Crack propagation tests: analytical and numerical approaches

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Abstract

Crack propagation tests are often used to identify adhesion parameters,helping to evaluate the quality of bonded joints. The critical energy release rate G_C is one of the basics parametres that caracterizes a bonded joint. One can predict the behavior of the bonded plates during a crack propagation test by obtaining the propagation curves for a given G_C value. This paper aims to consider the usefullness of two methods for obtaing such curves. In a classic analytical approach, the adhesion between the plates is considered perfect. In such case the interface stiffness is not taken into account, where both plates behave as one when the joint is undamage and as two separated plates in the cracked zone. The second approach is numerical. The bonded interface is now consider elastic. The interface stiffness is also a parametre that caracterize the bonded joint. With the aide of the finit element code Cast3m, a diferent method is proposed to obtain the propagation curves. A good quality of fitting was achieved when both analytical and numerical based values are compared. Finally, some exemples applying numerical method are presented to show the advantage of such approach.

Keywords: bonded joints, adhesive, mechanical tests, propagation curves.

1 Introduction

In the last years, industrial use of adhesive has been growing over other conventional joining techniques. This growth can be mainly explained by the low weight among other engineering effectiveness of this type of structural joint.

Crack propagation tests are normally used to identify the adhesion parameters, helping to evaluate the quality of bonded joints [1, 2]. In classic tests, a initial crack between two bonded plates propagates when a flexure load is applied. This tests are classified by the propagation modes defined in the fracture mechanics theory. In Fig. 1, DCB (Double Cantilever Beam) and ENF (End Notched Flexure) are pure mode I and pure mode II tests, respectively. There are also some tests that combine pure propagation modes, as the MMF (Mixed Mode Flexure).

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Two diferents methods to obtain this curves are considered here. In an classic analytical approach, the adhesion between the plates is considered perfect. In such case the interace stiffness is not taken into account. Both plates behave as one when the joint is undamage and as two separated plates in the cracked zone. By applying the classic beam theory one can study the behavior of the plates during the delamination in the framework of the Linear Elastic Fracture Mechanics. This approach was originally presented by Allix, Ladeveze and Corigliano [3]. They have showed how to calculate the stability conditions for load and displacement control, wich is very important to identify snap-back problems.

The second approach is numerical. The bonded interface is now consider elastic. The interface stiffness is also a parametre that caracterize the bonded joint. The propagation curves are abtained with the aide of the finit element code Cast3M, developed by the CEA (*Commisariat a l'Energie Atomique*, France), by taking the structural response for a given value of initial crack at a time.

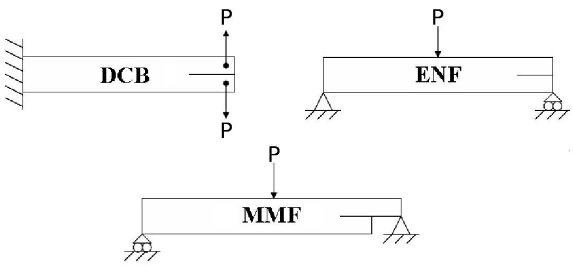

Figure 1: Crack propagation tests.

2 Analytical approach

The critical energy release rates G_c can be derived from linear beam theory using the classical Irwin-Kies expression for the fracture energy [4]:

$$G_c = \frac{P^2}{2B}\frac{dC}{da} \tag{1}$$

where P is the load applied to the specimen, B the specimen width and a the crack length.

The compliance C is defined as:

$$C = \frac{u}{P} \tag{2}$$

where u is the displacement.

Allix, Ladeveze and Corigliano [3] presents the expressions for the compliance and the propagation curves for DCB and ENF specimens (Tab. 1). In this table the expressions for a mixed mode test (MMF) were found following the same methode presented by Allix, Ladeveze and Corigliano [3]. The expressions were calculated for two beans of length L, width B, thickness H, Young's modulus E and moment of inertia of section I.

Table 1: Compliance and propagation curves for pure mode tests

Test	a	C	$f(P,u)$
DCB	$0 \leq a \leq L$	$\frac{2a^3}{3EI}$	$u = \frac{1}{3P^2}\sqrt{EI}\,(BG_c)^{3/2}$
ENF	$0 \leq a \leq L/2$	$\frac{L^3+12a^3}{384EI}$	$u = \frac{PL^3}{384EI} + \frac{16}{P^2}\sqrt{EI}\left(\frac{BG_c}{3}\right)^{3/2}$
ENF	$L/2 \leq a \leq L$	$\frac{L^3+3(L-a)^3}{96EI}$	$u = \frac{PL^3}{96EI} - \frac{16}{P^2}\sqrt{EI}\left(\frac{BG_c}{3}\right)^{3/2}$
MMF	$0 \leq a \leq L/2$	$\frac{L^3+28a^3}{384EI}$	$u = \frac{PL^3}{384EI} + \frac{112}{3P^2}\sqrt{EI}\left(\frac{BG_c}{7}\right)^{3/2}$
MMF	$L/2 \leq a \leq L$	$\frac{2L^3-7(L-a)^3}{96EI}$	$u = \frac{PL^3}{48EI} - \frac{112}{3P^2}\sqrt{EI}\left(\frac{BG_c}{7}\right)^{3/2}$

Figure 2 shows the crack propagation curves for a MMF test and the lines that mark the stability zones for load control ($0.5L < a < L$) and for displacement control ($0.261L < a < L$). The curves were obtained using the following values: Gc = 0.4 N/mm; E = 81000 MPa; L = 120 mm; B = 20 mm and H = 3 mm.

3 Numerical approach

Flexure tests can also be studied by numerical simulations [5, 6]. The simplest way to simulate a flexure test is to use a finite elements model with an elastic interface representing the adhesive. The simulations here were performed in Cast3M software developed by the CEA - (*Commisariat a l'Energie Atomique*, France). The elastic interface behavior can be simulated in this software, in which the elastic stiffness is defined by the constants $k(i)$ (i = 1,2 or 3). The use of a simple elastic interface does not allow observing the evolution of the crack automatically. It should be necessary to use a damage interface model to be able to represent a crack propagation through the interface[7]. However, when sharp snap-back problems are encountered in the structural response it's necessary to follow some special procedures to overcome it [8]. In this work we propose to obtain the propagation curves by using simple elastic interface and by taking the structural response for a given value of initial crack

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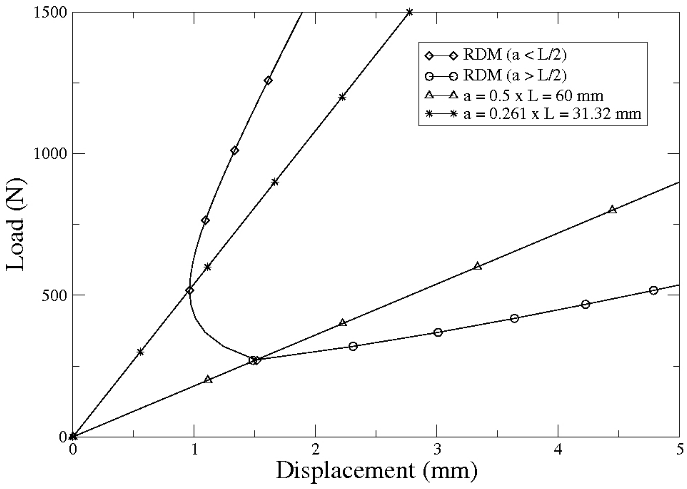

Figure 2: Crack propagation curves and stability zones for a MMF test.

at a time. Figure 3 is a scheme representing the crack evolution. The energy G necessary to make a crack propagate represented by the gray area is calculated by Eq. (3).

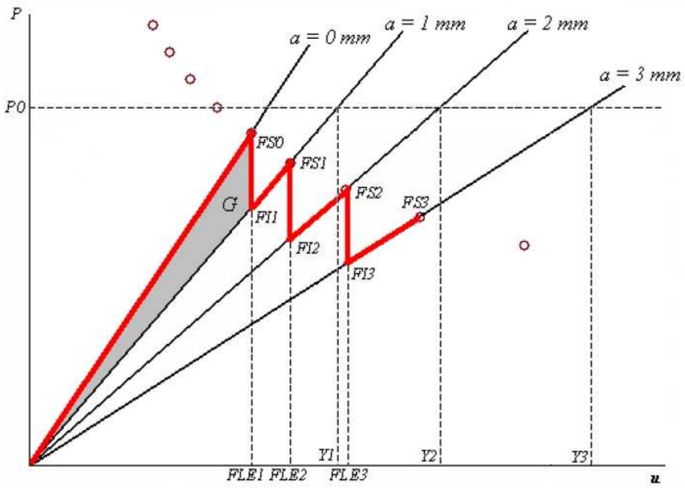

Figure 3: Crack propagation scheme.

$$G = \frac{FLE(i)}{2}\left(FS(i-1) - FI(i)\right) \tag{3}$$

Index i represents the instant observed in relation to the initial crack length a. For instance, *FLE(2)* and *FS(1)* represent the displacement and load at the time the crack advances from 1mm to 2mm, and *FI(2)* is the load at the time when energy begins to accumulate again, to make the crack advance from 2mm to 3mm. The 1mm crack step is used as a reference to simplify analysis, however, from the numerical standpoint, smaller steps can be used.

With the FE model it is possible to obtain the displacement *Y(i)* corresponding to a load *P0* applied for a given crack length a. The propagation curve for a given critical energy value G_c is the curve that contains all the *FS(i)* points. Based on Fig. 3, the following relations can be written:

$$\frac{F1}{Y(i)} = \frac{FS(i)}{FLE(i+1)} = \frac{FI(i)}{FLE(i)} = R(i) \tag{4}$$

Using Eq. (3) one can find all displacements *FLE(i)* for each value of the rigidity *R(i)*:

$$FLE(i) = \sqrt{\frac{2G_c}{R(i-1) - R(i)}}$$

A routine was performed in CAST3M to calculate loads *FS(i)* and *FI(i)* and the displacements *FLE(i)* and *Y(i)* for a given load *P0*. Figure 4 shows some examples of curves obtained in an ENF test for different values of G_c. The curves were obtained using the following values: $k(i) = 10^{17} N/m^3$; $E = 81000$ MPa; $L = 120$ mm; $B = 20$ mm and $H = 3$ mm.

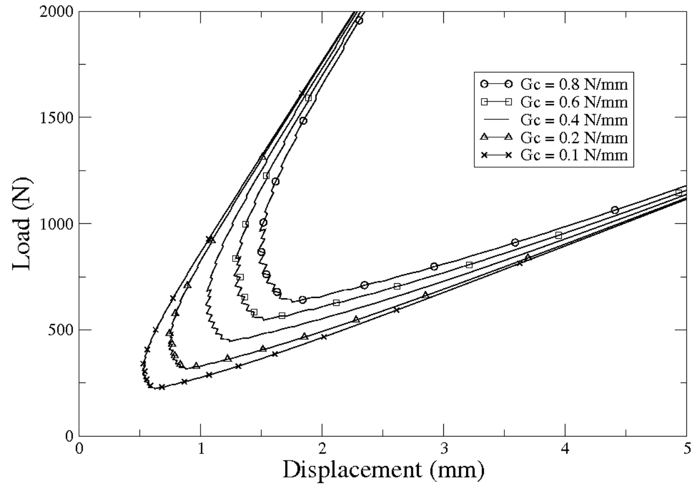

Figure 4: Crack propagation curves.

4 Comparison of results

The graph in Fig. 5 compares the numerically obtained curve to those obtained analytically for a ENF test. The two methods present very close curves, with a small difference when the crack enters the stability zone for displacement control ($a = 41.64mm$). This difference is due to the fact that the analytic method considers that the plates behave like two beams without any contact in the crack region, while the finite elements model takes the unilateral contact between the plates in the crack region into account.

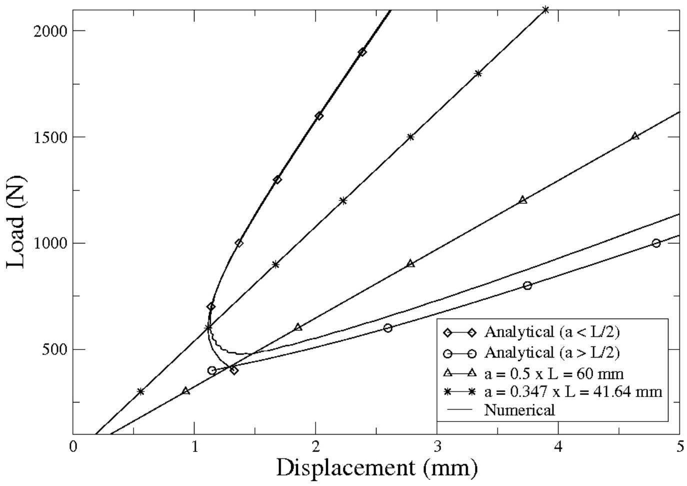

Figure 5: Analytical and numerical curves for ENF tests.

Figures 6 and 7 confirms the good agreement between the two methods now for mode I and mixed mode tests.

5 Different test geometries

More than an easier method to obtain propagation curves, the numerical method presented here is an important tool to selection of optimized adhesion tests geometries. The Tapered Double Cantilever Beam test (TDCB) is a good example of the difficulties associated with the analytical approach due to the complexity of its geometry [9]. Qiao, Wang and Davalos [10] have shown that it can be even more complex in the case of different materials for adherend and contour portions. With the numerical method presented here, it is possible to obtain the propagation curves for TDCB tests with the same material or different materials as well. One can also study the influence of the specimen geometry by changing the FE mash (Fig. 8). Figure 9 shows an example of propagation curves for TDCB tests with

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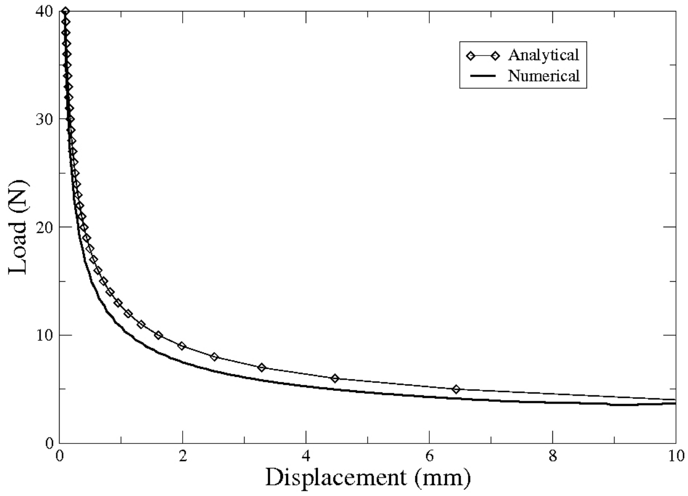

Figure 6: Analytical and numerical curves for DCB tests.

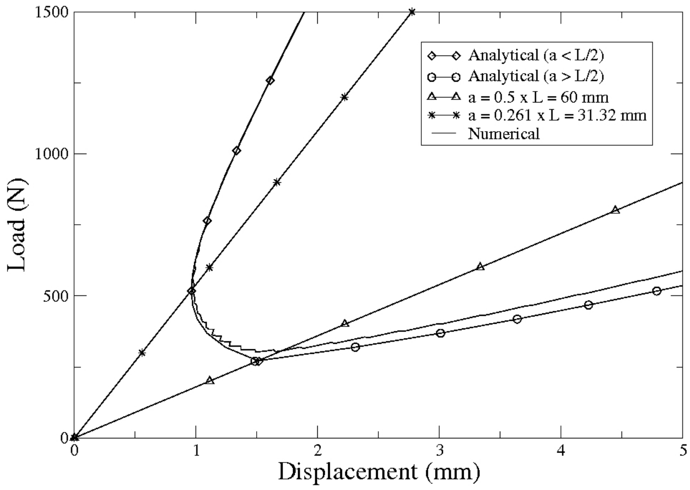

Figure 7: Analytical and numerical curves for MMF tests.

different values of the H1. The curves were obtained using the following values: $k(i) = 10^{17} N/m^3$; *E* = 81000 MPa; *L1* = 300 mm; *L2* = 40 mm; *H2* = 15 mm and width *B* = 20 mm. One can see the rigidity increasing specially for a crack lenght bigger than *L2*. Also, the closest *H1* is to *H2* the closest the curve is to a DCB test one.

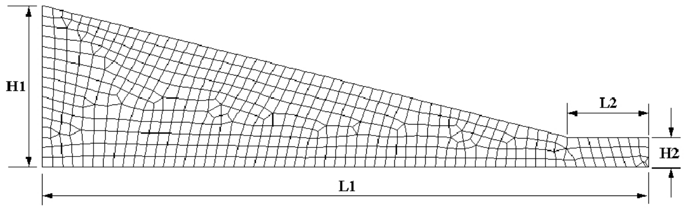

Figure 8: TDCB FE mash.

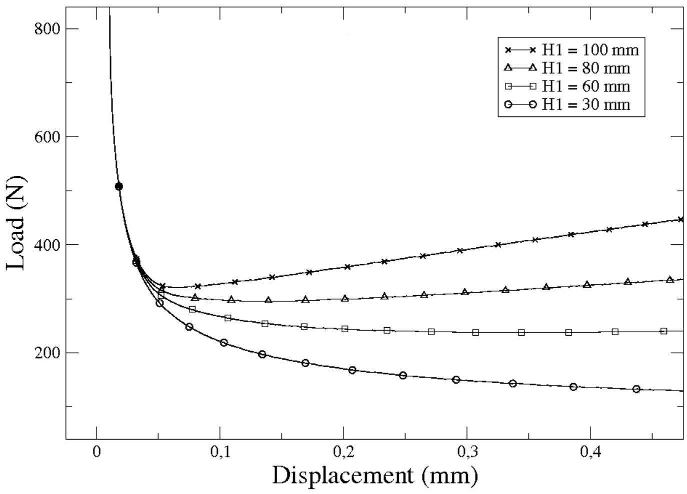

Figure 9: Crack propagation curves for TDCB tests.

6 Interface stiffness

Damage interface models usually take the interface stiffness redution as a parameter to indicate the degradation of the interface [11–14]. The problem using this models is to obtain the initial value of the stiffness. It is impossible to obtain the undamaged interface stiffness from classical mechanical tests with accuracy. Figure 10 shows the propagations curves for a ENF test by changing the interface stiffness in the FE model. For the values of $k(i)$ above $10^4 N/m^3$, one cannot see the difference because the curves are too close. Usually, the interface stiffness is higher than such value. That is why an ultrasonic method was proposed to measure it [15].

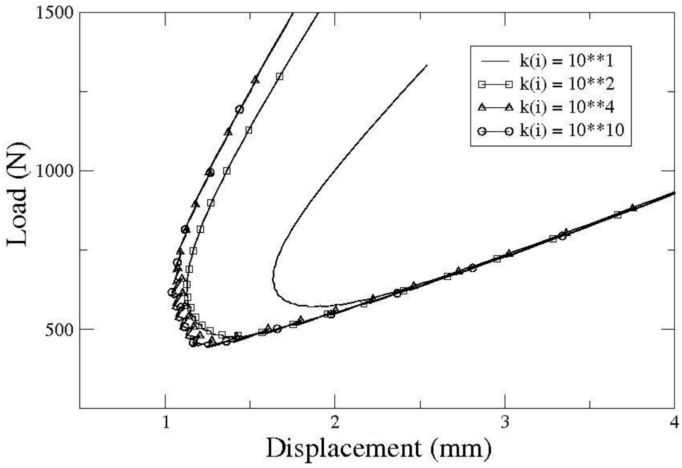

Figure 10: Influence of the interface stiffness.

7 Conclusion

The crack propagation in adhesion tests was studied in the framework of linear elastic fracture mechanics by two different and complementary approaches. The first, purely analytical, allows the equations of propagation curves to be obtained. The second is numerical and allows taking elastic interface stiffness into account. It was seen that the two approaches generate compatible results. The results obtained in an initial analysis using the methods presented here, provide an idea of the behavior of bonded plates before to perform the mechanical tests. The prior knowledge of the stability zones allows planning the test so that the crack propagation occurs in a stable manner. It can be done by using a correct value for the initial crack length. The advantage of the numerical method is that it can supply the propagation curves easier than the analytical. It can also be used for more complex geometries different from the classical tests. Depending on the specimen used in the test, the analytical calculation may take a lot of work or even be imprecise when the profiles used are no longer in the domain of the classical beam theory.

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