Study of flexible wall acoustic cavities using Beam Finite Element

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Abstract

The problems related to coupled fluid-structure are found in various fields of engineering. Technology enhancements, new materials, and the development of various sectors of industry led to the raise of complex problems, involving, for example, those related to acoustic-mechanical coupling, which appear in cases of cavities containing fluid, limited by elastic walls. This work analyses some cases of coupled fluid-structure problems on acoustic cavities with flexible walls, using two classic Eulerian formulations to fluid-structure interaction problems: U-ϕ-P_0 and U-P, represented, respectively, by the programs: FEDYFE (GDFE/UnB) and ANSYS. The theoretical formulation and the finite element discretization are presented for the first formulation, not both. In this fluid-structure interaction problem, the solid will be treated using beam finite elements, while for the fluid 2D acoustic fluid finite elements are used. Cases involving free vibration for coupled systems in 2D, for conditions of regular contour of the cavity with walls open and/or closed, rigid and/or flexible were studied. The frequencies and the vibration modes obtained have presented a good agreement when compared with the two numerical methods, both for the uncoupled and coupled problems. Analytical solutions are also used and indicate good agreement with these results.

Keywords: fluid-structure, finite element, vibration modes, cavity, beam.

1 Introduction

The coupled problems related to fluid-structure can be found in various branches of engineering. The studies of these problems can appear in several cases of cavities containing fluid and it is associated to structures that can be elastic, flexible/rigid, as well as in pipe with fluid on the outside/inside, and so on. Generally, the coupled systems are characterized by axisymmetric and ill-conditioned matrices. The elements of the global matrix are represented by physical and geometrical constant of the domains involved, which have very different magnitudes, which complicates the resolution of these systems. Therefore, a segment of the literature related to the subject is dedicated to the study of simplified models, to make them more applicable and to produce satisfactory results.

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The formulation used considers the fluid as acoustic (without flow), which makes the model more simplified. The movements are small around a position of equilibrium, where there are only waves of pressure. Among the classic formulations that deal with the coupled problem of fluid-structure has been: 1) The Lagrangian has been used, for example, by Zienkiewicz and Bettess [1], Wilson and Khalvati [2] and others; 2) the Eulerian has been caracterized by the pressure, a displacement or velocity potencial for the fluid, and the displacement as variable for the solid, such as described by Zienkiewicz and Newton [3], Daniel [4, 5], Everstine [6], Sandberg [7], Sandberg, Hansson and Gustavsson [8] and others.

This work presents the implementation of a Bernoulli beam element in the code FEDYFE (Finite Elements in Dynamics and Fluid-Structure) which was developed by a research Group for Dynamic and Fluid-Structure at UnB (GDFE). This code is based on the symmetrical potencial formulation ($\boldsymbol{U}$-$\boldsymbol{\phi}$-$\boldsymbol{P}_0$), proposed by the Olson and Bathe [9] and adapted by Barbosa [10]. The FEDYFE code uses simple triangular elements of the plane stress state, as well as triangular elements with linear interpolation for the fluid.

The implementation of the beam element in the FEDYFE code is done by simply replacing the stiffness and mass matrices of the triangular element of the plane stress state with the stiffness and mass matrices of the Bernoulli's beam element. The frequencies and vibration modes obtained by FEDYFE were compared with results of the ANSYS [11] software, that uses other Eulerian formulation (U-P). This formulation was also used by Sousa Jr [12] and Souza [13]. Despite the formulations of the coupled problem are different, the numerical results provided by the two softwares were satisfactory when compared with each other.

This work selects two cases of coupling, considering the beam element: The Case 1 shows a simple supported beam with open square cavity and Case 2 corresponds to a simple supported beam with closed square cavity. The results of frequencies and coupled vibration modes obtained by the FEDYFE code were compared with their output of the ANSYS [11]. These results presented a good agreement for cases of reservoir with flexible bottom which were analyzed. Moreover, the frequencies and uncoupled numeric vibration modes of the structure were also compared with the analytical solution given by Cough and Penzien [14], showing satisfactory values.

2 Coupled problem formulation (U-ϕ-P$_0$)

The formulation in question, proposed by Olson and Bathe [9], considers as the main hypotheses for the solid domain linear elastic behavior, isotropic and homogenous material with constant elasticity module and small displacements. It is assumed that the fluid is inviscid, compressible and having an adiabatic behavior. The variational analysis of the dynamic problem for the solid is given by:

$$\Pi_s = \int \left[\frac{1}{2} \int \varepsilon^T C_s \varepsilon dS - \frac{1}{2} \int \rho \dot{u}^T \dot{u} dS - \int u^{IT} f^I dI \right] dt \tag{1}$$

where:

$\boldsymbol{C}_s$ = material stress-strain matrix.

ρ_s = density of solid; u = displacement vector.

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f^I = surface (interface) force vector.

The first variation of the functional of the Eq. (1), dismembering surface forces (f^I) into fluid interface forces (f^F) and general external forces (f^E) will lead as to the Eq. (2). The Eq. (2) corresponds to the principle of virtual work for the solid.

$$\int \delta\varepsilon^T C_s \varepsilon dS + \int \rho \delta u^T \ddot{u} dS = \int \delta u^{IT} f^E dI + \int \delta u^{IT} f^F dI \tag{2}$$

The functional of the acoustic problem is given by:

$$\Pi_f = \int \left[\frac{1}{2} \int \frac{1}{\beta} \left(P_0 - \rho_f \dot{\phi} \right)^2 dF - \frac{1}{2} \int \rho_F \nabla\phi \nabla\phi dF - \int \left(P_0 - \rho_f \dot{\phi}^I \right) u_N dI \right] dt \tag{3}$$

where:

ρ_f = density of fluid.

ϕ = velocity potential in the fluid.

u_N = externally imposed displacement, normal to the fluid boundary and positive acting into the fluid.

β = bulk modulus of fluid.

P_0 = hydrostatic pressure

Doing Eq. (3) stationary in relation to the ϕ and P_0, we get:

$$\begin{aligned} &\int \frac{1}{\beta} \delta P_0 P_0 dF - \int \frac{\rho_F}{\beta} \delta P_0 \dot{\phi} dF + \int \frac{\rho_F}{\beta} \delta\phi \dot{P}_0 dF - \int \frac{\rho^2{}_F}{\beta} \delta\phi \ddot{\phi} dF - \int \rho_F \nabla\delta\phi . \nabla\phi dF \\ &= \int \delta P_0 u_N dI + \int \rho_F \delta\phi^I \dot{u}_N dI \end{aligned} \tag{4}$$

Obtaining the expressions of both domain and considering that they have the same interface I, therefore the Eq. (5) allows the coupling between these domains:

$$f^F = n \left(P_0 - \rho_F \dot{\phi}^I \right) \tag{5}$$

Replacing the Eq. (5) with Eq. (2), we obtain the final coupled equation for the solid:

$$\int \delta\varepsilon^T C_s \varepsilon dS - \int \rho \delta u^T \ddot{u} dS = \int \delta u^{IT} f^E dI - \int \delta u^{IT} n P_0 dI + \int \rho_F \delta u^{IT} n \dot{\phi}^I dI \tag{6}$$

Where the fluid displacements are related to solid displacements at the interface I:

$$u_N = n^T u^I \tag{7}$$

Applying the Eq. (7) into Eq. (4), we get to the coupling expression for the fluid:

$$\int \frac{1}{\beta}\delta P_0 P_0 dF - \int \frac{\rho_F}{\beta}\delta P_0 \dot{\phi} dF + \int \frac{\rho_F}{\beta}\delta\phi \dot{P}_0 dF - \int \frac{\rho^2{}_F}{\beta}\delta\phi\ddot{\phi} dF - \int \rho_F \nabla\delta\phi.\nabla\phi dF$$
$$= \int \frac{1}{\beta}\delta P_0 n^T u^I dI + \int \rho_F \delta\phi^I n^T \dot{u}^I dI \tag{8}$$

Therefore, the Eq. (6) and the Eq. (8) describe the fluid-structure coupling in variational terms.

2.1 Discretization used beam element

The classic beam element used to present two nodes and two degrees of freedom per node (V_1, θ_1, V_2, θ_2), as shown in Fig. 1:

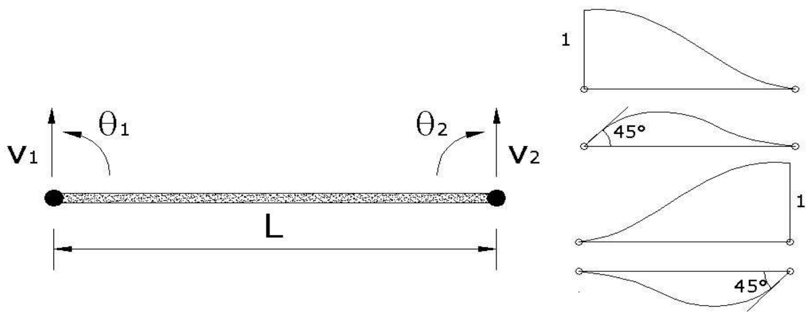

Figure 1: Beam element and shape functions.

The expression which provides the displacement in the inner part of the element is obtained by the shape functions (N) in Eq. (9):

$$V(x) = N_1 V_1 + N_2\theta_1 + N_3 V_2 + N_4\theta_2 = NV^e \tag{9}$$

The shape functions for the beam element are:

$$N_1 = \left[\frac{2x^3}{L^3} - \frac{3x^2}{L^2} + 1\right] \qquad N_3 = \left[\frac{3x^2}{L^2} - \frac{2x^3}{L^3}\right]$$
$$N_2 = \left[x - \frac{2x^2}{L} + \frac{x^3}{L^2}\right] \qquad N_4 = \left[\frac{x^3}{L^2} - \frac{x^2}{L}\right] \tag{10}$$

The vector deformation can be expressed in terms of nodal displacements, as:

$$\varepsilon = BV^e \tag{11}$$

where B is the matrix of the derivatives of the shape functions.

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The stiffness and mass matrices are given by the expressions $K_{SS} = \int B^T C_s B dV$ and $M_{SS} = \int \rho N^T N dS$, respectively. By replacing the shape function (10) in these expressions, we obtain the stiffness and mass classic matrices of a beam finite element, which are represented by Eq. (12) and Eq. (13):

$$K_{SS} = \frac{2EI}{L^3}\begin{bmatrix} 6 & 3L & -6 & 3L \\ 3L & 2L^2 & -3L & L^2 \\ -6 & -3L & 6 & -3L \\ 3L & L^2 & -3L & 2L^2 \end{bmatrix} \tag{12}$$

where:
E = Young' modulus of the element.
I = moment of inertia.
L = length of the element.

$$M_{SS} = \frac{\rho AL}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix} \tag{13}$$

where:
ρ = density of the solid.
A = cross sectional area of the element.
L = length of the element.

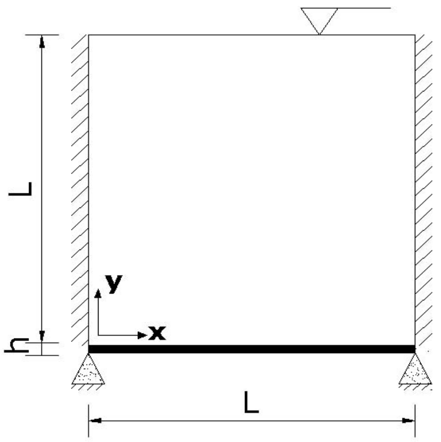
3 Results

3.1 Case 1: 2D cavity-beam with open boundary condition

Figure 2 illustrates the case 1. This example is a simple supported beam at the bottom, coupled to a square cavity of rigid side walls with an opening at the top, ie, a reservoir with rigid walls, without cover and flexible bottom. The physical and geometrical properties are showed as follows:

A) Analytical uncoupled frequencies and vibration modes for the structure

The frequencies and structure modes that describe the behavior of bending to a simple supported beam are defined by the Eq. (14) and Eq. (15), respectively:



Solid domain:

$L_x = 10m;\ b = 1m;\ h = 1m;$

$E = 2{,}1x10^{11}\dfrac{N}{m^2};\ \nu = 0{,}3;$

$\rho_s = 7800\dfrac{Kg}{m^3};$

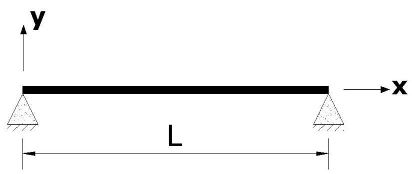
Fluid domain:

$L_Y = 10m;\ \beta = 2{,}25x10^9\dfrac{N}{m^2};$

$\rho_f = 1000\dfrac{Kg}{m^3};\ c = 1500\dfrac{m}{s};$

Figure 2: Case 1 - Open cavity with a simple supported beam at the bottom.

Description



Frequency (Hz): $$f_i = \frac{(i\pi)^2}{2\pi L^2}\left(\frac{EI}{m}\right)^{1/2} \qquad i = 1, 2, 3... \tag{14}$$

Mode: $$y = \sin\frac{i\pi x}{L} \tag{15}$$

where:
EI = bending stiffness.
m = mass of beam per unit length.
L = length of the element.

Figure 3 corresponds to the first four vibration bending modes for the simple supported beam with values of normalized displacement (y) in the direction (x).

B) Uncoupled numeric results (FEDYFE and ANSYS) for the structure

The mesh used in the discretization for the beam contains 20 elements with 21 nodes, that has two nodes for the finite element. There is a correlation between the modal deformed obtained by FEDYFE (Fig. 4) and ANSYS (Fig. 5), with their respective analytical vibration modes, as showed in Fig. 3.

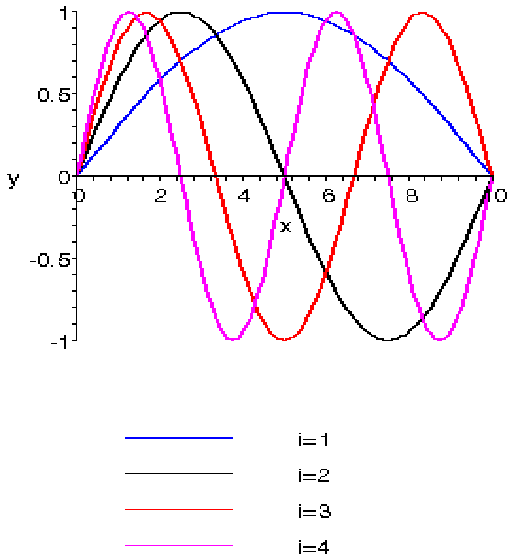


Figure 3: Analytic vibration modes for the simple supported beam.

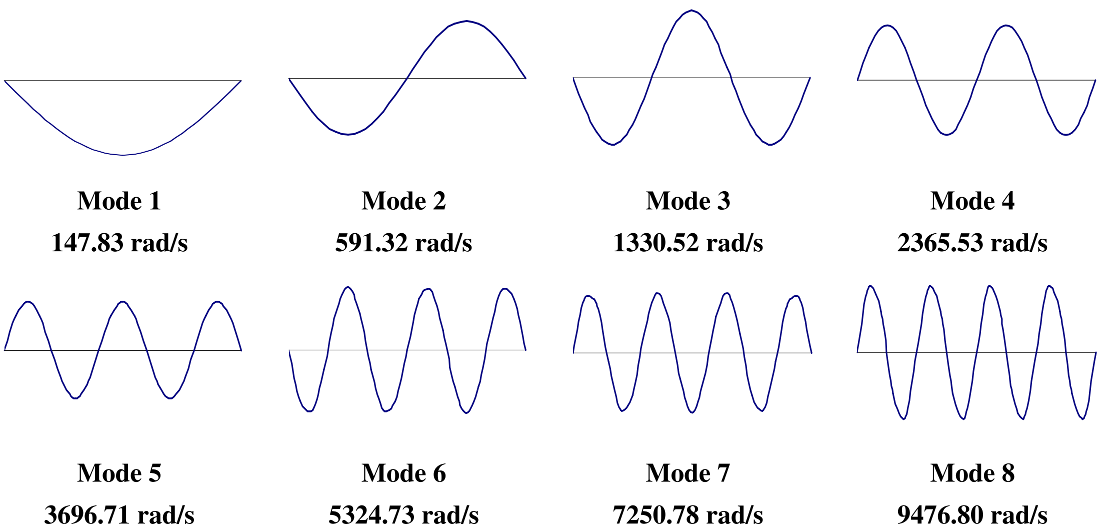


Figure 4: Numeric vibration modes for the simple supported beam obtained by the FEDYFE code.

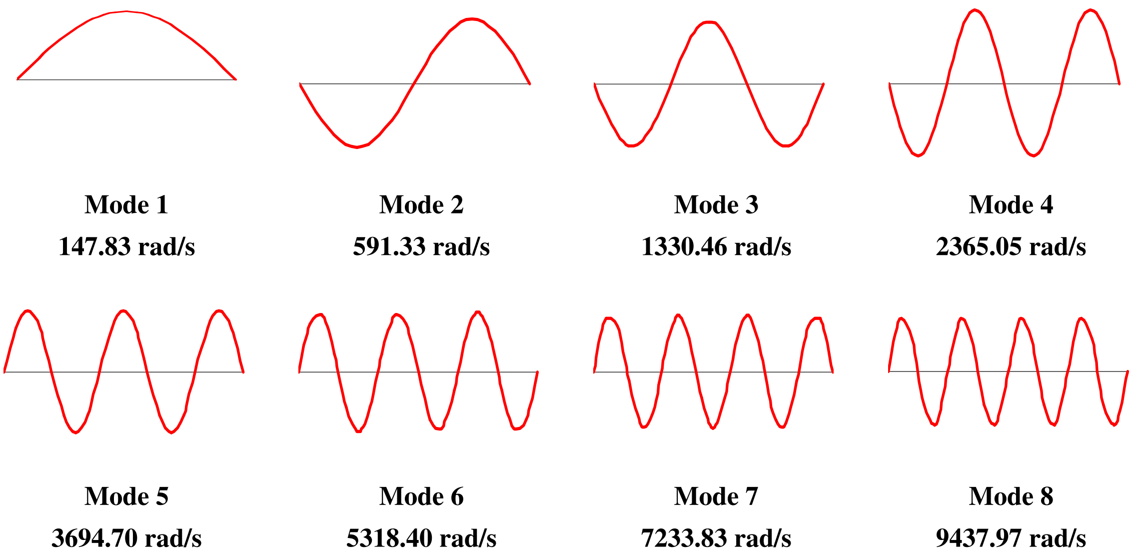


Figure 5: Numeric vibration modes for the simple supported beam obtained by the ANSYS software.

Table 1 summarizes the uncoupled numerical results for the structure obtained by FEDYFE and ANSYS softwares, as well as analytical results and the percentage of error between them.

C) Coupled numeric results (FEDYFE and ANSYS) for the Case 1

The discretization of the domain fluid consists of the 800 triangular linear elements with 441 nodes, while the beam has 20 elements with 21 nodes. In the coupled region, it used 20 interface elements with 21 nodes. Figure 6 shows the mesh used for the coupled case 1.

Table 2 shows the comparison of coupled frequencies obtained by the ANSYS and FEDYFE softwares, considering the open 2D cavity-beam problem, as illustrated in Fig. 2. Note that FEDYFE and ANSYS softwares present a good agreement in the results.

Figure 7 and Figure 8 illustrate the first five coupled modes of the 2D acoustic cavity-beam problem, respectively, got by the FEDYFE and ANSYS softwares. The frequencies and vibration modes showed a good agreement when compared with each other.

The analysis and interpretation of the modal forms associated to the fluid-structure coupled problem with acoustic cavities present some special characteristics: the appearance of modes with different nature and dominance, such as: added mass mode (MA), structure dominant modes (DE) and cavity dominant modes (DC). These modes present typical aspects of uncoupled modal deformed of the structure or the cavity, as analyzed by Souza [13].

The mode 1 represents the typical added mass mode with frequency lower (MA). In this case, the

Table 1: Analytic and numeric uncoupled frequencies for the structure in rad/s.

Mode	UNCOUPLED STRUCTURE				
	Analytic Eq. (14)	Numeric FEDYFE	Numeric ANSYS	Error (%) FEDYFE	Error (%) ANSYS
1	147.83	147.83	147.83	0.00	0.00
2	591.33	591.32	591.33	0.00	0.00
3	1330.5	1330.52	1330.46	0.00	0.00
4	2365.33	2365.53	2365.05	0.01	0.01
5	3695.83	3696.71	3694.70	0.02	0.03
6	5321.99	5324.73	5318.40	0.05	0.07
7	7243.67	7250.78	7233.83	0.10	0.14
8	9461.31	9476.80	9437.97	0.16	0.25
9	11974.48	12005.65	11922.34	0.26	0.44
10	14783.30	14841.35	14676.26	0.39	0.72

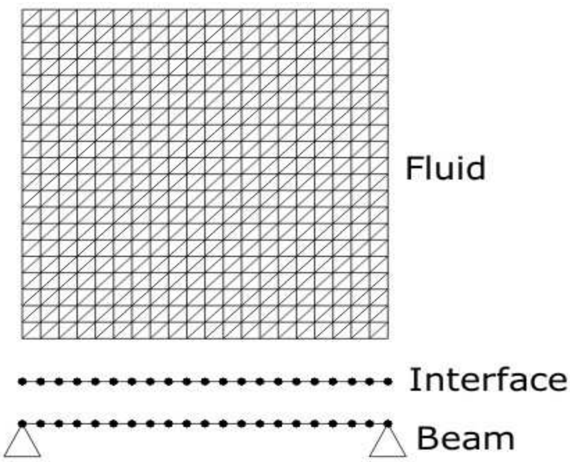


Figure 6: Mesh used in the discretization of the coupled case 1.

structure presents the first modal deformed for the cavity with incompressible fluid ($\omega_1 L/c = 0.65 < 1$). The modal deformed of the fluid presents a linear evolution.

Table 2: Comparison of coupled numeric frequencies obtained by the ANSYS and FEDYFE softwares for the Case 1.

		ω - rad/s	
	Mode	**ANSYS**	**FEDYFE**
Beam + Fluid	1	98.32	98.51
	2	315.61	316.16
	3	446.04	448.85
	4	620.58	625.67
	5	736.39	736.68
	6	885.87	889.83
	7	957.49	960.48
	8	1,127.83	1,140.48
	9	1,209.01	1,210.16
	10	1,291.76	1,296.81

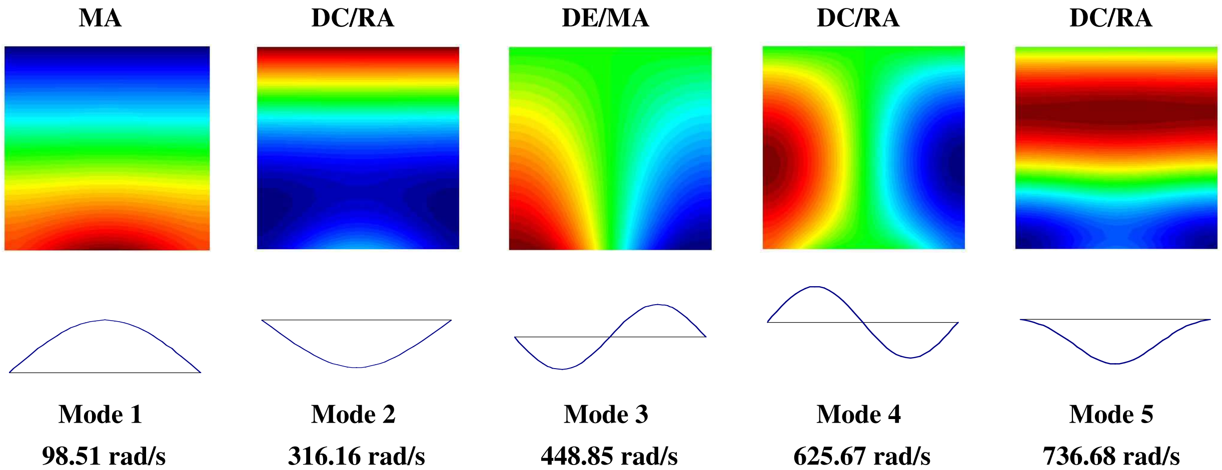


Figure 7: Representation of coupled vibration modes obtained by FEDYFE for Case 1.

The modes 2, 4 and 5 characterize the cavity modes with added stiffness (DC/RA). They reproduce the shape of the 2D uncoupled cavity modes with values higher than those uncoupled frequencies. The

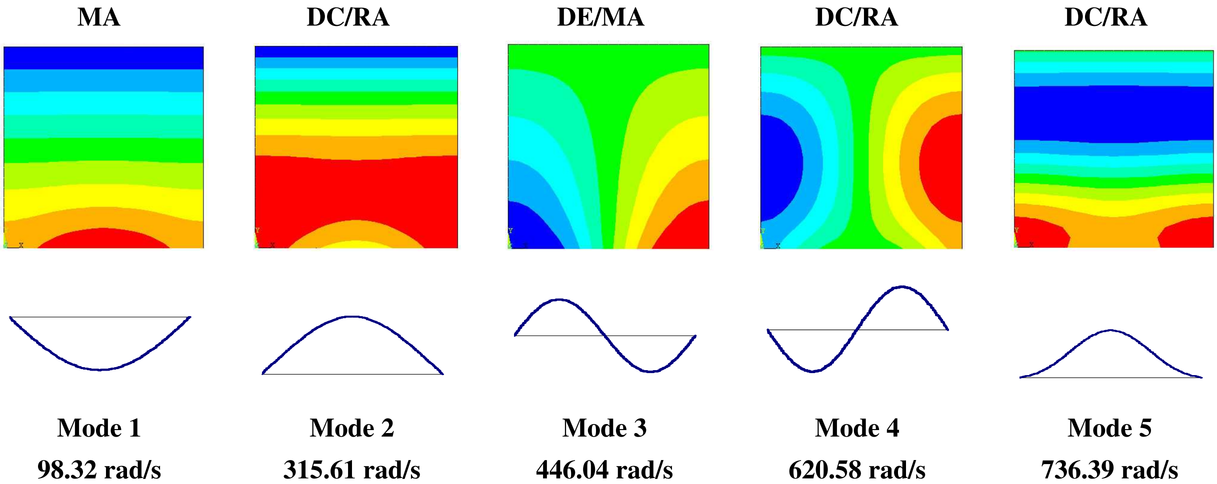


Figure 8: Representation of coupled vibration modes obtained by ANSYS for Case 1.

structure follows the modal forms of pressure, it conforms to those and follows some of its typical and well defined deformed, as shown and interpreted by Souza [13].

The mode 3 presents the characteristic of the structure dominant modes (structure modal form) with added mass (DE/MA). The coupled frequencies have values lower than uncoupled frequencies for the structure. The fluid is attached to the structure deformed. There isn't excitement of the modal forms of the cavity, because the fluid is disturbed only in the area of interface with the beam and acts with added mass in the structure.

3.2 Case 2: 2D acosutic cavity-beam with closed boundary condition

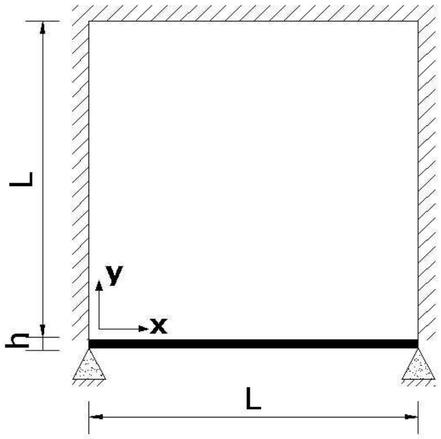
The case 2 has the same physical and geometrical characteristic of case 1, as illustrated in the Fig. 2, the exception of this example is a closing at the upper part of the cavity, as shown in Fig. 9.

A) Coupled numeric results (FEDYFE and ANSYS) for Case 2

The mesh used in the discretization for case 2 is the same mesh for case 1, as shown in Fig. 6. The difference is that model is closed at the top, while that one in case 1 is open at the top ($p = 0$).

Table 3 summarizes the coupled numerical results of the problem in Fig. 9, calculated by the ANSYS and FEDYFE softwares. There is a good correlation between these numerical results.

Furthermore, the first five frequencies correspond to Tab. 3, the FEDYFE code generates the first five coupled modes to the problem of case 2, as illustrated in Fig. 10. Thus, these modes present an excellent agreement when compared with the results provided by the ANSYS software, as illustrated in Fig. 11.



**** Solid domain:***

$$L_x = 10m;\ b = 1m;\ h = 1m;$$

$$E = 2{,}1x10^{11}\frac{N}{m^2};\ \nu = 0{,}3;$$

$$\rho_s = 7800\frac{Kg}{m^3};$$

**** Fluid domain:***

$$L_Y = 10m;\ \beta = 2{,}25x10^9\frac{N}{m^2};$$

$$\rho_f = 1000\frac{Kg}{m^3};\ c = 1500\frac{m}{s};$$

Figure 9: Case 2 - Closed cavity with a simple supported beam at the bottom.

Table 3: Comparison of coupled numeric frequencies got from the ANSYS and FEDYFE softwares for Case 2.

		ω - rad/s	
	Mode	**ANSYS**	**FEDYFE**
Beam + Fluid	1	0.0	0.0
	2	177.74	178.29
	3	431.05	431.68
	4	513.44	516.14
	5	544.13	546.76
	6	726.59	730.77
	7	930.35	931.84
	8	969.49	969.94
	9	1,038.23	1,043.74
	10	1,084.26	1,084.66

Observe that there is a zero frequency mode (low frequency) and modal deformed with constant pressure throughout the cavity, as described in zero mode (M0). As the cavity is closed and symmetric, we can easily observe the presence of this mode, which in the ANSYS software appears with a residue of low pressure in the cavity (small change in pressure).

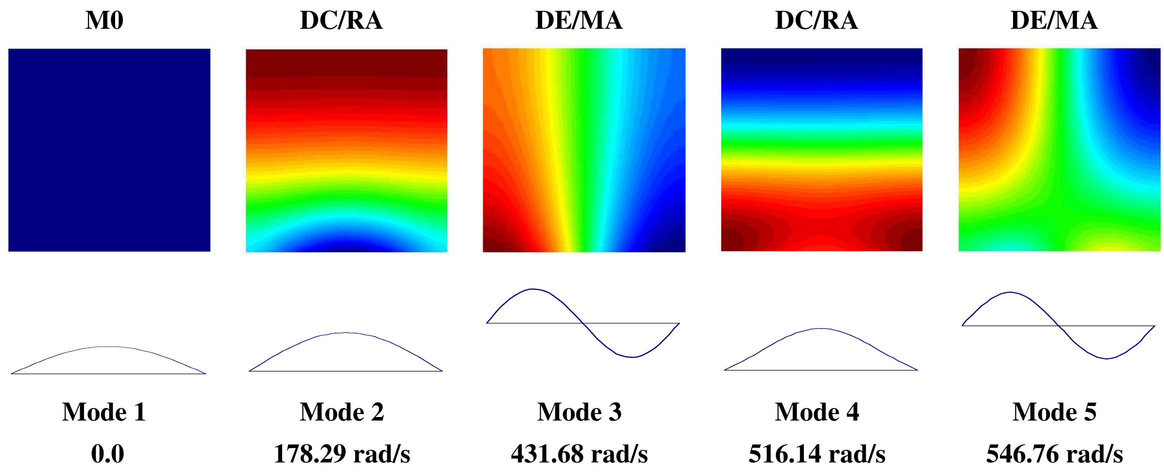


Figure 10: Representation of coupled vibration modes obtained by FEDYFE software for Case 2.

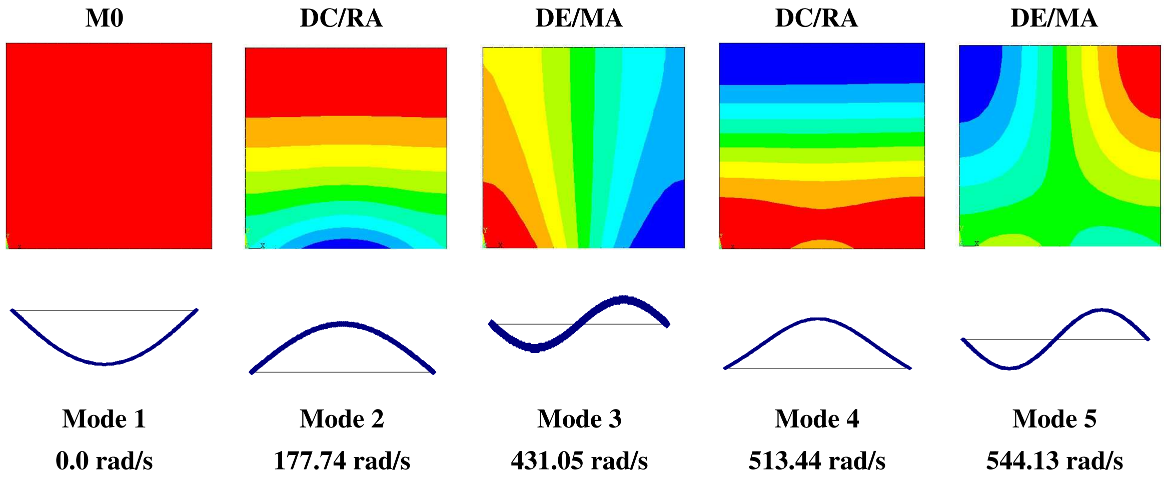


Figure 11: Representation of coupled vibration modes obtained by ANSYS software for Case 2.

Using the same methodology proposed by Souza [13], we observe the rising of modes with characteristic of the structure and the cavity. They reproduce the same typical vibration modes for the structure and cavity uncoupled problem.

The structure dominant mode with added mass (DE/MA) is characterized by disturbed fluid that acts with the effect of added mass in the structure. It has the value of coupled frequency lower than uncoupled frequency for the structure. In this problem, the modes 3 and 5 represent the structure dominant modes with added mass.

The cavity dominant mode with added stiffness (DC/RA) is characterized by frequency higher than those obtained for the uncoupled problem (cavity and structure). This is due to the increasing of stiffness in the coupled system. In this example, the DC/RA mode is represented by the modes 2 and 4. There is a clear correlation in the characteristics of coupled modes with the uncoupled cavity mode for the frequencies of the cavity. We suggest reading Souza [13], for better understanding of the methodology used to identify coupled modes.

4 Conclusion

In this work, the fluid-structure coupled problems using the beam element through the potential symmetrical formulation, obtained by the use of the FEDYFE software, showed excellent results, both for the values of the frequencies and for the fluid-structure coupled vibration modes, considering the open and closed cavities.

When a 2D acoustic cavity-beam coupled problem was used, there were modes difficult to identify (structure and cavity), because the beam element accentuates the effects of bending, which disturb the pattern of coupled deformed modal, in particular, in the cavity domain.

The frequencies and vibration modes results of the FEDYFE code showed good agreement compared with the ANSYS software, even though they have different Eulerian formulations. Therefore, these results validate the implementations made in the FEDYFE code. The interpretation and methodology used to identify the coupled vibration modes in the cavity-beam problem follow the same procedure developed by Souza (2007) for the finite element of plane stress state.

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