APPLICATION OF NURBS FOR OBTAINING FREE FORM CURVES AND SURFACES BY USING COORDINATE MEASURING MACHINES (CMMs)

Esly César Marinho da Silva, eslymarinho@ifba.edu.br
Instituto Federal de Educação da Bahia – IFBA, Via Universitária, s/n. Pitanguinha. CEP 43700-000 - Simões Filho – BA, Brasil

Igor Lucena Peixoto Andrezza, igorlpa90@hotmail.com
Verílton Nunes da Silva, verilton@yahoo.com.br
Luiz Henrique Melo Silva Nóbrega, rikinobrega@hotmail.com
José Carlos de Lima Júnior, limajrcarlos@hotmail.com
João Bosco de Aquino Silva, jbosco@ct.ufpb.br
Universidade Federal da Paraíba, Centro de Tecnologia, Bloco "F" – Campus I – CEP 58.059-900 – João Pessoa – PB, Brasil

Abstract. *The increasing demand for products with high accuracy and geometric dimensional measures are required high accuracy measurement system. Additionally, parts of complex geometry and free form parts have been a common practice in industries such as: automotive, aeronautics, bioengineering, among others. It is necessary inspection systems more accurate and flexible to meet these demands. The Coordinate Measuring Machines (CMMs) are an important tool in the design, fabrication and inspection of manufactured products. These machines are used by the engineer, whose main purpose is to produce an accurate digital model in a virtual space for later use in CAD / CAE / CAM / CAI. It is known, however, that the accuracy of the modeling process of a workpiece depends on the number of points which are measured on the surface of the inspected part. Thus, CMMs which incorporated Laser system are the best tool for reverse engineering process. However, this type of machine is very expensive. CMMs that use touch trigger probe are less expensive but they require considerable time to get a lot of points. This paper present an approach based on NURBS (Non-Uniform Rational B-Splines) for obtaining free form curves and surfaces from a set of points (Xi, Yi, Zi) collected on the part to be measured. NURBS are an important mathematical tool and consist of generalizations of Bezier curves and surfaces B-splines. The NURBS present many advantages, simplicity and facility of data handling, which tends to minimize the problems of randomness and inaccuracy of the cloud of points obtained by the CMM. Thus, the NURBS can contribute for modeling free-form curve and surfaces and is very useful in reverse engineering applications. Simulations results showed the effective of the proposed approach. Additionally, experimental results demonstrated that it is of practical use, non time consuming and an alternative way to apply CMMs that incorporated touch trigger probe in modeling processes.*

Keywords: CMM, Inspection, free form surfaces

1. INTRODUCTION

In general, inspection operations of workpieces and products in the industries have become essential to reach the quality and reliability. All of them can be done manually or by computerized machines. This last one is more used to achievements more complex operations, which the execution carried out by a human being would mean a greater use of time and can occur various errors. Nowadays, stringent inspections and accurate are needed, mainly when it refers to workpieces for been used in the aeronautical industries, space, medicine etc, and therefore, it is considered a major ally to conduct inspections of complex surfaces, the Coordinate Measuring Machines (CMMs), Silva (2009). Not only for the inspection the CMMs are used, but also to reverse engineering, that consist to produce new workpieces, products or tools from existing components, Varady *et al* (1997). These machines are be able to measure a large number of points that represents characteristics of complex components, often by performing a simple operation, Silva (2002). The CMMs contributions reduce inspection time and allow us to identify errors resulting the manufacturing process with greater efficiency and speed.

Over the past few years has been developed several techniques for acquisition three dimensional information using contact or non-contact measure sensor. The accuracy inspection has been widely developed in the industrial field and, actually, the survey that has been highlight is about the measurement of free surfaces and complex shapes. According Li and Gu (2004) for measures of regular surfaces the use of the CMMs' touch trigger probe can reach better results like tolerance and accuracy. In the other hand, the use of non contact measurement technologies are widely used as an important tool for measuring free formats surface or complex ones.

Among different techniques for data acquisition, the following can be highlighted: echos detection, photogrammetry, laser, etc. A major disadvantage of this type of performing measurements is the high cost of the equipment, which makes it impossible for the acquisition of the majority of industries and/or companies that work with measurement technology. At the moment, what can be realized in major of companies is the use of CMMs' contact probe due to its great versatility of operations and low cost when compared to non-contact measuring system.

Following the changes in the shapes of products on the environment metrological is not an easy task, it must use a specific software like a tool for design. The modeling can be considered as accurate and detailed creating of the free and

complex forms. These are considered free forms when they are different from models with known characteristics and geometry, for example, planes, circles, cylinders, etc. Actually, CAD systems placed in most measuring equipment working to international standards for representations graph, design and information exchange of geometric data processed by computer. The Coordinate Measuring Machines have an own CAD that it are able to identify, visualize and interpret the data collected and thus characterize known geometries such as circles, cylinders, etc. For measurements of regular surfaces with CMMs, the results are obtained with narrow ranges of tolerances and accuracy, but for free form workpiece the inspections become more complicated. For this reason there has been increasing the number of searches for data acquisition of surfaces of complex geometry using CMMs or some other equipment. According to Li and Gu (2004), It is considered complex surface the format presented by piece and it has not the features of known geometry, within the mathematical analysis, but defined by control points through Bélzier, B-spline, NURBS (Non Uniform Rational B-splines) techniques, etc. The tool for reconstruction and acquisition of surfaces used in this paper was the NURBS, which it has great advantages such as simplicity and easy data manipulation, representation of the analytical forms and free-forms, processing time is fairly small, are generalizations of Bélzier and B-splines curves and surfaces, can represent complex shapes with few data, (Piegl, 1997, Aquino e Carvalho, 2003, Minneto, 2003).

Then, this paper aimed to present the results of a methodology associated with the use of the NURBS tool, modeling a free-form surface, i.e., with no know geometry, and the use of CMMs' touch trigger probe in order to meet the design specifications for inspection and reverse engineering.

2. CURVES AND SUFACES NURBS

Although B-spline are widely used in curves design, they cannot represent curves of simple shapes, as in the case of circles. Therefore, to represent shapes such as circles, ellipses, etc., it is necessary an extension of B-splines curves. For solving these problems is necessary to generalize B-spline curves using rational homogeneous coordinates, which it is called Non-Uniform Rational B-splines, NURBS, Minetto (2003).

Mathematically, a NURBS curve of n degrees can be defined by Eq. (1) below,

$$C(u) = \frac{\sum_{i=0}^{n} N_{i,p}(u)\omega_i P_i}{\sum_{i=0}^{n} N_{i,p}(u)\varpi_i}, \qquad \text{com } a \leq u \leq b \tag{1}$$

Where $\{P_i\}$ are the control points of the curve, $\{N_{i,p}\}$ are the B-spline basic functions of p degree and each real value ω_i is a weigth associated of the control point P_i. It is known that rational functions are defined by Eq. (2),

$$R_{i,p}(u) = \frac{N_{i,p}(u)\omega_i}{\sum_{j=0}^{n} N_{j,p}(u)\omega_j} \tag{2}$$

Therefore, from Eq. (1) and (2), the equation of the NURBS curve can be represented by Eq. (3) below.

$$C(u) = \sum_{i=0}^{n} R_{i,p}(u)P_i, \qquad \text{com } a \leq u \leq b \tag{3}$$

According Piegl (1997) an elegant way to represent a rational curve or surface by using coordinate functions are using homogeneous coordinates that facilities the compression of the stored data and computational processing. In the three-dimensional Euclidean space the coordinates can be defined as $P = (x,y,z)$, but the space where the coordinates are homogeneous, see Fig. 1, they are being written as:

$$P^w = (wx, wy, wz, w) \tag{4}$$

Where w are the weights.

In this case, for the NURBS definition is necessary to keep the control point as a column vector, written with four components, as shown in Eq. (5) below:

$$P_i^w = (w_i x, w_i y, w_i z, w) \tag{5}$$

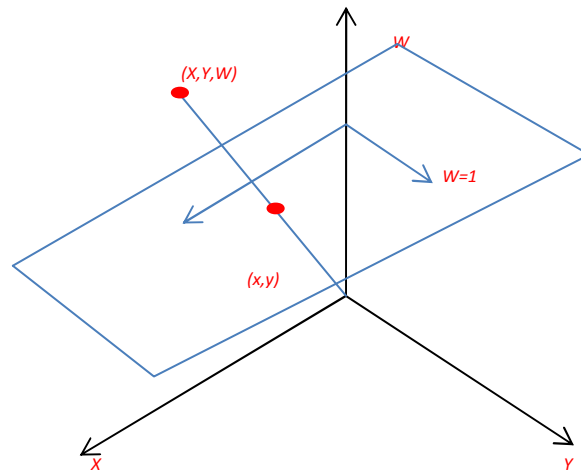Figure1. Representation of points in Euclidean space to homogeneous coordinates

The representation of NURBS curves is very similar with the development of parametric representation of NURBS curves. Considering a NURBS surface of p degree in direction u and q degree in direction v is defined by a two-dimensional system of control points $P_{i,j}$ with their respective weights $\omega_{i,j}$ and B-spline basic functions. The general expression to represent a NURBS surface is given by Eq. (6),

$$s(u,v) = \frac{\sum_{i=0}^{n}\sum_{j=0}^{m} N_{i,p}(u)N_{j,q}(v)\omega_{i,j}P_{i,j}}{\sum_{i=0}^{n}\sum_{j=0}^{m} N_{i,p}(u)N_{j,q}(v)\omega_{i,j}}, \quad 0 \leq u, v \leq b \tag{6}$$

The major of the operations to be done on these surfaces can occur first in one direction, obtaining a curve as a result and then do them in the other direction, thereby reducing the complexity of working directly with the surface studied. The Eq. (6) can be rewritten as follows:

$$s(u,v) = \sum_{i=0}^{n}\sum_{j=0}^{m} R_{i,j}(u,v)P_{i,j} \tag{7}$$

Where,

$$R_{i,j}(u,v) = \frac{N_{i,p}(u)N_{j,q}(u)\omega_{i,j}}{\sum_{i=0}^{n}\sum_{j=0}^{m} R_{i,j}(u,v)P_{i,j}} \tag{8}$$

In general, an expression to represent the NURBS surfaces is given by Eq. (9):

$$s^{\omega}(u,v) = \sum_{i=0}^{n}\sum_{j=0}^{m} N_{i,p}(u)N_{j,q}(v)P_{i,j}^{\omega} \tag{9}$$

3. GENERATION OF CURVES AND SURFACES USING NURBS

With the competition between industries, the products offered tend to have several characteristics to attract new customers, with no loss of functionality, quality and cost. Thus, the shapes and designs of the products such as automotive, aeronautical etc, tends to be innovative, with complex geometrical shapes and high efficiency. Following the changes in the shapes of products on the environment metrological is not an easy task, it must use a specific software like a tool for design. The modeling can be considered an accurate and detailed creation of free and complex

forms. These are considered free forms when they are different from models with known characteristics and geometry, for example, planes, circles, cylinders, etc.

This work made use of a MATLAB toolbox to represent shapes and surfaces of various geometries. The toolbox used is not inserted into the commercial package MATLAB, but it was developed by Zang (2006) and put the download to use in the Matlab Central site for download. This toolbox was used for the treatment of the simulated points generated using analytical equations to obtain curves and surfaces known as well as tools used in the experimental producing in this paper. This toolbox was used for treatment of simulated points generated using analytical equations to obtain curves and surfaces known as well as tools and experimental procedure used in this work.

The methodology for the generation and acquisition of curves and surfaces was developed by Silva (2011).

3.1. Development for construction of a known geometry surface using NURBS

In this work, for generate the 3D surface was used a second order model. That one was used by Silva (2002), as show below,

$$z = -4,6 - 3,1x - 2,8y + 1,4x^2 - 2,5y^2 + 0,9xy \tag{10}$$

Using Eq. 10, the points coordinates was calculated to generate the NURBS surface and was assigned to the axes x and y values ranging from -1 to 1, spaced at 0.25 mm, ie,

Coordinates x: [-1, -0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75, 1] (11)
Coordinates y: [-1, -0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75, 1]

Substituting these values in Eq. (10), it was reached the following value matrix of z, for each corresponding pair (x,y), as show below.

$$z(x, y) = \begin{bmatrix} z(-1,1) & \dots z(1,1) \\ \vdots & \\ z(-1,-1) \dots z(1,-1) \end{bmatrix} \tag{12}$$

The values for Eq. (12) are:

$$z(x, y) = \begin{bmatrix}
-6,900 & -6,281 & -5,975 & -5,981 & -6,300 & -6,931 & -7,875 & -9,131 & -10,700 \\
-6,513 & -5,950 & -5,700 & -5,763 & -6,138 & -6,825 & -7,825 & -9,138 & -10,763 \\
-5,950 & -5,444 & -5,250 & -5,369 & -5,800 & -6,544 & -7,600 & -8,969 & -10,650 \\
-5,213 & -4,763 & -4,625 & -4,800 & -5,288 & -6,088 & -7,200 & -8,625 & -10,363 \\
-4,300 & -3,906 & -3,825 & -4,056 & -4,600 & -5,456 & -6,625 & -8,106 & -9,900 \\
-3,213 & -2,875 & -2,850 & -3,138 & -3,738 & -4,650 & -5,875 & -7,413 & -9,263 \\
-1,950 & -1,669 & -1,700 & -2,044 & -2,700 & -3,669 & -4,950 & -6,544 & -8,450 \\
-0,513 & -0,288 & -0,375 & -0,775 & -1,488 & -2,513 & -3,850 & -5,500 & -7,463 \\
1,100 & 1,269 & 1,125 & 0,669 & -0,100 & -1,181 & -2,575 & -4,281 & -6,300
\end{bmatrix} \tag{13}$$

The input data to obtain the surface are the control points, Piegl (1997), in this case correspond to the point obtained from Eq. (10). From Eq. (5), the the input coordinates for the procedure becomes,

$$coordinates = \begin{bmatrix} x_1 \cdots x_{81} \\ y_1 \cdots y_{81} \\ z_1 \cdots z_{81} \\ 1 \cdots 1 \end{bmatrix} \tag{14}$$

As the control points (coordinated points), other input data for construction a NURBS surface is the knot vector. A knot vector non-uniform, can be defined as a non-decrease sequence of real numbers as shown below,

$$u = \{ \underbrace{a, \ldots, a}_{p+1}, \ldots, u_{m-p-1}, \underbrace{b, \ldots, b}_{p+1} \} \qquad (15)$$

In other words, $u = \{ 0, \ldots, 0, \ldots, u_{m-p-1}, 1, \ldots, 1 \}$, where $a = 0$, $b = 1$ e p is the function degree.

Les Piegl (1997) mentions that for the construction of an unknown curve or surface, a 3 degree function (*p = 3*) can have a good modeling. Therefore, this study has used functions of degree 3 to represent the most several free forms. Thus, the knot vector adopted to obtain the curve was,

$$u = \{0 \ 0 \ 0 \ 0 \ 0.15 \ 0.25 \ 0.5 \ 0.75 \ 0.85 \ 1 \ 1 \ 1 \ 1\}; \qquad (16)$$

$$v = \{0 \ 0 \ 0 \ 0 \ 0.15 \ 0.25 \ 0.5 \ 0.75 \ 0.85 \ 1 \ 1 \ 1 \ 1\};$$

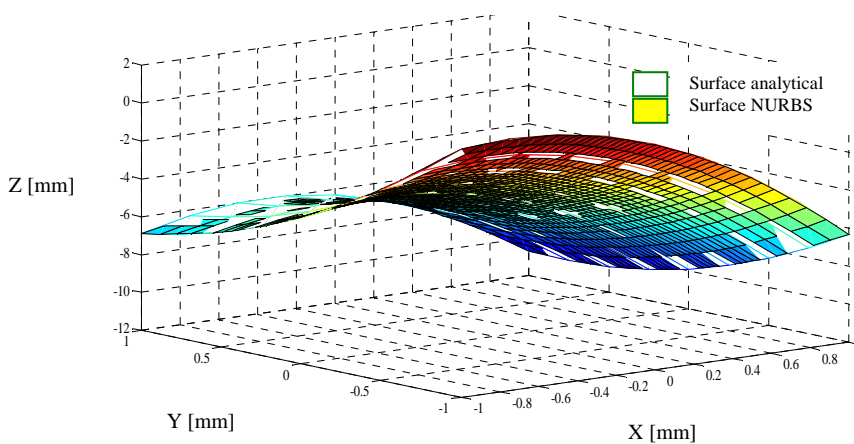The Fig. 2 presents in the same graphic ambient the construction of NURBS analytical surface.

Figure 2. Analytical surface x NURBS surface representation

The matrix below present the error matrix obtained from difference between the points measured by Coordinate Measuring Machine and the points obtained by the use of NURBS tools, and the matrix values are expressed in millimeters.

$$Error = \begin{bmatrix}
0,000 & 0,076 & 0,054 & 0,053 & 0,065 & 0,053 & 0,055 & 0,079 & 0,000 \\
0,040 & 0,035 & 0,013 & 0,011 & 0,024 & 0,013 & 0,009 & 0,037 & 0,042 \\
0,030 & 0,043 & 0,024 & 0,024 & 0,035 & 0,026 & 0,025 & 0,047 & 0,030 \\
0,030 & 0,046 & 0,025 & 0,023 & 0,035 & 0,024 & 0,028 & 0,046 & 0,030 \\
0,036 & 0,038 & 0,017 & 0,017 & 0,029 & 0,017 & 0,016 & 0,041 & 0,036 \\
0,030 & 0,042 & 0,022 & 0,023 & 0,035 & 0,023 & 0,026 & 0,043 & 0,030 \\
0,031 & 0,048 & 0,022 & 0,020 & 0,036 & 0,023 & 0,024 & 0,051 & 0,031 \\
0,046 & 0,028 & 0,009 & 0,010 & 0,021 & 0,014 & 0,005 & 0,035 & 0,045 \\
0,000 & 0,074 & 0,053 & 0,053 & 0,065 & 0,053 & 0,055 & 0,079 & 0,000
\end{bmatrix} \qquad (17)$$

Analyzing all values presented in the differences matrix among the measured points and those obtained by NURBS, it was detected that the values found are in the range of the 0 µm and 100 µm. The ISO 10.360 dictates the range of accuracy of the measuring equipment, including the CMMs. For the case of equipment with CMM Laser head, these one have measuring accuracy ranging from 20 µm to 100 µm, such as manufactures (Metris, 2011).

3.2. Generation of an Airplane Wing surface using NURBS

Also, an experimental procedure was realized to verify the effectiveness of the developed methodology by Silva (2011). This experiment consisted in obtaining the surface profile of a miniature model airplane wing MIG. The wing profile was divided in order to build a 8 x 8 matrix a total of 64 points collected. Planning / strategy, assembling and fixing the model and performing the measurements operations took about 4 hours. To perform the data collection was

not necessary to offset an additional task of the probe, i.e., moves to change angles and rotations were not used. The measurement procedure started with the manual settings to determining the coordinates system to be realized later the automatic setting. The measurement sequence by the probe started from wing base to the extremity. The Fig. 3 shows a measurement sequence used in this process.
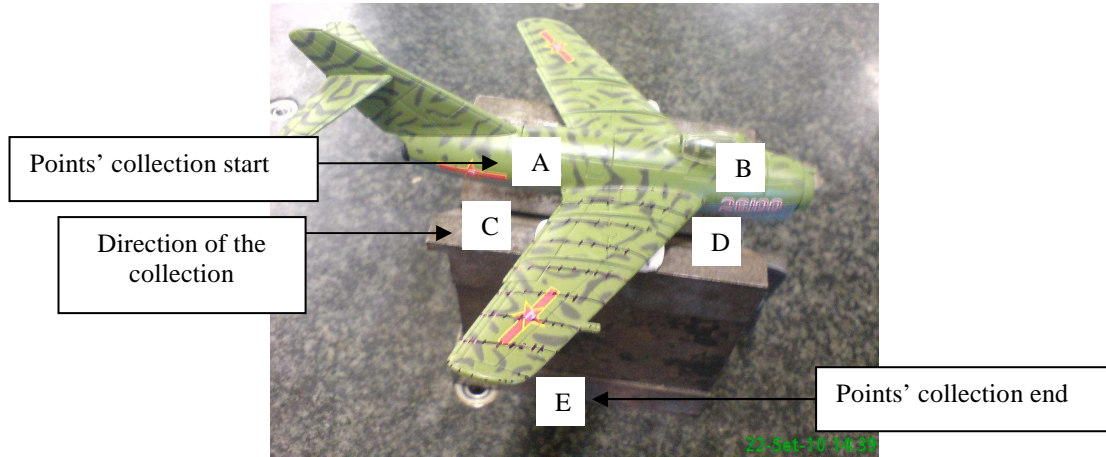

Figure 3. Direction of the points collection used by probe

The sequence to collect points used by machine was in the following directions $A \rightarrow B$, $C \rightarrow D$, and ending E. According to the sequence of sampling points established by Fig. 3 (from A to E), it is presented below the matrix for each coordinates found on any surface measured.

The X coordinates measured were:

$$X_{MEASURED} = \begin{bmatrix} -34,973 & -43,701 & -48,276 & -51,665 & -56,151 & -61,517 & -72,469 & -78,405 \\ -42,169 & -47,652 & -51,931 & -55,218 & -59,815 & -64,683 & -73,811 & -79,283 \\ -47,699 & -52,907 & -56,973 & -59,202 & -64,427 & -67,729 & -76,552 & -82,368 \\ -53,539 & -59,551 & -62,176 & -64,713 & -68,908 & -72,415 & -80,813 & -86,088 \\ -59,212 & -64,562 & -67,610 & -69,597 & -73,763 & -77,569 & -85,537 & -90,337 \\ -65,488 & -69,540 & -72,724 & -75,307 & -78,643 & -82,373 & -90,474 & -93,594 \\ -77,015 & -80,359 & -82,345 & -85,204 & -87,243 & -90,473 & -98,270 & -100,684 \\ -82,496 & -86,271 & -88,270 & -93,090 & -96,385 & -98,929 & -101,542 & -102,682 \end{bmatrix}$$

(18)

The Y coordinates measured were:

$$Y_{MEASURED} = \begin{bmatrix} -12,060 & -12,109 & -12,298 & -12,204 & -11,866 & -12,568 & -12,100 & -12,374 \\ -20,516 & -19,071 & -18,606 & -18,362 & -18,033 & -17,601 & -17,965 & -17,296 \\ -28,937 & -27,280 & -26,959 & -26,794 & -26,405 & -26,168 & -25,510 & -25,048 \\ -37,259 & -35,689 & -35,100 & -34,912 & -34,600 & -34,340 & -33,177 & -32,675 \\ -45,108 & -44,638 & -44,408 & -43,898 & -43,592 & -43,833 & -43,263 & -42,833 \\ -53,651 & -52,477 & -52,996 & -52,626 & -52,398 & -52,300 & -52,230 & -52,222 \\ -69,381 & -68,897 & -68,763 & -68,849 & -68,697 & -68,457 & -67,778 & -67,496 \\ -76,457 & -78,257 & -78,697 & -78,503 & -77,941 & -77,842 & -76,310 & -74,871 \end{bmatrix}$$

(19)

The Z coordinates measured were:

$$Z_{MEASURED} = \begin{bmatrix} 0,567 & 2,176 & 2,430 & 2,430 & 2,511 & 1,896 & 0,887 & -0,659 \\ 0,313 & 1,948 & 2,390 & 2,433 & 2,283 & 1,839 & 0,591 & -1,618 \\ -0,122 & 1,656 & 2,212 & 2,305 & 2,150 & 2,071 & 0,947 & -0,345 \\ -0,434 & 1,782 & 2,109 & 2,190 & 2,283 & 2,130 & 1,044 & -0,980 \\ -0,459 & 1,575 & 2,072 & 2,253 & 2,342 & 2,247 & 1,336 & 0,464 \\ 0,055 & 1,791 & 2,194 & 2,409 & 2,541 & 2,422 & 1,483 & 0,399 \\ 1,312 & 2,177 & 2,522 & 2,852 & 3,088 & 3,204 & 2,474 & 1,676 \\ 1,388 & 2,560 & 2,541 & 3,529 & 3,468 & 3,000 & 2,648 & 2,158 \end{bmatrix}$$

(20)

For the construction of these surface was set a function of degree 3. The knot vector chose was:

$$u = \{ 0\ 0\ 0\ 0\ 0.2\ 0.4\ 0.6\ 0.8\ 1\ 1\ 1\ 1\} \tag{21}$$
$$v = \{ 0\ 0\ 0\ 0\ 0.2\ 0.4\ 0.6\ 0.8\ 1\ 1\ 1\ 1\}$$

The points found for the Z coordinates by the NURBS tool were:

$$Z_{NURBS} = \begin{bmatrix} 0,567 & 2,010 & 2,385 & 2,441 & 2,404 & 1,933 & 0,605 & -0,659 \\ 0,395 & 1,822 & 2,309 & 2,387 & 2,262 & 0,669 & 0,400 & -1,172 \\ -0,145 & 1,699 & 2,153 & 2,261 & 2,160 & 2,046 & 0,774 & -0,739 \\ -0,386 & 1,639 & 2,085 & 2,210 & 2,236 & 2,043 & 0,758 & -0,647 \\ -0375 & 1,523 & 2,013 & 2,256 & 2,331 & 2,153 & 1,249 & 0,212 \\ 0,115 & 1,852 & 2,206 & 2,408 & 2,539 & 2,379 & 1,334 & 0,638 \\ 1,161 & 2,124 & 2,469 & 2,820 & 3,027 & 3,062 & 2,275 & 1,603 \\ 1,388 & 2,451 & 2,555 & 2,555 & 3,387 & 3,359 & 2,681 & 2,158 \end{bmatrix} \tag{22}$$

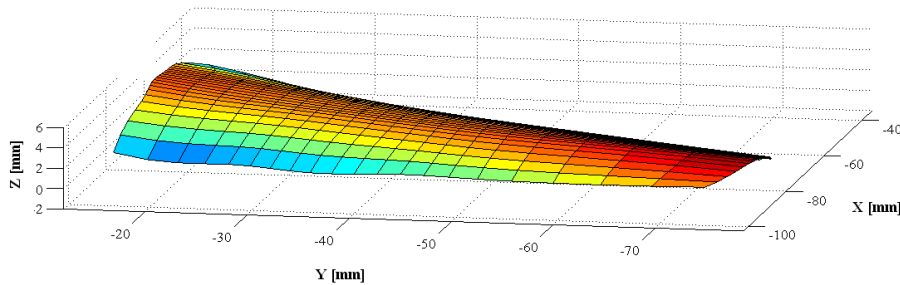The Fig. 4 shows the modeling surface using NURBS tools.

Figure 4. Wing surface obtained using NURBS

Based on corresponding x, y and z values measured, it was calculated the difference between the z coordinate points measured and the values for that coordinate obtained by NURBS. From these values was created an error matrix for further analysis of these parameters. The following can be seen the error matrix in which its values are expressed in millimeters.

$$Error = \begin{bmatrix} 0,000 & 0,165 & 0,045 & -0,011 & 0,106 & -0,037 & 0,281 & 0,000 \\ -0,106 & 0,126 & 0,081 & 0,046 & 0,021 & 0,015 & 0,149 & 0,446 \\ 0,023 & -0,043 & 0,054 & 0,043 & -0,010 & 0,024 & 0,173 & 0,393 \\ -0,048 & 0,143 & 0,024 & -0,020 & 0,047 & 0,087 & 0,286 & 0,333 \\ -0,083 & 0,050 & 0,058 & -0,003 & 0,011 & 0,093 & 0,086 & 0,251 \\ -0,060 & -0,061 & -0,012 & 0,001 & 0,002 & 0,043 & 0,148 & -0,238 \\ 0,150 & 0,053 & 0,054 & 0,032 & 0,060 & 0,141 & 0,199 & 0,072 \\ 0,000 & 0,109 & -0,013 & -0,014 & 0,142 & 0,109 & -0,033 & 0,000 \end{bmatrix} \tag{23}$$

The Fig. 5 shows the error found and distributed on the full surface, as shown in the error matrix.
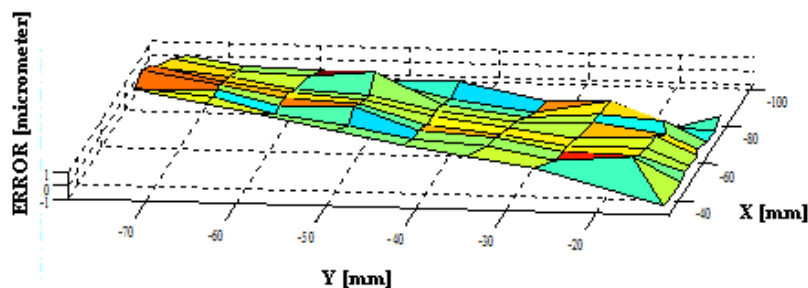

Figure 5. Distribution of errors found in the entire length of the plane's wing

Analyzing the error matrix obtained from the Z coordinate measured by CMM and the Z coordinate obtained by NURBS, observe that most of the points are below 100 µm. The distribution of values of the error matrix is the difference between the Z coordinate measured by the machine and its corresponding value found by NURBS function, following the sequence established by Fig. 3.

Two cases in this matrix can be seen: Approximated values between 0 and 100 µm in the first six collected points, complying the scanning direction (direction $A \rightarrow B$, $C \rightarrow D$, etc.) and the above values 100 µm, obtaining a maximum value of 286 µm, in more remote regions of the beginning of the scans. Analyzing the results, it is noticed the need for more collection points on the right side of the wing surface.

Table 1. Results obtained for the values used to perform validation of the methodology applied in the wing miniature

Xmeasured	Ymeasured	Zmeasured	Znurbs	Error (mm)
-66,442	-12,225	1,444	1,439	0,005
-70,260	-17,291	1,198	1,086	0,112
-73,194	-25,762	1,469	1,331	0,138
-76,559	-33,621	1,696	1,544	0,152
-80,966	-43,604	1,969	1,833	0,136
-86,179	-52,374	2,117	2,029	0,088
-94,507	-68,154	3,005	2,723	0,282
-98,929	-77,842	3,000	3,094	-0,094

Table 1 presents the results obtained with point values used to perform validation of the developed methodology. The values found for the errors are around 100 µm and a not typical one of 282 µm. In addition, study was conducted to verify the influence if the weights. The Fig. 6 shows the point chosen to perform this analysis.
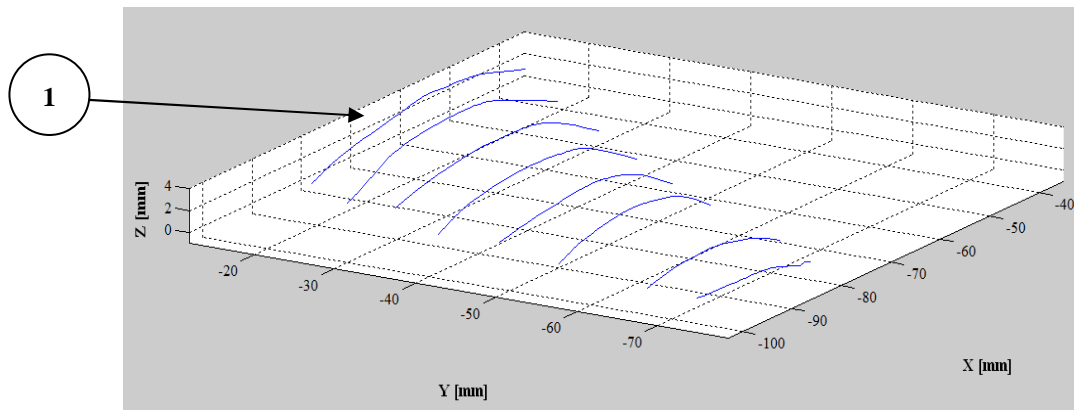

Figure 6. Point chosen to verify the effect of weight function

The first line of Fig. 6 was used to the construction of NURBS curve. It's representation is in Tab. 2. The errors value obtained in this generation, differ form the values found in the error matrix for the generation of NURBS surface. For this situation (curves creation), the influence of the weights for the construction of NURBS curves are only in one direction, i.e., the points over the right and left side of the point whose weight value is changed. In general, after many manipulations in the values of the weight functions, a satisfactory solution was found. The new weights is present in Tab. 2. The Tab. 2 shows that the values of the errors more discrepant after the weight changed, were about of 100 µm, showing a better result when compared to the result obtained without changing the weights in the process of obtaining the curve.

Table 2. Comparative values for Z coordinates obtained by CMM and NURBS to build the curve 1 corresponding to wing plane after the change of the weights

Analysis curves: Line 1								
Coordinates / Measured Points	1	2	3	4	5	6	7	8
X	-34,973	-43,701	-48,276	-51,665	-56,151	-61,517	-72,469	-78,4050
Y	-12,060	-12,109	-12,298	-12,204	-11,866	-12,568	-12,100	-12,3740
Z	0,567	2,176	2,430	2,430	2,511	1,896	0,887	-0,6590
Weigths	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000
Znurbs	0,567	2,306	2,475	2,416	2,523	1,892	1,135	-0,659
Error (mm)	**0,000**	**-0,130**	**-0,045**	**0,014**	**-0,012**	**0,004**	**-0,248**	**0,000**
Change of the weights for coordinates points above								
New weights	1,000	1,000	1,000	1,005	0,950	1,018	0,980	1,000
Znurbs	0,567	2,436	2,522	2,390	2,521	1,789	1,219	-0,659
Error (mm)	**0,000**	**-0,130**	**-0,047**	**0,026**	**0,002**	**0,103**	**-0,084**	**0,000**

The Figure 7 presents the NURBS curve with end without the change of the weights for line 1.
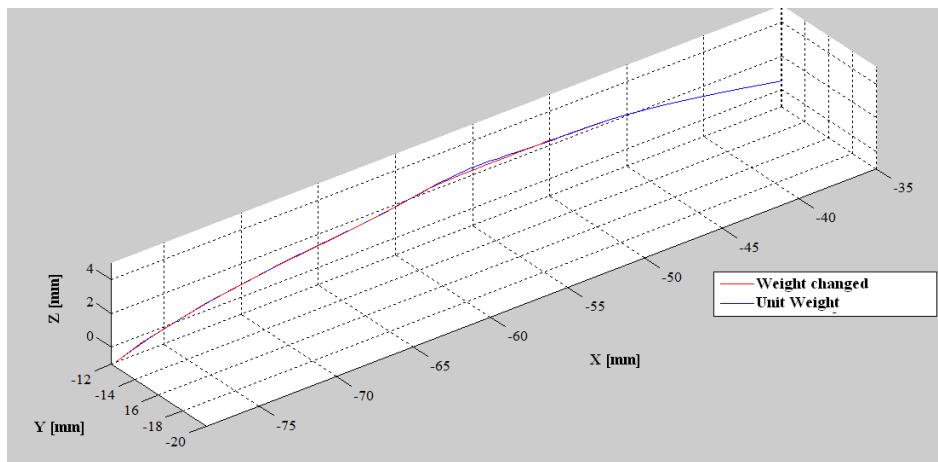

Figure 7. NURBS curve obtained for the line 1 with and without weight changes

The values of weights was choose randomly and serve only to illustrate the importance of the study of surface modeling. It is intended in future to develop an algorithm to find the best values for the weights for many different shapes and geometries.

4. CONCLUSION

This article aimed to present the results obtained experimentally with a Coordinate Measurement Machine using a touch trigger probe sensor. The first part consisted of an examination to verify the simulation method to the build curves and surfaces using NURBS developed by Silva (2011). The experimental work consisted to obtain a modeling of a curve of a miniature model of a wing plane (MIG). It is important highlight that the accurate of laser scanning system, supplied by manufacturers vary from 20 to 100 µm that the methodology proposed in this papar was efficient. The results found for the difference between the measured points and the NURBS points varied from 0 to 100 µm in the most of the wing. The results, however, also showed the need for more collection points in the right region of the plane, taking as base the scanning direction, where the errors showed was 286 µm. The results obtained showed the effective of the proposed approach.

REFERENCES

Aquino, D. M. G and Carvalho, J. C. M., 2003. "Malhagem de Superfícies Tridimensionais Utilizando-se de NURBS E B-Splines". XII Seminário de Iniciação Científica. UFU.
Li, G. and Gu, P., 2004. "Free form surface inspection techniques state of the art review". Computer aided design, pp 1395-1417, Canada.
Metris, 2011. <http://www.metris.com/catalog>, acesso em janeiro.

Minneto, C. F., 2003. "Um Estudo sobre Curvas NURBS". Dissertação de Mestrado. UFRGS.

Piegl, L. and Tiller, W., 1997. "The NURBS Book". Second Edition, Springer-Verlag.

Silva. J. B. A and Burdekin, M., 2002. "A modular space frame for assessing the performance of co-ordinate measuring machines (CMMs)". Journal of the International Societies for Precision Engineering and Nanotechnology. Vol 26. Pag 37 - 48.

Silva, E.C.M, 2009. "Medição de superfícies de formas livres: Um estudo comparativo com uso de medição com contato e sem contato para fins de uso de inspeção e engenharia reversa". V Congresso Brasileiro de Metrologia, Salvador, BA.

Silva. E.C.M., 2011. "Aplicação de NURBS e MMCs, com apalpador touch trigger para escaneamento de superfícies de formas livres e geometrias complexas". Tese de doutorado.UFPB.

Varady, T., Martin R. R & Coxt, J., 1997. "Reverse Engineering of geometric models – an introduction". Computer Aided Design. Elsevier Science Ltd. Vol. 29. Pag. 255 – 268.

Zhang, P., 01 feb 2008. http://www.mathworks.com/matlabcentral/fileexchange/authors/82397. "NURBS Toolbox by D.M. Spink".