With respect to the previous work by Bernaud *et al.* [5], the main contribution lies in the fact the present model accounts for deformation by pressure-solution. Due to large porosity changes induced by compaction process, the model is formulated within the framework of large poro-visco-plasticity. Effect of large irreversible strains on the stiffness increase, on the hardening law as well as on the viscous law are accounted for through the reduction of the pore volume. The constitutive model is specifically devised for the sediment material subjected to mechanical or/and chemo-mechanical compaction.

The main features of the model have been illustrated through the analysis of a simplified compaction model. Depending on the loading level, different phases related to the response of the representative elementary volume could be predicted. The study was intended to provide a qualitative insight on

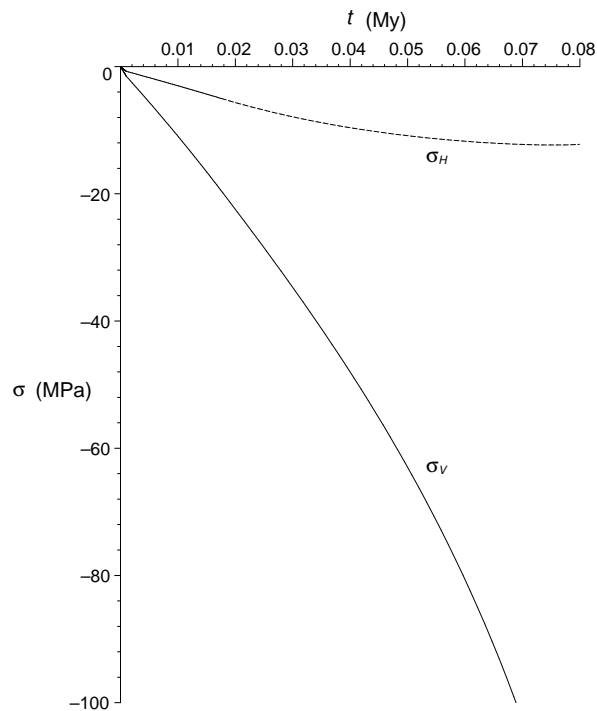


Figure 3: Evolution in time of the vertical and horizontal stresses in the r.e.v.

stress and deformation of the sediment material during burial.

The theoretical model is now being incorporated within a finite element tool dedicated to simulate sedimentary basins undergoing 2D evolutions. The extension to 3D situations such as that induced by tectonic loading is also one of the main issues to be foreseen in the future.

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References

- [1] Angevine, C.L. & Turcotte, D.L., Porosity reduction by model for quartz arenites pressure solution: A theoretical model for quartz arenites. *Geol Soc Am Bull*, **94**, pp. 1129–1134, 1983.
- [2] Tada, R. & Siever, R., Pressure solution during diagenesis. *Ann Rev Earth Planet Sci*, **17**, pp. 89–118,

- 1989.
- [3] Evans, B., Bernabé, Y. & Zhu, W., Evolution of pore structure and permeability of rocks in laboratory experiments. *Growth, Dissolution and Pattern Formation in Geosystems*, eds. B. Jamtveit & P. Meakin, Kluwer Academic Publishers: Dordrecht, The Netherlands, pp. 327–344, 1999.
 - [4] Schmidt, V. & Mc Donald, D.A., The role of secondary porosity in the course of sandstone diagenesis. *Aspects of diagenesis*, eds. P.A. Scholle & P.R. Schluger, Society of Economic paleontologists and Mineralogists Special Publication, volume 26, pp. 175–207, 1979.
 - [5] Bernaud, D., Dormieux, L. & Maghous, S., A constitutive and numerical model for mechanical compaction in sedimentary basins. *Computers and Geotechnics*, **33**, pp. 316–329, 2006.
 - [6] Hellmann, R., Renders, P.J.N., Gratier, J.P. & Guichet, R., Experimental pressure solution compaction of chalk in aqueous solutions. part 1. deformation behavior and chemistry. *Water-rock interaction, Ore Deposits, and Environmental Geochemistry*, eds. R. Hellmann & S.A. Wood, The Geochemical Society Special Publication, volume 7, pp. 129–152, 2002.
 - [7] Rutter, E.H., Pressure solution in nature, theory and experiment. *J Geol Soc London*, **140**, pp. 725–740, 1983.
 - [8] Rutter, E.H., The kinetics of rock deformation by pressure solution. *Phil Trans R Soc, London, Ser A*, **283**, pp. 203–219, 1976.
 - [9] Raj, R., Creep in polycrystalline aggregates by matter transport through a liquid phase. *J Geophys Res*, **87**, pp. 4731–4739, 1982.
 - [10] Lehner, F.K., Thermodynamics of rock deformation by pressure solution. *Deformation Process in Minerals, Ceramics and Rocks*, eds. D.J. Barber & P.G. Meredith, Unwin-Hyman: London, England, pp. 296–333, 1990.
 - [11] Schutjens, P.M.T.M. & Spiers, C.J., Intergranular pressure solution in NaCl: grain-to-grain contact experiments under the optical microscope. *Oil & Gas Science and Technology - Revue de l'IFP*, **54**, pp. 729–750, 1999.
 - [12] Houseknecht, D.W., Assessing the relative importance of compaction processes and cementation to reduction of porosity in sandstones. *Am Assoc Petroleum Geologist Bull*, **71**, pp. 633–642, 1987.
 - [13] Laubsher, H.B., Viscous components in jura folding. *Tectonophysics*, **27**, pp. 239–254, 1975.
 - [14] Fletcher, R.C., Coupling of diffusional mass transport and deformation in tight rocks. *Tectonophysics*, **83**, pp. 275–292, 1982.
 - [15] Schneider, F., Potdevin, J.L., Wolf, S. & Faille, I., Modèle de compaction élastoplastique et viscoplastique pour simulateurs de bassins sédimentaires. *Oil & Gas Science and Technology - Revue de l'IFP*, **49**, pp. 141–148, 1994.
 - [16] Schneider, F., Potdevin, J.L., Wolf, S. & Faille, I., Mechanical and chemical compaction model for sedimentary basin simulators. *Tectonophysics*, **263**, pp. 307–317, 1996.
 - [17] Schneider, F. & Hay, S., Compaction model for quartzose sandstones application to the garn formation, haltenbanken, mid-norwegian continental shelf. *Marine Petrol Geol*, **18**, pp. 833–848, 2001.
 - [18] Lehner, F.K. & Leroy, Y.M., Sandstone compaction by intergranular pressure solution. *Mechanics of fluid-saturated rocks, Int. Geophys. Series*, eds. Y. Gueguen & M. Bouteca, Elsevier Academic press: London, England, pp. 115–168, 2004.
 - [19] Dormieux, L. & Maghous, S., Poroelasticity and poroplasticity at large strains. *Oil & Gas Science and Technology - Revue de l'IFP*, **54**, pp. 773–784, 1999.
 - [20] Dormieux, L. & Maghous, S., Evolution of elastic properties in finite poroplasticity. *C R Acad des Sci Paris*, **328(IIb)**, pp. 593–600, 2000.
 - [21] Bernaud, D., Deudé, V., Dormieux, L., Maghous, S. & Schmitt, D.P., Evolution of elastic properties in

- finite poroplasticity and finite element analysis. *Int J Numer Anal Meth Geomech*, **26(9)**, pp. 845–871, 2002.
- [22] Perzyna, P., Fundamental problems in viscoplasticity. *Adv Appl Mech*, **9**, pp. 243–277, 1966.
- [23] De Meer, S. & Spiers, C.J., Uniaxial compaction creep of wet gypsum aggregates. *J Geophys Res*, **102**, pp. 875–891, 1997.
- [24] De Meer, S. & Spiers, C.J., Mechanisms and kinetics of creep by intergranular pressure solution. *Growth, Dissolution and Pattern Formation in Geosystems*, eds. B. Jamtveit & P. Meakin, Kluwer Academic Publishers: Dordrecht, The Netherlands, pp. 345–363, 1999.
- [25] Zhang, X. & Spiers, C.J., Compaction of granular calcite by pressure solution at room temperature and effects of pore fluid chemistry. *Int J Rock Mech Min Sci*, **42**, pp. 950–960, 2005.
- [26] Lehner, F.K., A model for intergranular pressure solution in open systems. *Tectonophysics*, **245**, pp. 153–170, 1995.
- [27] Dewers, T. & Ortoleva, P., Differentiated structures arising from mechano-chemical feedback in stressed rock. *Earth-Sci Rev*, **29**, pp. 283–298, 1990.
- [28] Deudé, D., Dormieux, L., Maghous, S., Barthélémy, J.F. & Bernaud, D., Compaction process in sedimentary basins: the role of stiffness increase and hardening induced by large plastic strains. *Int J Numer Anal Meth Geomech*, **28**, pp. 1279–1303, 2004.
- [29] Muir Wood, D.M., *Soil behavior and critical state soil mechanics*. Cambridge University Press: Cambridge, England, 1990.
- [30] Renard, F., Park, A., Ortoleva, P. & Gratier, J.P., An integrated model for transitional pressure solution in sandstones. *Tectonophysics*, **312**, pp. 97–115, 1999.
- [31] Ghoussoub, J. & Leroy, Y.M., Solid-fluid phase transformation within grain boundaries during compaction by pressure solution. *J Mech Phys Solids*, **49**, pp. 2385–2430, 2001.
- [32] Charlez, P.A., *Rock Mechanics, vol. 2: Petroleum applications*. Technip: Paris, France, 1997.
- [33] Tada, R., Maliva, R. & Siever, R., A new mechanism for pressure solution in porous quartzose sandstone. *Geochim Cosmochim Acta*, **51**, pp. 2295–2301, 1987.
- [34] Shimizu, T., Kinetics of pressure solution creep in quartz: theoretical considerations. *Tectonophysics*, **245**, pp. 121–134, 1995.
- [35] Durney, D.W., Early theories and hypotheses on pressure-solution-redeposition. *Geology*, **9**, pp. 419–424, 1978.