Natural vibration frequency of classic MEMS structures

Zacarias E. Fabrim
PGCIMAT, UFRGS, Porto Alegre, RS, Brazil
Wang Chong, Manoel Martín Pérez Reimbold
DeTec, UNIJUI, Ijuí, RS, Brazil

Abstract

This work addresses the determination of natural frequencies of three typical MEMS (Microelectromechanical Systems) microresonators comb-drive. The microstructures are understood as modular systems, composed by rigid masses coulped to elastic beam elements. The structure stiffness is calculated on the base of deflection deformation energy of elastic beams. The equivalent mass is given by the equality of the kinetic energy of the microstructure system with the one of a simple mass-spring system in harmonic vibration. Then the natural frequencies are obtained analytically in terms of the structure sizes and materials parameters. Numerical modal analysis with the software ANSYS showed good agreement with analytical results. Also it was discussed the influence of the rigidities of structural elements such as plates and beams on the frequencies and vibration modes. To achieve the expected frequencies and vibration modes its important the rigidity of microstructures and stiffness of elastic elements, therenfore the plates and beams shall be move in the same plane as expected. The methods presented in this work could be good for the determination of dynamic parameters of various MEMS structures.

Keywords: MEMS comb-drive, natural frequency, equivalent stiffness, equivalent mass.

1 Introduction

Microelectromechanical Systems (MEMS) are micro-manufactured structures, enabling functions of control, sensing and actuation [1]. The dynamic behavior of mechanical structures of MEMS has influence on the electrical responses through energy transduction principle [2]. Therefore, the determination of mechanical parameters such as elasticity, natural frequency and damping are of great technological importance to characterization and optimization of these devices. In this work we present a method under the framework of solid vibration theory [3] and strength of materials to determine natural frequency of laterally drive resonator MEMS *comb-drive* [4]. The analytical results are compared with numerical ones of finite element method (FEM) using the commercial software ANSYS. Three typical structures of MEMS (Figure 1) are analyzed: Typology 1 (T1), composed of two cantilever beams and

Mechanics of Solids in Brazil 2009, H.S. da Costa Mattos & Marcílio Alves (Editors)
Brazilian Society of Mechanical Sciences and Engineering, ISBN 978-85-85769-43-7

a central mass (Fig. 1 (a)), Typology 2 (T2), four cantilever beams and a central mass (Fig. 1(b)) and Typology 3, a central mass, two lateral masses and eight beams (Fig 1 (c)).

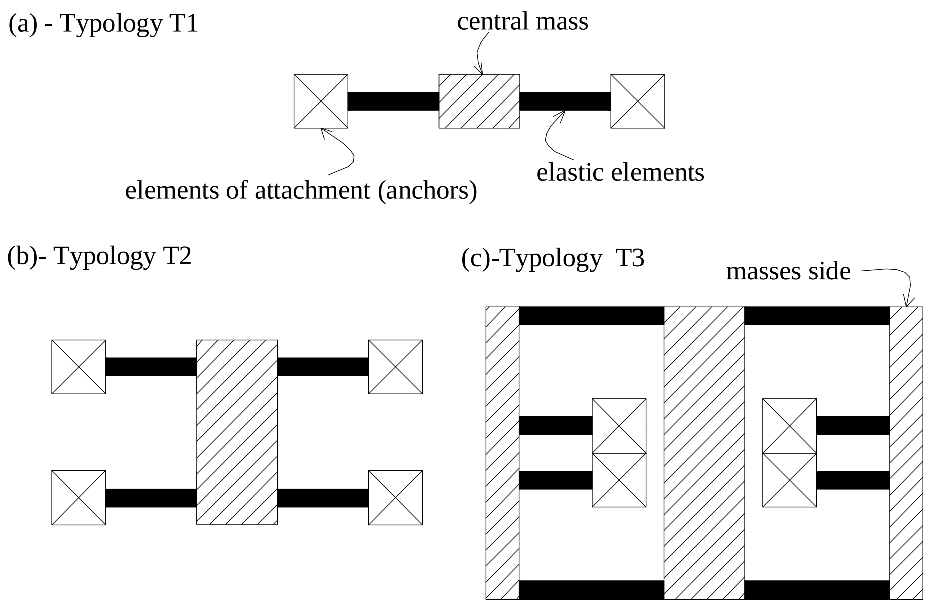

Figure 1: Typologies examined and basic elements: central mass, anchors, elastic elements and masses side.

2 Methodology

The natural frequency of a mass-spring system is expressed by the Equation (1):

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{1}$$

where k is the stiffness constant and m the mass.

Planar MEMS resonators are made of continuous structures in which the determination of resonant frequency is relatively complex. However, it is possible to substitute the original system for a simple mass-spring system by calculating structure stiffness and kinetic energy once are establish a kinematical equality for the displacements of systems, typology and mass spring (Fig. 2).

The total kinetic energy of a typology structure can be obtained by the sum of kinetic energy of the *mass-beam* elements. As the symmetry of structural geometry and boundary conditions, the parameters K and M_E can be calculated considering the typologies as modular systems, composed by coupling of basic mass-beam elements, as illustrated in the Figure 3.

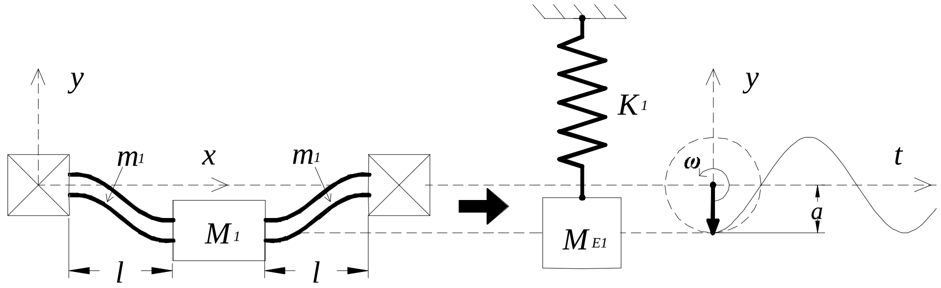

Figure 2: Kinematical equivalency between original system of typology T1 with a mass-spring system; m_1 is the mass of elastic element, M_1 is the mass of rigid element, K_1 the stiffness constant of spring, M_{E1} the equivalent mass, ω is the angular speed, a the maximal displacement and t time.

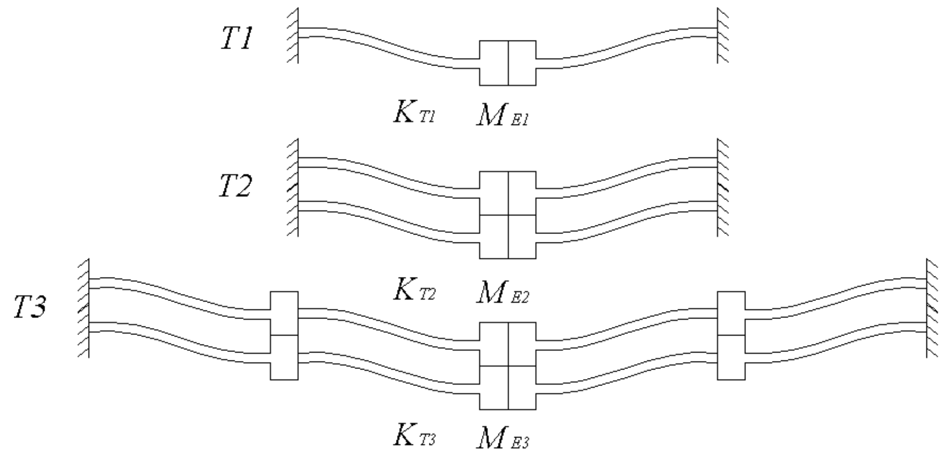

Figure 3: Modular structures of T1, T2 and T3.

3 Stiffness constant of mass beam element

The total stiffness constant of one typology, K, is obtained by the deflection of the beam end subjected to a concentrated force and connected to the central mass. The displacements in direction y are given as the deflection curve of the elastic beam. The concepts of connecting beams in series and in parallel allow the stiffness of modular systems to be a function of several basic mass-beam elements. The stiffness constant of a basic mass-beam element illustrated in the Fig. 4 is obtained by division of the force by the displacement at the position in the direction of the applied force, that is $K_e = P_e / |y_B|$.

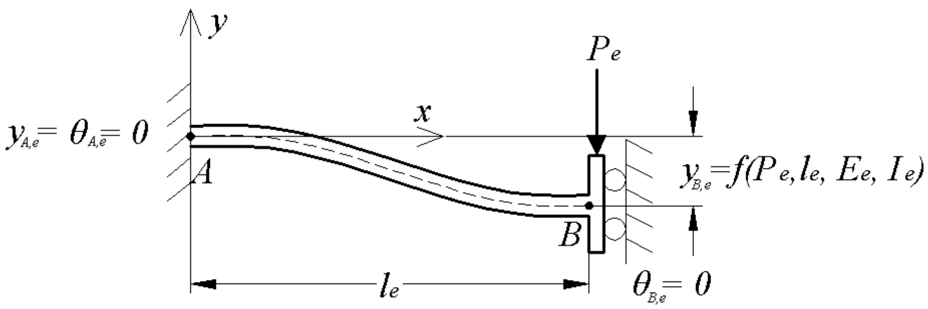

Figure 4: Load, deflection and boundary conditions for the elastic element beam AB. The displacement y_B will be a function of load P_e, length l_e, elasticity module E_e and inertial moment of the transversal section I_e.

When the force P_e is applied, the energy potential is stored in the form of elastic deformation energy in the entire beam. According to the elastic beam theory, the deflection curve is given by:

$$y(x) = -\frac{P_e\, x^2}{E_e I_e 6}\left(\frac{3}{2} l_e\, x\right) \tag{2}$$

From above, we can get the stiffness constant of basic element beam:

$$K_e = \frac{12 E_e I_e}{l_e^3} \tag{3}$$

4 Equivalent mass of mass-beam element

A simple method to obtain the equivalent mass, M_E, is to make a kinematics analogy between the original system and the equivalent system (Fig. 5). The equivalence principle will be that the original system and equivalent system have the same dynamic effect, i.e. the same kinetic energy. The kinetic energy of the original system can be obtained by the sum of the kinetic energy of moving parts.

Mechanics of Solids in Brazil 2009, H.S. da Costa Mattos & Marcílio Alves (Editors)
Brazilian Society of Mechanical Sciences and Engineering, ISBN 978-85-85769-43-7

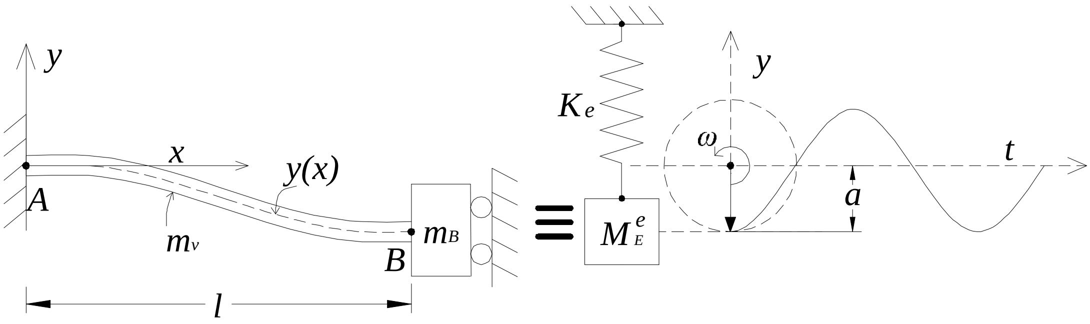

Figure 5: Equivalence between the basic mass-beam element to a simple mass-spring. m_v - beam mass and m_B - rigid mass.

We separate the kinetic energy of vibration elements into two components: $E_C^{m_B}$, kinetic energy of the rigid body at the end of the beam and E_C^l, kinetic energy of the elastic beam. For the rigid body:

$$E_C^{m_B} = \frac{1}{2} m_B v_B^2 \tag{4}$$

The kinetic energy of the elastic beam AB can be described by the sum of the kinetic energy of the small elements, that is:

$$E_C^l = \frac{1}{2} \sum_{i=1}^{n} v_i^2 \Delta m_i \cong E_C^l(t) = \frac{\rho \mathrm{A}}{2} \int_0^l v(x,t)^2 dx \tag{5}$$

where A is the area of cross section of the beam and ρ the density of the material, supposedly constants. Assuming that the beam movement in the direction y is of the form described as the static deflection curve, Eq. (2), the total kinetic energy of vibration elements is

$$E_C^e = E_C^{m_B} + E_C^l = \frac{P^2 l^6}{288 E^2 I^2} \omega^2 \cos^2(\omega t + \theta) \left(\frac{13 m_v}{35} + m_B \right) \tag{6}$$

where θ is the phase angle and ω the angular velocity of vibration. From above, we get the mass equivalent of the basic element of Figure 7 by assuming that the mass spring system vibrates with the magnitude equal to the maximum deflection:

$$M_E^e = \left(\frac{13 m_v}{35} + m_B \right) \tag{7}$$

5 Analytical results

We obtained the stiffness constant of each typology using the stiffness constant of the basic beam element (Eq. (3)) and the equivalent stiffness concept of beams connected in series and in parallel. The separation of a continuous structure into same modular elements, shown as Figure 5, simplifies the calculation of equivalent stiffness and equivalent mass of a typology.

The natural frequency of each typology is obtained through substituting the equivalent stiffness and equivalent mass into the Equation (1). Tab. 1 lists the analytic results of stiffness constant, K, equivalent mass M_E and natural frequency f_n in terms of the dimensions, mass e materials of typologies.

Table 1: Systematization of analytic results: stiffness constant, equivalent mass and natural frequency. Where $w, h$and l are the width, thickness and length respectively; M_V is the mass of all beams of a given type; M_C is the central mass; M_L is the sum of the two lateral bodies of the typology T3.

Typology	K		M_E	f_n
T1	$\frac{24EI}{l^3}$	$\frac{2Ehw^3}{l^3}$	$\frac{13M_V}{35} + M_C$	$\frac{1}{2\pi}\sqrt{\frac{2Ehw^3}{l^3\left(\frac{13M_V}{35} + M_C\right)}}$
T2	$\frac{48EI}{l^3}$	$\frac{4Ehw^3}{l^3}$	$\frac{13M_V}{35} + M_C$	$\frac{1}{2\pi}\sqrt{\frac{4Ehw^3}{l^3\left(\frac{13M_V}{35} + M_C\right)}}$
T3	$\frac{24EI}{l^3}$	$\frac{2Ehw^3}{l^3}$	$\frac{12}{35}M_V + M_C + \frac{M_L}{4}$	$\frac{1}{2\pi}\sqrt{\frac{2Ehw^3}{l^3\left(\frac{12}{35}M_V + M_C + \frac{M_L}{4}\right)}}$

6 FEM simulation

Using the software ANSYS, we got the natural frequencies from modal analysis. The simulations were conducted for silicon monocrystalline microstructures (Tab. 2). The parameters of size, mass and excitement harmonic are presented in Tab. 3.

We are interested in frequencies of resonance and respective modes of vibration. Two cases for T3 were simulated: T3 (a) and T3 (b), having differences in the dimensions and weight of the lateral and central masses, but same masses. However, the dimensions, weights and thicknesses of beams are same for each case, so that their analytical results are identical. Since obtaining of analytical formulations considered the central mass and lateral masses as lumped ones, no deformation energy of these elements was considered. The purpose of making comparison between two conceptions of T3 is to check the influence of the rigidity of mass central and lateral masses on the vibration modes and respective natural frequencies.

Table 2: Mono-crystalline silicon data.

Description	*Symbol*	*Value*	*Conversion*
Young Module	E	140 [GPa]	1,4 x 10^5 [kg.μm^{-1}.s^{-1}]
Density	ρ	2,33 [g.cm^{-3}]	2,33 x 10^{-15} [kg.μm^{-3}]

Table 3: Dimensions and loading.

Description	*Symbol*	*Value*
Length of the beams	l	200 μm
Width of the beams	w	2 μm
Thickness of the all structure	h	2,1 μm
Central mass	M_C	5,0887 x 10^{-11} kg
Lateral mass of T3	M_L	2.6911 x 10^{-12}kg
Harmonic excitement amplitude	F_0	0,14 μN

Fig. 6 shows the graphic results of the ANSYS for the first mode of vibration of T1 and T2. It can be seen that the deflections of beams are in agreement with the analytical models for the first mode of vibration, that is, the deflection curves of the two beams are close well to the curve described by the Eq. (2).

The modal analysis of T3 (a) showed a distinct behavior from T1 and T2. The first mode (the left illustration in Fig. 7) is characterized by a rotational movement around the mass center; also the strain of sidebars is considerable. For T3 (a), the vibration mode that best matches the analytical models was the 3 rd mode, as shown by the right illustration in Figure 7. However with the size of T3 (b), the first mode of vibration was in agreement with the analytical result (Figure 8).

The numerical and analytical results of natural frequency are compared in Table 4. We can see good coincidences between the analytical results and the numerical ones, except for T3(a). The large difference for T3 (a) is owing to the low rigidity of the lateral rods, whose horizontal displacement x is greater than the one of T3(b) (Figure 9).

Calculation of elastic strain energy confirmed that the lateral rods of T3(a) have experienced notable deformation. The ratio of the elastic strain energy of the lateral rods to the total elastic strain energy

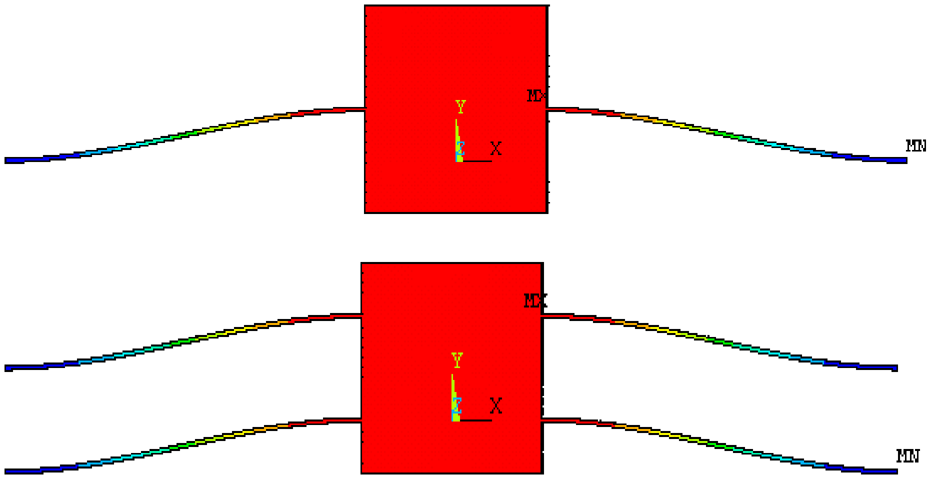

Figure 6: First vibration mode of T1 and T2.

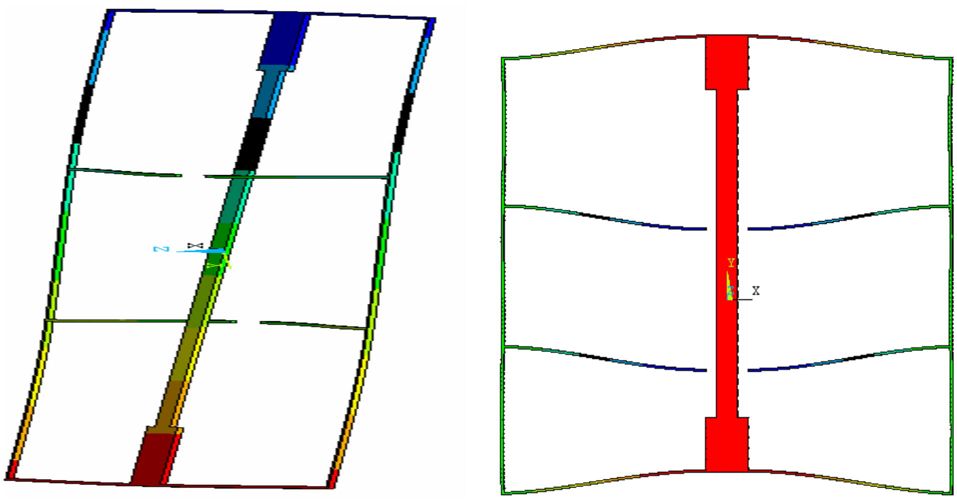

Figure 7: First (left) and third (right) modes of vibration of T3 (a).

of the structure is about 6.7% for the case of T3(a) but only 0.68% for T3(b). The notable deformation violated the assumption of rigidity masses for the analytical models and resulted in the large difference in the analytical and numerical natural frequencies.

Mechanics of Solids in Brazil 2009, H.S. da Costa Mattos & Marcílio Alves (Editors)
Brazilian Society of Mechanical Sciences and Engineering, ISBN 978-85-85769-43-7

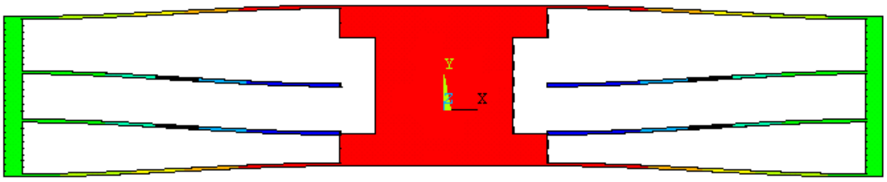

Figure 8: First vibration mode of T3 (b).

Table 4: Comparison of natural frequencies.

Typology	f_r*analítical* [Hz]	f_r *ANSYS* [Hz]	*difference* [Hz]	*relative difference* [%]
T1	16869	16843	26	0,15
T2	23532	23495	37	0,16
T3 (a)	15896	14905	991	6,23
T3 (b)	15896	15887	9	0,05

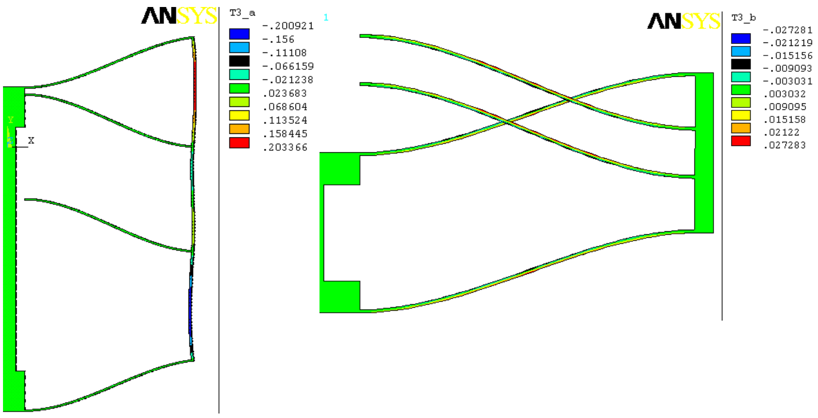

Figure 9: Displacement towards x in T3 (a), left, and shifts towards x in T3 (b), right. The scale is 16 times higher in relation to the values of the legend.

7 Conclusion

The MEMS sided resonator *comb-drive* can be modulated by such a structure consisting of elastic beams and rigid masses. The natural frequency can be obtained by equivalent mass-spring systems.

Depending on the structure typology, the equivalent stiffness can be determined from springs connected in parallel and in series. The results obtained by the software ANSYS confirm the validation of analytical formulation, deduced from equivalent systems. The analytical formulation obtained in this work can be used in the design of MEMS. The method and the concept of equivalent systems proposed can be extended to other types of MEMS. However, to achieve the desired mode of lateral vibration, the mass elements must have sufficient rigidity. This should be paid high attention in the design or manufacture of MEMS.

References

[1] Maluf, N. & Willians, K., *An Introduction to Microelectromechanical System Engineering*. Artech House, Inc.: MA, US, 2004. 282 p.
[2] Varadan, V.K., Vinoy, K.J. & Gopalakrishnan, S., *Smart Materil Systems and MEMS: Design and Development Methodologies*. John Wiley & Sons Ltd.: England, 2005.
[3] Meirovitch, L., *Elements of vibration analysis*. McGraw Hill: Singapore, 2nd edition, 1986.
[4] Tang, W.C. et al., Electrostatic-comb drive of lateral polysilicon resonators. *Sensors and Actuators,* A21-A23, pp. 328–331, 1990.

Mechanics of Solids in Brazil 2009, H.S. da Costa Mattos & Marcílio Alves (Editors)
Brazilian Society of Mechanical Sciences and Engineering, ISBN 978-85-85769-43-7