DEVELOPMENT OF A FORCE MODEL REPRESENTATIVE OF A MILLING PROCESS

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Abstract: *This paper considers the problems of machining tasks involving removal of material by manufacturing. Normally a machining process is performed using industrial machines, but more recently the use of robot manipulators has been reported on the literature. In such tasks, the tool position, located at the end effector of the robot manipulator, and the interaction forces between this tool and the environment, have to be simultaneously controlled. With the intention to solve this problem, one needs to have the knowledge of the robot and environment model, beyond the contact characteristics. In this context, it is presented here a simple description of the interaction forces in machining tasks involving removal of material by manufacturing, as well as the parameters of relevance to compute them. The main objective is to develop a novel mathematical model, representative of the interaction forces that arise between the tool and the environment surface. To develop this model a milling task that uses a peripheral mill as tool is considered. With the intention to validate the developed model, experimental results obtained on a milling machine are presented, which shows the expected magnitudes for the interaction forces reached at a specific milling task. These experimental results, in turn, are compared to simulation results of the developed model.*

Keywords: *machining process, milling task, interaction forces, mathematical modeling, experimental and simulation results.*

1. INTRODUCTION

This paper provides a description of the forces present in a milling process, as well as the relevant parameters for its determination, with the aim of developing a mathematical model representative of the interaction forces that arise between the tool and the environment. The description begins in section 2, where the components of the total machining force, acting on the tool and meaningful for the process, are detailed. Section 3, in turn, gives emphasis to the most significant component of the total machining force and highlights some of the models present in the literature, representative of this component, that composed the basis for the formulation of the developed model for the interaction forces, whose description begins in section 4. Section 5 presents a classic mathematical model, representative of the interaction forces, present in the literature and that was added to the developed model as one of the portions representative of the interaction forces, followed by section 6, where a full version of the developed mathematical model is exposed. Finally, in sections 7 and 8 are respectively presented some results obtained experimentally and by simulation that prove the applicability and effectiveness of the model and a list of conclusions taken with respect to this work.

2. DESCRIPTION OF THE MACHINING FORCES

Regarding the mechanics of the cutting process one has, in the general case of the machining processes, that the total force acting between the tool and the workpiece during the machining process, called the *total machining force* $\mathbf{F_u}$, is a spatial force that can be considered as consisting of:

- Geometric components resulting from the decomposition of the total force with respect to any arbitrary frame;
- Physical components, due to specific physical actions in certain directions (friction, thrust, shear etc.), whose simultaneous action produces the total force.

ISO 3002/1 (1982) states that the most important plan for defining the machining geometry is the plan containing the feedrate direction and the cutting direction of the tool, through the reference point of the cutting edge. This plan is called *work plan* and that's where all movements that contribute to the chip generation are performed. This same standard also assumes, in order to simplify the representation of the forces involved in a machining process, that they act on a single reference point, chosen on the active part of the main edge and set on the work plan.

According to the literature (FERRARESI, 1970; MANGONI, 2004; STEMMER, 2007) and to the standards governing the subject (DIN 6580, 1963; DIN 6581, 1966; ISO 3002, 1977; DIN 6584, 1982; ISO 3002/1, 1982), the total machining force can be decomposed in other force components that will depend on the current frame. In this paper the adopted force decomposition is that one sketched in Fig. 1:

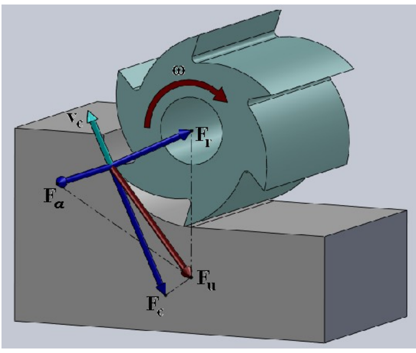

Figure 1 - Perspective of the adopted decomposition for $\mathbf{F_u}$.

whose components, among others resultant of different decompositions (CRUZ, 2010), appear in detail in Tab. 1:

Table 1 - Components of the total machining force.

$\mathbf{F_a}$	Active force (projection of $\mathbf{F_u}$ on the work plan).
$\mathbf{F_\alpha}$	Axial force (projection of $\mathbf{F_u}$ along a direction parallel to the tool's axis, taken on the reference point).
$\mathbf{F_c}$	Cutting force (projection of $\mathbf{F_u}$ on the cutting direction, given by the direction of the cutting velocity $\mathbf{v_c}$).
$\mathbf{F_f}$	Feedrate force (projection of $\mathbf{F_u}$ on the feedrate direction, given by the direction of the feedrate velocity $\mathbf{v_f}$).
$\mathbf{F_e}$	Effective cutting force (projection of $\mathbf{F_u}$ on the effective cutting direction, given by the direction of the effective cutting velocity $\mathbf{v_e}$, where $\mathbf{v_e} = \mathbf{v_c} + \mathbf{v_f}$).
$\mathbf{F_r}$	Radial force (projection of $\mathbf{F_u}$ on a direction perpendicular to the effective cutting direction).

As can be noticed from Fig. 1, the relation of forces acting on the tool is three dimensional. This makes it impossible to represent the force components in a plan, complicating the analysis of the correlations between the various components of the total machining force. To solve this problem, a particular case of orthogonal machining is adopted (FERRARESI, 1970), from which one can extract the following relation of decomposition of the total machining force, that, in this case, coincides with the active force $\mathbf{F_a}$:

$$\mathbf{F_u} = \mathbf{F_a} = \mathbf{F_c} + \mathbf{F_r} \qquad (1)$$

It is also emphasized the fact that all components of the total machining force located in the work plan contributes to the machining power $\mathbf{P_u}$ and among these components, the most significant, contributing with the largest part of the machining power, is the effective cutting force $\mathbf{F_e}$. Additionally, in the analyzed case it is verified that the effective cutting force $\mathbf{F_e}$ coincides with the cutting force $\mathbf{F_c}$, a condition fulfilled always the angle η between $\mathbf{v_e}$ and $\mathbf{v_c}$ is negligible.

Moreover, the components of the total machining force outside of the work plan may not contribute to the machining power, but this does not imply that these force components can be neglected in the process analysis and on the development of the mathematical model representative of these forces.

To determine the magnitude and direction assumed by each component of the total machining force, it was employed a piezoelectric platform, whose results are shown in section 7 in conjunction with the results obtained by simulation. The model of the piezoelectric platform used in the experiments as well as an illustrative photo of these experiments can be seen in Cruz (2010).

3. REPRESENTATIVE MODELS OF THE ACTIVE FORCE

For purposes of developing the mathematical model being proposed, representative of the interaction forces that arise between the tool and the environment, the components of the total machining force belonging to the work plan were resumed to the active force $\mathbf{F_a}$ that, as defined previously, is the projection of $\mathbf{F_u}$ on the work plan.

With no major damage, the condition set in Eq. (1) was considered to represent $\mathbf{F_a}$ in the analyzed milling process.

According to Eq. (1), this component of $\mathbf{F_u}$ can be decomposed into two other orthogonal vector components of force. Figure 2 illustrates this force decomposition with greater clarity.

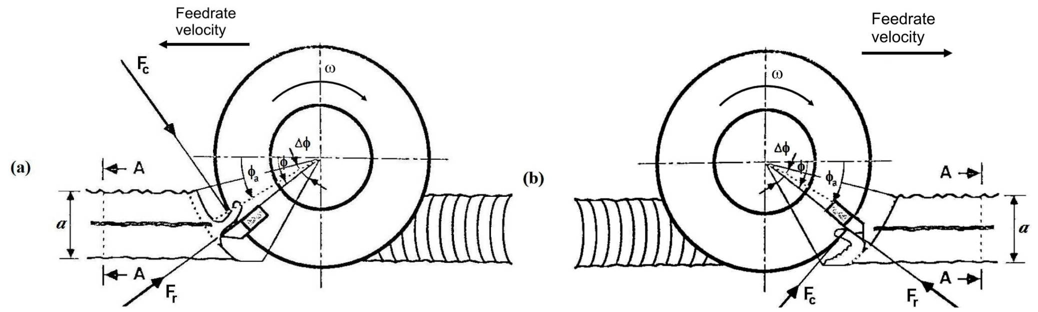

Figure 2 - Forces acting on the tool during a cutting process: (a) Up milling process, (b) Down milling process. (Adapted from Duelen *et. al.* (1992)).

According to this decomposition, it is joined to the contact point, considered unique, a cutting force $\mathbf{F_c}$ acting tangentially to the tool's surface, oriented against the direction of its angular velocity $\boldsymbol{\omega}$ and a radial force $\mathbf{F_r}$ acting in the normal direction to the tool's surface.

The magnitude F_c of the cutting force can be expressed, according to Ziliani *et. al.* (2005, 2007), as:

$$F_c = \frac{ead}{\omega} v_f \qquad (2)$$

where e represents the magnitude of the material's specific energy, a and d are respectively the magnitudes of the cutting depth and width of the cutting section, v_f is the magnitude of the feedrate velocity and ω is the magnitude of the tool's angular velocity.

This same magnitude of the cutting force can be expressed also, according to Duelen *et. al.* (1992), as:

$$F_c = K_c v_f \qquad (3)$$

with

$$K_c = \frac{K_z zad \cos\phi}{\omega r \cos\phi_{\mathrm{a}}} \qquad (4)$$

where K_z represents the magnitude of the specific cutting force, r is the tool's radius, z is the number of teeth or contact points between the tool and the environment (workpiece) cutting at the same time and ϕ and ϕ_{a} are respectively the instantaneous and average angular positions of the edge.

The magnitude F_r of the radial force, in turn, is usually assumed to be proportional to the magnitude of the cutting force and, according to Ziliani *et. al.* (2005, 2007), can be given by:

$$F_r = \tan(\beta + \gamma) F_c \qquad (5)$$

where β and γ are respectively the angles of the friction and of the tool's front chamfer.

Duelen *et. al.* (1992), in turn, establishes an empirical relationship for the magnitude of this radial component of force expressed by:

$$F_r = \xi F_c \qquad (6)$$

where ξ is the rate of the cutting force that depends of the *working conditions*, ie, workpiece (environment) and tool materials, area of the cutting section, geometry and angular position of the tool's edge, sharpening of the tool, lubrification, feedrate and cutting velocities, among others.

4. PROPOSAL OF A REPRESENTATIVE MODEL FOR THE ACTIVE FORCE

We begin this section by proposing a small change, but significant, on the coefficient K_c given in Eq. (4):

$$\overline{K}_c = \frac{\overline{K}_z zad}{\omega sign(\boldsymbol{\omega}_{|i}) r} sign(\cos\phi_{\mathrm{a}|i}) \qquad (7)$$

where $\overline{K}_z = \dfrac{K_z \cos\phi}{\cos\phi_{\mathrm{a}}}$ and the term $sign\left((*)_{|i}\right)$ indicates the sign of the quantity $(*)$ in the direction of the component

i, with $i = \vartheta^c$ ou ψ^c, both angular components of the position vector $\mathbf{x^c}$, to be defined later in Eq. (22). The value of i to be considered in Eq. (7) will depend on the orientation assumed by the tool's axis which may be parallel or perpendicular to the desired final geometry for the environment (workpiece) surface.

If, for simplification, in accordance with ISO 3002/1, it is considered that there is a single tooth or contact point of the tool with the environment (workpiece), results $z = 1$ and Eq. (7) passes to be described by:

$$\overline{K}_c = \frac{\overline{K}_z a d}{\omega sign(\boldsymbol{\omega}_{|i}) r} sign(\cos\phi_{\mathrm{a}|i}) \tag{8}$$

With no major damage to the description of the system's dynamic behavior, it is considered, as well as Duelen *et. al.* (1992), an average value to represent the behavior of ϕ $(\phi = \phi_{\mathrm{a}})$, although it is known that the depth of the burrs varies directly with that angle. As a direct result of this consideration, we also get an average value for $\mathbf{F_c}$.

In addition, as part of the proposed model, it can be verified that the active force can be represented also by a second orthogonal decomposition, where it appears a tangential component of force $\mathbf{F_t}$, exerted in the direction of the tool's feedrate path, tangent to the desired final geometry for the environment (workpiece) surface, and a normal or binormal component of force ($\mathbf{F_n}$ or $\mathbf{F_b}$), depending on the orientation assumed by the tool's axis (parallel or perpendicular to the desired final geometry for the environment (workpiece) surface), performed in a direction perpendicular to the first one, ie:

$$\mathbf{F_a} = \mathbf{F_t} + \mathbf{F_n} \tag{9}$$

or

$$\mathbf{F_a} = \mathbf{F_t} + \mathbf{F_b} \tag{10}$$

The tangential component of the active force is approximately proportional to the removal rate and to the composition of the material (KAZEROONI, 1988). In turn, the removal rate of material is a function of the product between the cutting section area, which varies with the position, and the feedrate velocity of the tool. One can therefore expect large variations of this component of the active force. It is also known that if the material of the environment (workpiece) presents a homogeneous composition, the tangential component of the active force varies proportionally just with respect to the volume of material to be removed.

Moreover, the value of the normal or binormal component of the active force varies directly with the depth of cut.

It is important to make clear that regardless of the active force be composed by a tangential component and a normal component or a tangential component and a binormal component, it will be always verified the presence of these three components in the spatial analysis of the process, even if one of them is zero. The component that is not part of the orthogonal decomposition of the active force will represent the component of the total machining force belonging to a plan perpendicular to the work plan.

Based on Eq. (1) and on the assertion made earlier that the effective cutting force $\mathbf{F_e}$ is the force component most significant to the machining power and making use of the consideration that $\mathbf{F_e} = \mathbf{F_c}$ always the angle η between $\mathbf{v_e}$ and $\mathbf{v_c}$ is negligible $(\eta = 0)$ (FERRARESI, 1970; STEMMER, 2007), one can assume, hereafter, that the active force $\mathbf{F_a}$ is equal to the cutting force $\mathbf{F_c}$, leading to Eqs. (9) and (10):

$$\mathbf{F_a} = \mathbf{F_e} = \mathbf{F_c} = \mathbf{F_t} + \mathbf{F_n} \tag{11}$$

$$\mathbf{F_a} = \mathbf{F_e} = \mathbf{F_c} = \mathbf{F_t} + \mathbf{F_b} \tag{12}$$

Thus, choosing to base the mathematical models of these vector components of $\mathbf{F_a}$ on a model similar to that of Duelen *et. al.* (1992) because of the increased number of parameters that it presents when compared to the model of Ziliani et. al. (2005, 2007), one can obtain for the tangential component:

$$\mathbf{F_t} = sen\phi_{\mathrm{a}} \overline{K}_c \mathbf{v_f} \tag{13}$$

while for the normal component, or binormal component if applicable, one gets:

$$\mathbf{F_n} = -\cos\phi_{\mathrm{a}} \overline{K}_c \mathbf{v_f} \tag{14}$$

or

$$\mathbf{F_b} = -\cos\phi_{\mathrm{a}} \overline{K}_c \mathbf{v_f} \tag{15}$$

The term $sign(\cos\phi_{a|i})$ added to the expression of $\overline{K}_c$, Eq. (7), allows Eqs. (13), (14) and (15) to represent any direction assumed by these force components, regardless of the relative motion manifested between tool and environment (down milling or up milling). Figure 3 illustrates the possibilities of occurrence, all satisfied by these equations, when considering the clockwise (positive) rotation of the tool. Consistent results are also obtained when considering the counter-clockwise (negative) rotation of the tool.

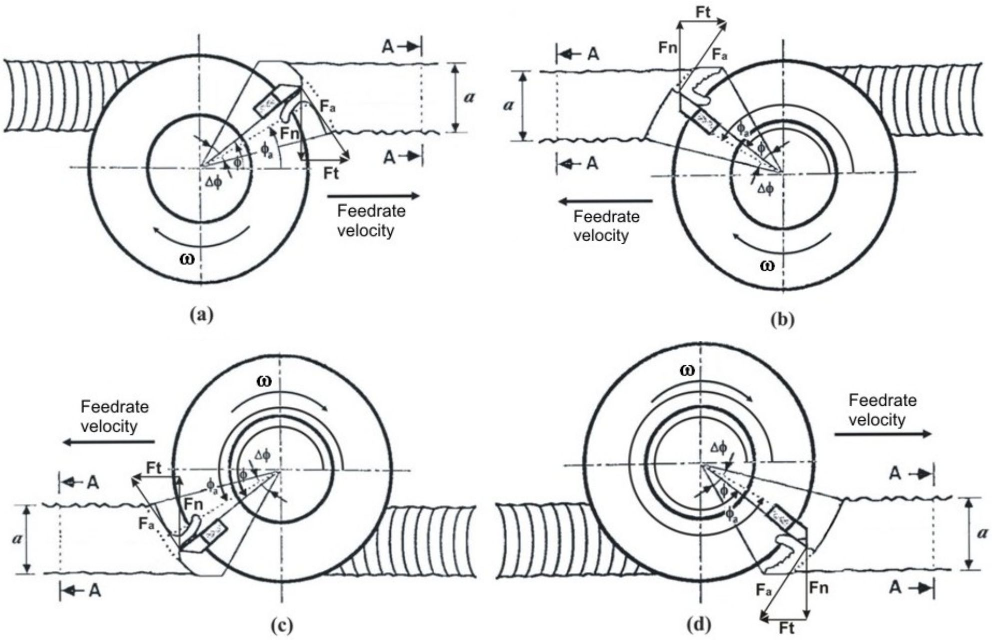

Figure 3 - Forces exerted by the tool on the environment during a cutting process when it attacks with: (a) the first quadrant, (b) the second quadrant, (c) the third quadrant and (d) the fourth quadrant. (Adapted from Duelen *et. al.* (1992)).

In case of an up milling process, as shown in Fig. 3.a and in Fig. 3.c, $\mathbf{F_t}$ will be oriented in the same direction of the feedrate velocity while $\mathbf{F_n}$, or $\mathbf{F_b}$ if applicable, will be oriented from the environment (workpiece) surface to the tool in a direction perpendicular to the feedrate direction.

The reaction forces acting on the tool have an opposite direction to these. Note that, in this case, the normal or binormal force of reaction tends to pull the tool into the environment (workpiece) surface.

On the other hand, in case of a down milling process, as shown in Fig. 3.b and in Fig. 3.d, $\mathbf{F_t}$ passes to point in the opposite direction of the feedrate velocity and $\mathbf{F_n}$, or $\mathbf{F_b}$ if applicable, will be oriented from the tool to the environment (workpiece) surface in a direction perpendicular to the feedrate direction.

The normal or binormal reaction force acting on the tool has, as mentioned earlier, the opposite direction of the normal or binormal force exerted on the environment (workpiece) and, thus, the tool tends, in this case, to depart from the environment as it is pushed by the same.

5. CLASSICAL MODEL REPRESENTATIVE OF THE INTERACTION FORCES

In Siciliano and Villani (1999) and Sciavicco and Siciliano (2004) is presented a model for the interaction forces when the environment is considered elastic and unbound. This model is described in terms of the environment's stifness and is given by:

$$\mathbf{h}_{\mathbf{Rig}} = \mathbf{K_A}(\mathbf{x})(\mathbf{x} - \mathbf{x_e}) \qquad (16)$$

where $\mathbf{K_A}(\mathbf{x})$ is a diagonal matrix of order (6×6), composed by nonnegative elements, representing the environment's stifness:

$$\mathbf{K_A}(\mathbf{x}) = diag\lfloor K_{Ax}(\mathbf{x}) \quad K_{Ay}(\mathbf{x}) \quad K_{Az}(\mathbf{x}) \quad 0 \quad 0 \quad 0 \rfloor \qquad (17)$$

and $\mathbf{x}$ and $\mathbf{x_e}$, given by:

$$\mathbf{x} = \begin{bmatrix} p_x & p_y & p_z & \varphi & \vartheta & \psi \end{bmatrix}^T \qquad (18)$$

and

$$\mathbf{x_e} = \begin{bmatrix} p_{ex} & p_{ey} & p_{ez} & \varphi_e & \vartheta_e & \psi_e \end{bmatrix}^T \quad (19)$$

are respectively the vectors of generalized coordinates of position and orientation of the end of the tool and of the undeformed and at rest environment.

All these quantities are defined in the operational space with respect to a frame fixed to a common base.

It is worth mentioning that the matrix $\mathbf{K_A}(\mathbf{x})$ can result positive semi-definite since, depending on the geometry of the environment and the tool, there may be directions in which the environment does not restrict the movement of the tool and, therefore, does not impose reaction forces in those directions.

To establish the dynamic behavior and the magnitude assumed by the components of the matrix $\mathbf{K_A}(\mathbf{x})$, it was carried out two compression tests on samples taken from the environment that provided the following values for the environment's stiffness: 420250N/m for forces ranging from 0 to 40N and 5815375N/m for forces between 50 and 200N (CRUZ, 2010).

The developed mathematical model took into account that the behavior of $\mathbf{K_A}(\mathbf{x})$ is uniform along the three directions of the operational space. Thus, all components of $\mathbf{K_A}(\mathbf{x})$ assume the same value, which will depend on the magnitude presented by the interaction forces.

In the next section it is proposed a novel mathematical model for the interaction forces present in a milling operation.

6. PROPOSAL OF A REPRESENTATIVE MODEL FOR THE INTERACTION FORCES IN A MILLING PROCESS

As previously stated, it is assumed that the forces act on a single point of the operational space, called here C (located on the workpiece). Consider a frame whose origin is that point where $\mathbf{x_t}$, $\mathbf{x_b}$ and $\mathbf{x_n}$ represent respectively the tangential direction (defined by the direction of the feedrate velocity), normal and binormal directions to the desired final geometry for the workpiece surface. The resulting frame is shown in Fig. 4:

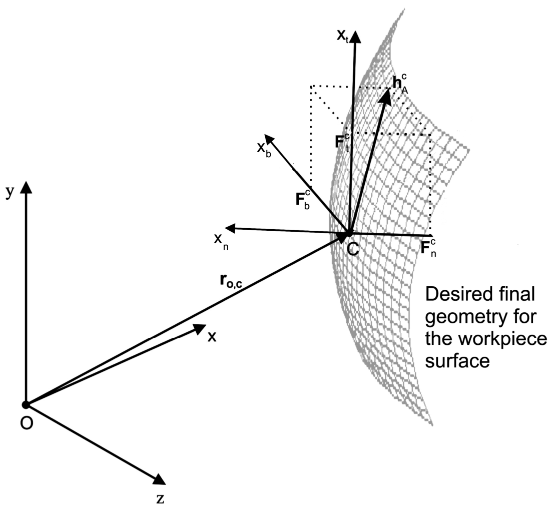

Figure 4 – Representation of the interaction forces on the $C - \mathbf{x_t x_b x_n}$ system.

In this frame, the vector of forces and moments applied by the tool is given by:

$$\mathbf{h_A^c} = \begin{bmatrix} F_t^c & F_b^c & F_n^c & M_t^c & M_b^c & M_n^c \end{bmatrix}^T \quad (20)$$

It is known, however, that forces acting on a point do not generate moments around it and, therefore, all the moments exposed in Eq. (20) are null, ie, $M_t^c = M_b^c = M_n^c = 0$.

Based on this information, the vector of forces and moments applied by the tool shall be given by:

$$\mathbf{h_A^c} = \begin{bmatrix} F_t^c & F_b^c & F_n^c & 0 & 0 & 0 \end{bmatrix}^T \quad (21)$$

In this same frame, the magnitudes of the cutting depth and width of the cutting section, represented respectively by a and d, also suffer simplifications in their notations and become $a = \left| p_b^c - p_{eb}^c \right|$ and $d = \left| p_n^c - p_{en}^c \right|$ if the tool axis is oriented in a direction perpendicular to the desired final geometry for the workpiece surface or $a=\left|p_n^c-p_{en}^c\right|$ and $d=\left|p_b^c-p_{eb}^c\right|$ if the tool axis is oriented in a direction parallel to the desired final geometry for the workpiece surface.

The terms p_b^c, p_n^c, p_{eb}^c and p_{en}^c represent respectively the position components in the binormal and normal directions to the desired final geometry for the workpiece surface of the vectors of generalized coordinates of position and orientation of the end of the tool and of the undeformed and at rest environment. These vectors are also defined with respect to this same frame (the $C-\mathbf{x_t x_b x_n}$ system) and are given respectively by:

$$\mathbf{x^c}=\begin{bmatrix}p_t^c & p_b^c & p_n^c & \varphi^c & \vartheta^c & \psi^c\end{bmatrix}^T \tag{22}$$

and

$$\mathbf{x_e^c}=\begin{bmatrix}p_{et}^c & p_{eb}^c & p_{en}^c & \varphi_e^c & \vartheta_e^c & \psi_e^c\end{bmatrix}^T \tag{23}$$

Knowing that the feedrate velocity $\mathbf{v_f}$ is set in the tangential direction of this frame, one can express it by:

$$\mathbf{v_f}=\mathbf{\dot{p}_t^c} \tag{24}$$

where $\mathbf{\dot{p}_t^c}$ is the time derivative of 1st order of $\mathbf{p_t^c}$.

Based on what was exposed, one can conclude that the resulting expressions representative of the tangential, binormal and normal components of the total machining force acting on the environment are described by:

$$\mathbf{F_t^c}=sen\phi_a\overline{K}_c\mathbf{\dot{p}_t^c}+K_{At\%}\left(\mathbf{x^c}\right)\left(\mathbf{p_t^c}-\mathbf{p_{et}^c}\right) \tag{25}$$

$$\mathbf{F_b^c}=-\cos\phi_a\overline{K}_c\mathbf{\dot{p}_t^c}+K_{Ab\%}\left(\mathbf{x^c}\right)\left(\mathbf{p_b^c}-\mathbf{p_{eb}^c}\right) \tag{26}$$

and

$$\mathbf{F_n^c}=K_{An\%}\left(\mathbf{x^c}\right)\left(\mathbf{p_n^c}-\mathbf{p_{en}^c}\right) \tag{27}$$

if the tool axis is oriented in a direction perpendicular to the desired final geometry for the workpiece surface or:

$$\mathbf{F_t^c}=sen\phi_a\overline{K}_c\mathbf{\dot{p}_t^c}+K_{At\%}\left(\mathbf{x^c}\right)\left(\mathbf{p_t^c}-\mathbf{p_{et}^c}\right) \tag{28}$$

$$\mathbf{F_b^c}=K_{Ab\%}\left(\mathbf{x^c}\right)\left(\mathbf{p_b^c}-\mathbf{p_{eb}^c}\right) \tag{29}$$

and

$$\mathbf{F_n^c}=-\cos\phi_a\overline{K}_c\mathbf{\dot{p}_t^c}+K_{An\%}\left(\mathbf{x^c}\right)\left(\mathbf{p_n^c}-\mathbf{p_{en}^c}\right) \tag{30}$$

if the tool axis is oriented in a direction parallel to the desired final geometry for the workpiece surface, noticing that the components of $\mathbf{K_{A\%}}\left(\mathbf{x^c}\right)$ that appear in these expressions represent only a percentage of the components of the original matrix $\mathbf{K_A}\left(\mathbf{x^c}\right)$, similar to that given by Eq. (17), and whose value (CRUZ, 2010) will depend on the operating conditions of the employed machining process. Due to the change of frame, it can be assumed that $K_{At}\left(\mathbf{x^c}\right)=K_{Ax}\left(\mathbf{x}\right)$, $K_{Ab}\left(\mathbf{x^c}\right)=K_{Ay}\left(\mathbf{x}\right)$ and $K_{An}\left(\mathbf{x^c}\right)=K_{Az}\left(\mathbf{x}\right)$.

Using a matrix notation, the proposed force model can also be written as:

$$\mathbf{h_A^c}=\begin{bmatrix}F_t^c\\F_b^c\\F_n^c\\M_t^c\\M_b^c\\M_n^c\end{bmatrix}=\begin{bmatrix}sen\phi_a\overline{K}_c & 0&0&0&0&0\\-\cos\phi_a\overline{K}_c & 0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\end{bmatrix}\begin{bmatrix}\dot{p}_t^c\\\dot{p}_b^c\\\dot{p}_n^c\\\dot{\varphi}^c\\\dot{\vartheta}^c\\\dot{\psi}^c\end{bmatrix}+\begin{bmatrix}K_{At\%}\left(\mathbf{x^c}\right)&0&0&0&0&0\\0&K_{Ab\%}\left(\mathbf{x^c}\right)&0&0&0&0\\0&0&K_{An\%}\left(\mathbf{x^c}\right)&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\end{bmatrix}\begin{bmatrix}p_t^c-p_{et}^c\\p_b^c-p_{eb}^c\\p_n^c-p_{en}^c\\\varphi^c-\varphi_e^c\\\vartheta^c-\vartheta_e^c\\\psi^c-\psi_e^c\end{bmatrix} \tag{31}$$

if the tool axis is oriented in a direction perpendicular to the desired final geometry for the workpiece surface or:

$$\mathbf{h}_\mathbf{A}^\mathbf{c}=\begin{bmatrix}F_t^c\\F_b^c\\F_n^c\\M_t^c\\M_b^c\\M_n^c\end{bmatrix}=\begin{bmatrix}sen\phi_a\overline{K}_c & 0&0&0&0&0\\0&0&0&0&0&0\\-\cos\phi_a\overline{K}_c&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\end{bmatrix}\begin{bmatrix}\dot{p}_t^c\\\dot{p}_b^c\\\dot{p}_n^c\\\dot{\varphi}^c\\\dot{\vartheta}^c\\\dot{\psi}^c\end{bmatrix}+\begin{bmatrix}K_{At\%}(\mathbf{x^c})&0&0&0&0&0\\0&K_{Ab\%}(\mathbf{x^c})&0&0&0&0\\0&0&K_{An\%}(\mathbf{x^c})&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\end{bmatrix}\begin{bmatrix}p_t^c-p_{et}^c\\p_b^c-p_{eb}^c\\p_n^c-p_{en}^c\\\varphi^c-\varphi_e^c\\\vartheta^c-\vartheta_e^c\\\psi^c-\psi_e^c\end{bmatrix} \quad (32)$$

if the tool axis is oriented in a direction parallel to the desired final geometry for the workpiece surface.

Regardless of the orientation assumed by the tool axis with respect to the desired final geometry for the workpiece surface, the proposed force model can be rewritten in a simplified form as:

$$\mathbf{h}_\mathbf{A}^\mathbf{c}=\mathbf{C}_\mathbf{A}\left(\mathbf{x}_\mathbf{e}^\mathbf{c},\mathbf{x}^\mathbf{c},\dot{\mathbf{x}}^\mathbf{c}\right)\dot{\mathbf{x}}^\mathbf{c}+\mathbf{K}_{\mathbf{A}\%}\left(\mathbf{x}^\mathbf{c}\right)\left(\mathbf{x}^\mathbf{c}-\mathbf{x}_\mathbf{e}^\mathbf{c}\right) \quad (33)$$

where $\mathbf{C}_\mathbf{A}\left(\mathbf{x}_\mathbf{e}^\mathbf{c},\mathbf{x}^\mathbf{c},\dot{\mathbf{x}}^\mathbf{c}\right)$ is a matrix, function of the system state variables, that appears post multiplied by the vector of generalized coordinates of velocity, ie, the time derivative of 1st order of Eq. (22), and which, working together, gives the first part of the right side of Eqs. (31) and (32) that describes a portion of the relationship between $\mathbf{h}_\mathbf{A}^\mathbf{c}$ and the tool's dynamics, representative of the viscous behavior assumed by the system. $\mathbf{K}_{\mathbf{A}\%}\left(\mathbf{x}^\mathbf{c}\right)$, in turn, as mentioned earlier, is related to a percentage matrix of the original matrix $\mathbf{K}_\mathbf{A}\left(\mathbf{x}^\mathbf{c}\right)$, similar to that given by Eq. (17), and describes the other portion of the relationship between $\mathbf{h}_\mathbf{A}^\mathbf{c}$ and the tool's dynamics, representative of the elastic behavior assumed by the system. Finally, the vectors $\mathbf{x}^\mathbf{c}$ and $\mathbf{x}_\mathbf{e}^\mathbf{c}$ are given by Eqs. (22) and (23).

Due to the non-linear and multivariable characteristic of the $\mathbf{C}_\mathbf{A}\left(\mathbf{x}_\mathbf{e}^\mathbf{c},\mathbf{x}^\mathbf{c},\dot{\mathbf{x}}^\mathbf{c}\right)$ matrix, one can notice, based on Eq. (33), that the force model proposed to represent the interaction between tool and environment also results non-linear.

Finally, Tab. 2 highlights the main differences between the model being proposed and three of the models found in the literature, namely, the models employed by Duelen *et. al.* (1992), Ziliani *et. al.* (2005, 2007), Siciliano and Villani (1999) and Sciavicco and Siciliano (2004).

Table 2 – Comparative analysis of representative models of the interaction forces in surface machining processes.

	Viscous behavior	Elastic behavior	Spatial analysis of the force	Feedrate velocity	Machining force
Proposed model	$\mathbf{C}_\mathbf{A}\left(\mathbf{x}_\mathbf{e}^\mathbf{c},\mathbf{x}^\mathbf{c},\dot{\mathbf{x}}^\mathbf{c}\right)\dot{\mathbf{x}}^\mathbf{c}$	$\mathbf{K}_{\mathbf{A}\%}\left(\mathbf{x}^\mathbf{c}\right)\left(\mathbf{x}^\mathbf{c}-\mathbf{x}_\mathbf{e}^\mathbf{c}\right)$	2D or 3D	Constant	Limited
Duelen *et. al.* (1992)	$\frac{K_z zad\cos\phi}{\omega r\cos\phi_a}v_f$		2D	Variable	Limited
Ziliani *et. al.* (2005, 2007)	$\frac{ead}{\omega}v_f$		2D	Variable	Limited
Siciliano and Villani (1999) and Sciavicco and Siciliano (2004)		$\mathbf{K}_\mathbf{A}(\mathbf{x})(\mathbf{x}-\mathbf{x}_\mathbf{e})$	2D or 3D		

Analyzing Tab. 2, one can notice that all models, except the proposed model, choose to represent the interaction forces taking into account that all the effects have the same nature, or of a viscous behavior (viscous friction) or of an elastic behavior (elastic deformation of the system based on the environment's stiffness and considering a rigid tool). However, what actually happens is that the two portions of distinct nature coexist and, depending on the tool (mill) and on the operating conditions employed, become more or less significant with respect to one another.

Moreover, among the models shown in Tab. 2, the proposed model is one of the few that allow to perform a spatial analysis (3D) of the interaction forces.

There are still some effects, not listed in Tab. 2, caused by the terms added to the model presented in Duelen *et. al.* (1992), namely, $sign(\cos\phi_{a|i})$ and $\omega sign(\boldsymbol{\omega}_{|i})$. The term $sign(\cos\phi_{a|i})$ added to the expression of $\overline{K}_c$, Eq. (7), in order to induce direction to the force components and described with respect to the tool's active quadrant, ie, the quadrant that maintains contact with the environment, allows Eqs. (13), (14) and (15) to represent any direction assumed by these force components, regardless of the relative motion manifested between tool and environment (down milling or up milling). In turn, the term $\omega sign(\boldsymbol{\omega}|_i)$, that appears replacing the term ω in Eq. (7), is employed in order to take into account the orientation of the tool's rotation.

Finally, unlike the models of Duelen *et. al.* (1992) and Ziliani *et. al.* (2005, 2007) that adapt the tool's feedrate velocity to the force limits considered tolerable to the subsystem composed by the tool, in the proposed model one has early access to an estimate of the expected magnitudes for the components of the total machining force, obtained on the basis of pre-selected values for the working conditions and of reference values for the system state variables, allowing the designer to abort the process if any of these components of the total machining force exceeds the allowed magnitude. This procedure guarantees not only the integrity of the tool but also of the workpiece being subjected to the employed machining process, since a variation of the feedrate velocity, which, in this case, coincides with one of the system state variables, could affect the final quality of the workpiece.

Based on what was discussed, it can be noticed that the proposed model covers a larger number of situations and possible effects that may show up, becoming more expressive than the other models with which it was compared.

7. RESULTS

As presented throughout this article, the developed force model represents the interaction forces that show up as a result of performing a milling task, regardless of the orientation assumed by the tool axis with respect to the desired final geometry for the workpiece surface.

Both, the experiments and the developed algorithm, consider the behavior of a peripheral mill, ie, a carbide-tipped cutter as the tool being employed. This tool is described by Fig. 5.

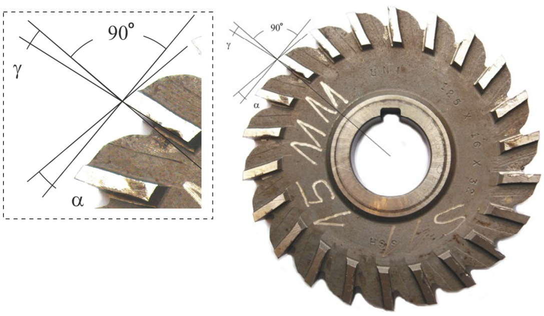

Figure 5 – Peripheral mill.

where the outside diameter has $\phi 125{,}00mm$, the internal diameter has $\phi 32{,}00mm$, the width of the tool is $16{,}00mm$, the departure angle is $(\gamma) = 8^o$ and the incidence angle is $(\alpha) = 10^o$.

It was chosen to orient the axis of this mill parallel to the desired final geometry for the workpiece surface because it is the usual orientation adopted for this tool.

Finally, it is worth mentioning that an up milling process was employed in the performed experiments and simulations, though the developed force model is also prepared to represent the behavior of the force components when a down milling process is employed.

The performed experiments (CRUZ, 2010) provided the behavior shown in Fig. 6(a).

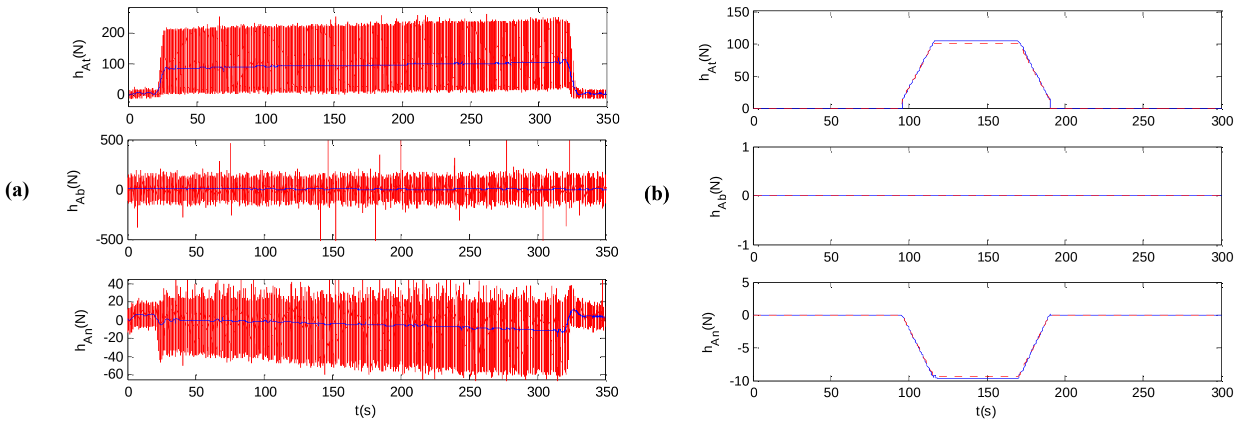

Figure 6 – (a) Force components obtained experimentally; (b) Force components obtained by simulation.

The mathematical model representative of the involved forces is that presented in section 6, which provides the behavior displayed in Fig. 6(b).

Note that the behavior assumed by each of the force components shown in Fig. 6(b) is consistent with the actual force component obtained experimentally and shown in Fig. 6(a).

8. CONCLUSIONS

The goal of this paper lies on the analysis of the forces acting on the environment (workpiece) and of the respective components of the reaction force arising on the cutting tool (mill) while performing a milling task. Based on this analysis, it has been developed a mathematical model representative of these forces. This model was developed from existing models with which it is compared in terms of range in describing the effects of the interaction forces.

To ensure that this model consistently represents the forces present in the task being performed, it is necessary to have an understanding of the overall behavior assumed by some of its parameters and coefficients, namely: K_z, $\mathbf{K}_{A\%}(\mathbf{x})$ and $\overline{K}_c$. The first two are related respectively to the magnitude of the specific cutting force and to an percentage matrix of the original stiffness matrix $\mathbf{K}_A(\mathbf{x}^c)$ and the latter one is a proportionality factor. All of them can be variable and are present in the calculation of other parameters that establish the relationships between the force components and the tool's dynamics. In this paper these coefficients were considered constant, whereas, in reality, they represent the unmodeled dynamics of the interaction forces. The results needed to describe the global behavior of K_z, $\mathbf{K}_{A\%}(\mathbf{x})$ and $\overline{K}_c$ can only be achieved through a statistical and behavioral analysis of the developed force model and this task remains as a proposal for future work.

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10. RESPONSIBILITY NOTICE

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