EXTENDED JACOBIAN FOR REDUNDANT ROBOTS OBTAINED FROM THE KINEMATICS CONSTRAINTS

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Abstract. *This paper presents a proposal for a new extended Jacobian method based on kinematic constraints, exploring only singularities of the kinematic chain. It is presented the development of the new extended Jacobian as well as their properties. These method can be applied to analyze the behavior of redundant robots on perfoming a task. Redundant robots are used to perform tasks which require some type of extra mobility, for example when it is necessary to avoid obstacles inside their workspace. In general the kinematic redundancy condition does not allow to find the solutions the solution of inverse kinematics directly. Methods based on pseudoinverse matrix and extended Jacobian are generally useful for solving inverse kinematics for redundant robots. However, these methods have limitations like metric problems and algorithmic singularities that do not belong to the kinematic chain. These limitations decrease the robot's ability to perform movements, while away from their kinematic singularities. Based on screw theory, the method of kinematic constraints consists in add Assur virtual chains to perform tasks that retricts movements, such as trajectory generation, collision avoidance, among others. To validate the proposal an example for redundant robot $P3R$ is developed.*

Keywords: *Redundant robot, extended Jacobian, Strutural singularities*

1. Introduction

A robotic system typically consists of a mechanical manipulator, an end-effector, a microprocessor-based controller and a computer. A mechanical manipulator comprises several links connected by joints forming a kinematic chain. Some of the joints in the manipulator are actuated; the others are passive. Typically, the number of actuated joints is equal to the degrees of freedom (Tsai, 1999).

Parallel robots are a class of manipulator that become more complex as a growing number of joints and circuits. This complexity is evident in the kinematic and dynamic models. Another factor that have to take in account, this kind of robot is classified as redundant (Tsai, 1999).

The kinematics modeling requires a systematic strategy that should be attend, as possible as, all these aspects related to the complexity of the kinematic chains of robot manipulators. To surround such complexities connected to the kinematic manipulator modeling has been used the Davies method associated with the Assur virtual chains. This methodology is also called method of kinematic constraints Campos *et al.* (2009).

The Davies method are extensively studied in Davies (1981), and discussed in Campos *et al.* (2009) and Simas *et al.* (2009), provides to achieve the differential kinematic model for closed kinematic chains with several loops. The method equation relate the velocities of the passive joints and those actuated joints. So, the kinematic chain can be now classified as a system with virtual and real joints.

Using the method of kinematic constraints, in some cases not all virtual joint are actuated, and as consequence the passive joints belonging to the virtual chains will be part of the secondary joints. In practice, kinematic analysis of the parallel robots imply to use only "real" passive joints, while the velocities and positions computed for the secondary virtual joints have no use. The presence of secondary virtual joints on the kinematics model requiring greater computational effort in such analysis. It is interesting to set up strategies to eliminate the secondary virtual joint from the model. The differential model based on the screw theory allows to eliminate the screws of secondary virtual joints, through reciprocal screws.

Reciprocal screw $\$^r$ represents a set of forces and moments applied over a rigid body, that moves along of a infinestesimal screw and that doesn't produce work (Gibson and Hunt, 1990). In this paper the concept of reciprocal screw is used in a way to eliminate of equation model the screws of the virtual secondary joints from the kinematics model. The proposed method is applied in a simplified redundant robot, where the final results is an extended Jacobian with only structural singularities. The classical extended Jacobian presented in Baillieul (1985) and applied in Antonelli and Chiaverini (1998) to URV's, is a strategy where additional tasks are included in structural Jacobian making it invertible. The classical extended Jacobian developed in Baillieul (1985) has, generally, singularities that don't belong to kinematic structure.

The main contribution of this work is present a systematic method to eliminate secondary virtual joints or its screws from differential kinematic model, obtained from the method of kinematic constraints. Also, it will be shown an new extended Jacobian to redundant robots with only structural singularities as result. The present method and its validation will be proved using a $P3R$ planar redundant robot with a obstacle inside in its workspace.

2. Fundamental tools

The approach, here proposed, is based on the Davied method, where the screw displacement are successively applied (Tsai, 1999), together with the Assur virtual chain concept, which is briefly presented in following sections.

2.1 Screw theory

The general spatial differential movement of a rigid body consists of a differential rotation about an axis, and a differential translation along the same axis named the instantaneous screw axis. The complete movement of the rigid body, combining rotation and translation, is called screw movement or twist and is here denoted by $\$$. The ratio of the linear velocity to the angular velocity is called pitch of the screw denoted as h.

The twist may be expressed by a pair of vectors $\$ = \left[\omega^T; V_p^T\right]^T$, where ω represents the angular velocity of the body with respect to the inertial frame and V_p represents the linear velocity of a point P attached to the body which is instantaneously coincident with the origin O of the reference frame. A twist may be decomposed into its magnitude and its corresponding normalized screw. The twist magnitude $\dot{q}$ is either the magnitude of the angular velocity of the body, $\|\omega\|$, if the kinematic pair is rotative ($h = 0$) or helical, or the magnitude of the linear velocity, $\|V_p\|$, if the kinematic pair is prismatic ($h \to \infty$). The normalized screw $\hat{\$}$ is a twist of unitary magnitude, i.e.

$$\$ = \hat{\$}\dot{q} \tag{1}$$

The normalized screw coordinates $\hat{\$}$ is written as:

$$\hat{\$} = \begin{bmatrix} s_i \\ s_{oi} \times s_i + h s_i \end{bmatrix} \tag{2}$$

where $s_i = \left[s_{i_x}, s_{i_y}, s_{i_z}\right]$ denotes an unit vector along the direction of the screw axis, and vector s_{oi} represents the position vector of a point lying on the screw axis.

Thus, the twist in Eq. (2) expresses the general spatial differential movement (velocity) of a rigid body relative to an inertial reference frame $O - xyz$. The twist can also represents the movement between two adjacent links of a kinematic chain. In this case, twist $\$_i$ represents the movement of link i relative to link $(i - 1)$.

More details of the screw theory and its applications can be found in the following works: Hunt (2000) and Davies (1981).

2.2 Davies method

Davies method is a systematic way to relate the joint velocities in closed kinematic chains. Davies derived a solution to the differential kinematics of closed kinematic chains from Kirchhoff circulation law for electrical circuits. The resulting Kirchhoff-Davies circulation law states that "The algebraic sum of relative velocities of kinematic pairs along any closed kinematic chain is zero" (Campos *et al.*, 2009). This method is used to obtain the relationship between the velocities of a closed kinematic chain. Since the velocity of a link with respect to itself is null, the circulation law can be expressed as:

$$\sum_{0}^{n} \hat{\$}_i \dot{q}_i = 0 \tag{3}$$

where $\hat{\$}_i$, $\dot{q}_i$ represent respectively the normalized screw and the magnitude of twist $\$_i$ and n is the number of joints.

Equation (3) is the constraint equation which, in general can be written as

$$N\dot{q} = 0 \tag{4}$$

where $N = [\hat{\$}_1 \quad \hat{\$}_2 \quad \cdots \quad \hat{\$}_n]$ is the network matrix containing the normalized screws, with the signs of the screws depend on the definition of the circuit orientation (as will be presented later) (Campos *et al.*, 2009), and $\dot{q} = [\dot{q}_1 \quad \dot{q}_2 \quad \cdots \quad \dot{q}_n]$ is the magnitude vector of the velocities of each joint.

A closed kinematic chain has actuated joints, here named primary joints, and passive joints, named secondary joints. The constraint equation, Eq. (4), allows the computation of the secondary joint velocities as functions of the primary joint velocities. To achieve this, the constraint equation is rearranged highlighting the primary and secondary joint velocities and Eq. (4) is rewritten as follows:

$$\begin{bmatrix} N_p & \vdots & N_s \end{bmatrix} \begin{bmatrix} \dot{q}_p \\ \cdots \\ \dot{q}_s \end{bmatrix} = 0 \tag{5}$$

where N_p and N_s are the primary and secondary network matrices, respectively, and $\dot{q}_p$ and $\dot{q}_p$ are the corresponding primary and secondary magnitude vectors, respectively.

So, Eq. (5) can be rewritten as

$$N_p\dot{q}_p + N_s\dot{q}_s = 0 \tag{6}$$

The secondary joint position can be computed by integrating Eq. (6) as follows:

$$q_s(t) - q_s(0) = \int_0^t \dot{q}_s dt = -\int_0^t N_s^{-1} N_p \dot{q}_p dt \tag{7}$$

2.3 Assur virtual chains

The concept of Assur virtual kinematic chain, or just virtual chain, is essentially a tool to get information on the movement of a kinematic chain or to impose movements on a kinematic chain (Campos *et al.*, 2009).

This concept was first introduced by (Campos *et al.*, 2009), which defines the virtual chain as a kinematic chain composed of links (virtual links) and joints (virtual joints) which possesses three properties: a) the virtual chain is open; b) it has joints whose normalized screws are linearly independent; c) it does not change the mobility of the real kinematic chain.

From the the third property, the virtual chain proposed by (Campos *et al.*, 2009) is in fact an Assur group, i.e. a kinematic subchain with null mobility such that, when connected to another kinematic chain preserves its mobility (Artobolevskii, 1970-75).

2.4 The direct graph notation

Consider a kinematic pair composed of two links E_i and E_{i+1}. This kinematic pair has its relative velocity defined by a screw ${}^R\$_j$ (joint j) relative to a reference frame R. Joint j represents the relative movement of the link E_i with respect to the link E_{i+1}. This relation can be represented by a graph (Campos *et al.*, 2009), where the vertices represent links and the arcs represent joints.

Now, studying a simple graph, where joint j is part of two closed chains. For each closed chain the circuit direction is chosen (Campos *et al.*, 2009). In a direct mechanism graph, if the joint has the same direction as the circuit, the twist associated with the joint has a positive sign in the circuit equation (constraint equation on Eq.(3)), and a negative sign if the joint has the opposite direction to the circuit.

3. Redundant robots and its solutions for inverse kinematics

A robot is said redundant when the number of joints available to be actuated is greater than those needed to perform the task. This can be best understood by making the relationship between the joint space and Cartesian space.

The joint space is defined by the number of joints that compose the robot (here, is called n degree of control), while the Cartesian or operational space is defined by the number of coordinates used to describe it (connectivity r). If in a task only r coordinates are be used, and so $r < n$, then there are degree of redundancy for that task, where the degree of redundancy is $n - r$ (Siciliano *et al.*, 2009). Redundancy can be formally defined as the difference bbetween the degree of control and connectivity of a kinematic chain (Martins and Carboni, 2007).

The differential kinematic model expresses the end-effector linear velocity $\dot{p}$ and the angular velocity ω as a function of the joint velocities $\dot{q}$ by means of the Eq. (8).

$$v = \begin{bmatrix} \dot{p} \\ \omega \end{bmatrix} = J(q)\dot{q} \tag{8}$$

where the matrix J $(r \times n)$ is the robot *Jacobian* matrix and determining the differencial mapping between the joint space and Cartesian space and $\dot{q} = [\dot{q}_1, \cdots, \dot{q}_n]^T$ represents the joints velocities vectorÂt's.

Equation (8) can be inverted, allowing to compute the joints velocities according to desired end-effector velocity. Thus it can be written as in Eq. (9):

$$\dot{q} = J^{-1}(q)v \tag{9}$$

where $J^{-1}(q)$ is the Jacobian inverse matrix.

The Jacobian, in general, is function of the joint position q. Depending on the configuration of the robot, the Jacobian may not possess full rank and it implies that the robot is in a condition of *singularity* (Siciliano *et al.*, 2009), what yield to $J(q)$ not invertible. Under this condition, Eq. (8) can admit an infinite number and the robot loses their mobility or, like parallel robot, can increases their mobility.

The Jacobian matrix of redundant manipulator has largest number of columns, n than rows r, since the dimension of the space joint is greater than the dimension of the operating space, i.e., $n > r$. So the differential inverse kinematics, shown in Eq. 9, presents an infinite number of solutions. The problem is to find a systematic method to find among these infinite solutions an suitable solution to a particular task.

In the next section, are shortly discussed the methods for solving inverse kinematics for redundant robots.

3.1 Differential inverse kinematic through Moore-Penrose PseudoInverse

In a task planning, a simpler strategy is to distribute the motion needed to perform a task for all joints of the robot. The purpose of this distribution is to minimize the energy used by the joints in their movements. The solution can be formulated as an optimization problem whose solution is obtained using Lagrange multipliers method (Siciliano *et al.*, 2009).Thus the differential inverse kinematics can be expressed by the following relationship in Eq. (10).

$$\dot{\mathbf{q}} = J^{\dagger}\mathbf{v} \tag{10}$$

where the matrix $J^{\dagger} = W^{-1}(J^T(JW^{-1}J^T)^{-1}$ is defined as the Jacobian *pseudoinverse* matrix and the matrix W is a suitable ($n \times n$) diagonal positive definite weighting matrix.

Changes in the optimization function allows the inclusion of velocities in the joints that are projected into nullspace of the direct differential mapping (Siciliano *et al.*, 2009).

The pseudoinverse matrix of Jacobian is used in other resolution strategies, such as the *Task priority method* (Antonelli and Chiaverini, 1998), the *Task priority robust to singularities* (Chiaverini, 1997) and *Dumped least square* (Chiaverini and Siciliano, 1994).

The methods based on pseudoinverse have limitations such as: a) In the case of the matrix W, must to be set n variables in its main diagonal, through an empirical adjust; b) the numerical stability of the inverse kinematics depends on the trajectory and; c) problems occur caused by metric problems of the pseudoinverse in the case of robots with structure constituted by rotative and prismatic joints (Campos *et al.*, 2009).

3.2 Extended Jacobian method

The method of extended Jacobian solves the redundancy of robots through a non-redundant system. This solution is gotten by adding kinematic constraints in order to make the Jacobian matrix invertible.

Constraints are based on functions of the form: $h(q) = 0$. In general, it uses the function $h(q)$ as an approximation of the energy of motion. This choice aims at optimizing the distribution of the energy through the joints of the robot (Chiaverini, 1997).

Considering $h(q)$ differentiable on q, we obtain the derivative on Eq. (11):

$$\begin{bmatrix} v \\ 0 \end{bmatrix} = \begin{bmatrix} J(q) \\ \frac{\partial h(q)}{\partial q} \end{bmatrix} \dot{q} \quad \Rightarrow \quad v_e = J_e \dot{q} \tag{11}$$

where J_e is the extended Jacobian and v_e is the augmented vector of velocities of the end-effector.

The extended Jacobian method has a limitation, by inserting algorithms singularities into differential model of the robot, what difficult its implementation and uses. These new singularities are not part of the robot kinematic chain and should be also monitored. The singularities vary according to the function $h(q)$ chosen.

Next section presents the proposed extended Jacobian, obtained from the kinematic constraints.

4. Extended Jacobian from kinematic restrictions

This section presents a mathematical development based on the differential kinematic model and on kinematic constraints, yielding to a equivalent extended Jacobian.

At the end of this section a study of the singularities shows that the proposed extended Jacobian does not introduce algorithmic singularities as classical methods discussed on section 3.2

The method is implemented in a $P3R$ redundant robot with an obstacle inside its workspace. To impose the trajectory, a PPR virtual chain is attached between the base and the end-effector of the $P3R$ redundant robot; and to collision avoidance a RPR virtual chain is attached between the obstacle and the link 3 (near to joint C) of the $P3R$ redundant robot. Figure 1 depicts the $P3R$ redundant robot and Fig.2 depicts the $P3R$ robot with the virtual chains attached.

In the Fig (1) are showed the $P3R$ redundant robot composed by one prismatic joint: A and three rotative joints: B, $C$and D; with links 0 (base), 1, 2, 3 and 4. Joint A has its direction indicated by a fixed unit vector with coordinates $[P_a, Q_a, 0]^T$. The PPR trajectory virtual chain is composed by prismatic joints tx, and ty and a rotative joint rz and links 5 and 6. The RPR collision avoidance virtual chain is composed by two rotative joint r_{z1} and r_{z2}, a prismatic joint pr and the links 7 and 8. The circuits 1 and 2 give the direction needed in the differential model by Davies' method.

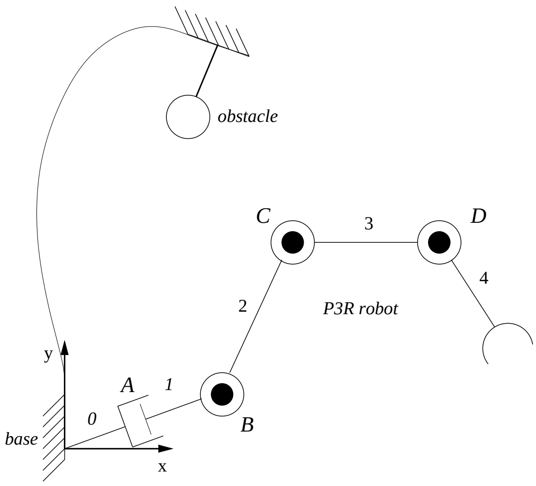

Figure 1. $P3R$ redundant robot with an obstacle inside its workspace.

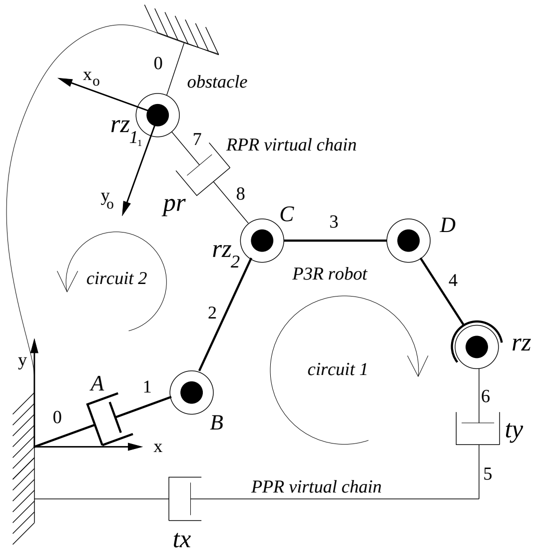

Figure 2. $P3R$ redundant robot with the virtual chains attached.

4.1 Methodology for elimination of the passive joint

The method of kinematic constraints have limitations as the kinematic chain becomes more complex, or according to the task. This limitation occurs because of the presence of screws from the virtual chains on N_s matrix. When the position of secondary joints are calculated, using Eq. (7), and it is obtained the displacement for secondary joints from the virtual chain which has no practical application. To simplify the inversion of the matrix N_s, it is necessary to eliminate the screws of the virtual chains.

The elimination of secondary virtual screws can be performed through reciprocal screws. The reciprocal are arranged in a matrix defined as annihilating matrix (Campos *et al.*, 2009).

The concept of annihilating matrix presented in (Campos *et al.*, 2009) to parallel manipulators is discussed below, using the $P3R$ robot.

By using the $P3R$ redundant robot with the virtual chains to impose trajectories and avoid collision shown on Fig.2, will be shown that it is possible to eliminate from equations, the virtual joints from secondary matrix N_s making it equivalent to the extended Jacobian.

Using the Davies method were obtained matrices N_p e N_s.

$$N\dot{q} = \begin{bmatrix} \hat{\$}_A & \hat{\$}_B & \hat{\$}_C & \hat{\$}_D & 0 & 0 \\ \hat{\$}_A & \hat{\$}_B & 0 & 0 & -\hat{\$}_{rz_1} & -\hat{\$}_{rz_2} \end{bmatrix} \begin{bmatrix} \dot{q}_A \\ \dot{q}_B \\ \dot{q}_C \\ \dot{q}_D \\ \dot{q}_{rz_1} \\ \dot{q}_{rz_2} \end{bmatrix} + \begin{bmatrix} 0 & -\hat{\$}_{rz} & -\hat{\$}_{px} & -\hat{\$}_{py} \\ -\hat{\$}_{pr} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_{pr} \\ \dot{q}_{rz} \\ \dot{q}_{px} \\ \dot{q}_{py} \end{bmatrix} = 0 \quad (12)$$

$$= N_s\dot{q}_s + N_p\dot{q}_p = 0$$

The robot has four joints, A, B, C and D, whose screws that are part of the secondary matrix, together with the screws of the virtual joints rz_1 and rz_2. The velocities of the joints rz_1 and rz_2 are not necessary to compute the position of the robot, so, it is useful to eliminate them from the secondary matrix N_s.

To eliminate these screws (columns) from secondary matrix, a second partition can be done as follows in Eq. (13).

$$N_s\dot{q}_s = \begin{bmatrix} \hat{\$}_A & \hat{\$}_B & \hat{\$}_C & \hat{\$}_D \\ \hat{\$}_A & \hat{\$}_B & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_A \\ \dot{q}_B \\ \dot{q}_C \\ \dot{q}_D \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -\hat{\$}_{rz_1} & -\hat{\$}_{rz_2} \end{bmatrix} \begin{bmatrix} \dot{q}_{rz_1} \\ \dot{q}_{rz_2} \end{bmatrix} = N_{sa}\dot{q}_{sa} + N_{sp}\dot{q}_{sp} \quad (13)$$

where N_{sa} corresponds to the screws of the joints of interest (here called active) and N_{sp} corresponds to the screws of the joints which there is no interest (here called passive).

The passive joints are eliminated using an annihilate matrix $\mathcal{K}$ which has the following structure on Eq. (14) (Campos *et al.*, 2009).

$$\mathcal{K} = \left[\begin{array}{c|c} I_{r\times r} & 0 \\ \hline 0 & {}^{ref}W_{N_{sp}(n-r)\times d} \end{array} \right] \tag{14}$$

where ${}^{ref}W_{N_{sp}}$, whose dimension is $(n-r)\times d$, is a set of reciprocal screws from secondary passive matrix N_{sp}. (Campos *et al.*, 2009)(Martins, 2002).

The reciprocal screws represent a set of external forces and torques that do not generate movements on secondary passive joints. Therefore pre-multiplying N_{sp} by $\mathcal{K}$, produces:

$$\mathcal{K}N_{sp} = 0 \tag{15}$$

To maintain equality is necessary that the Eq. (12) is rewritten, considering the Eq. (13), as follows in Eq. (16).

$$\mathcal{K}N_p\dot{q}_p + \mathcal{K}N_{sa}\dot{q}_{sa} + \mathcal{K}N_{sp}\dot{q}_{sp} = 0 \tag{16}$$

Using equality in Eq. (15) we have the Eq. (17).

$$\mathcal{K}N_p\dot{q}_p + \mathcal{K}N_{sa}\dot{q}_{sa} = 0 \tag{17}$$

The velocities of the primary joints are then obtained by Eq. (18).

$$\dot{q}_p = -(\mathcal{K}N_p)^{-1}\mathcal{K}N_{sa}\dot{q}_{sa} \tag{18}$$

So using the usual definition of the Jacobian (Eq. (8)), we have the Eq. (19).

$$J = -(\mathcal{K}N_p)^{-1}\mathcal{K}N_{sa} \tag{19}$$

Considering the expression in Eq. (8), it can be observed that in Eq. (18) that the vector $\dot{q}_p$ represents the magnitudes of the velocities of end-effector, increased with the magnitudes of the velocities of the actuated virtual joints as well as, $\dot{q}_{sa}$ represents the magnitudes of the velocities of active joints of the manipulator $P3R$. So, the Jacobian expressed by the Eq. (19) is a desired extended Jacobian matrix.

5. Application of the method and obtaining the new extended Jacobian

To evaluate the method of elimination of passive joints, an application was developed for the $P3R$ redundant robot.

Consider the problem of trajectory generation and collision avoidance for $P3R$ redundant robot shown in the previous and in the Fig. (2).

Taking as reference coordinate system O_r, it is obtained the normalized screws for each joint of the $P3R$ redundant robot and for the PPR trajectory virtual chain as following in Eq.(20).

$$\begin{array}{llll} \hat{\$}_A = \begin{bmatrix} 0 \\ P_a \\ Q_a \end{bmatrix} & \hat{\$}_B = \begin{bmatrix} 1 \\ L_aQ_a \\ -L_aP_a \end{bmatrix} & \hat{\$}_C = \begin{bmatrix} 1 \\ L_aQ_a + L_2s_2 \\ -L_aP_a - L_2c_2 \end{bmatrix} & \hat{\$}_D = \begin{bmatrix} 1 \\ x_d \\ y_d \end{bmatrix} \\ \hat{\$}_{rz} = \begin{bmatrix} 1 \\ x_e \\ y_e \end{bmatrix} & \hat{\$}_{px} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} & \hat{\$}_{py} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & \end{array} \tag{20}$$

where P_a and Q_a define the direction of the prismatic joint A; L_a represents the displacement of the prismatic joint A; L_2, L_3 and L_4 are the length of the links 2, 3 and 4; s_i is the $sin(\theta_i)$; c_i is the $cos(\theta_i)$; s_{ij} is the $sin(\theta_i + \theta_j)$; c_{ij} is the $cos(\theta_i + \theta_j)$; θ_2, θ_3 and θ_4 are the angles of the joints of the $P3R$ robot; the magnitudes x_d and y_d are given by: $x_d = L_aQ_a + L_2s_2 + L_3s_{23}$ and $y_d = -L_aP_a - L_2c_2 - L_3c_{23}$ and the magnitudes x_e and y_e are given by: $x_e = L_aP_a + L_2s_2 + L_3s_{23} + L_4s_{234}$ and $y_e = -L_aQ_a - L_2c_2 - L_3c_{23} - L_4c_{234}$.

The coordinates of the normalized screws of the RPR collision avoidance virtual chain, with respect to the coordinate system O_r are given as in Eq. (21):

$$\hat{\$}_{rz_1} = \begin{bmatrix} 1 \\ p_y \\ -p_x \end{bmatrix} \quad \hat{\$}_{pr} = \begin{bmatrix} 0 \\ c_{rz_1 p_1} \\ s_{rz_1 p_1} \end{bmatrix} \quad \hat{\$}_{rz_2} = \begin{bmatrix} 1 \\ p_y + L_r s_{rz_1 p_1} \\ -p_x - L_r c_{rz_1 p_1} \end{bmatrix} \tag{21}$$

where: p_x and p_y are the position coordinate of the base of the collision avoidance virtual chain (O_v) with respect to coordinate system of the base of the $P3R$ redundant robot (O_r); θ_{p_1} is the rotation angle between the systems O_v and O_r, taken in the direction z in relation to the base coordinate system (O_r); θ_{rz_1}, θ_{rz_2} are the angles of the rotative joints of the chain virtual RPR; and L_r represents the displacement of prismatic joint p_r chain RPR.

Aiming to simplify the development below, the angle θ_{p_1} will be considered equal to zero. This condition requires that the coordinate system of the collision avoidance virtual chain O_v will be parallel to the base coordinate system O_r.

Substituting the coordinates of $\hat{\$}_{rz_1}$ and $\hat{\$}_{rz_2}$ in N_{sp} we have as result the Eq. (22).

$$N_{sp} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -1 & -1 \\ -p_y & -p_y - L_r s_{rz_1} \\ p_x & p_x + L_r c_{rz_1} \end{bmatrix} \tag{22}$$

Campos (2004) develops a systematic methodology for obtaining the annihilating matrix. Using this procedure it can be obtained the annihilating matrix to N_{sp} as shown in Eq. (23).

$$\mathcal{K} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & L_r(p_x s_{rz_1} - p_y c_{rz_1}) & L_r c_{rz_1} & L_r s_{rz_1} \end{bmatrix} \tag{23}$$

So: a) knowing that $\mathcal{K}N_{sp} = 0$; b) applying the matrix obtained in Eq. (23) in the Eq. (17); it results in the matrices $\mathcal{K}N_p$ and $\mathcal{K}N_{sa}$, that can be written as Eq. (24):

$$\mathcal{K}N_{sa} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ P_a & L_a Q_a & L_a Q_a + L_1 s_2 & x_d \\ Q_a & -L_a P_a & -L_a P_a - L_1 c_2 & y_d \\ x_3 & x_4 & 0 & 0 \end{bmatrix} \qquad \mathcal{K}N_p = \begin{bmatrix} -1 & 0 & 0 & 0 \\ -x_e & -1 & 0 & 0 \\ -y_e & 0 & -1 & 0 \\ 0 & 0 & 0 & -L_r \end{bmatrix} \tag{24}$$

where $x_3 = L_r(P_a c_{rz_1} + Q_a s_{rz_1})$ and $x_4 = -L_r((p_y - LaQ_a)c_{rz_1} - (p_x - LaP_a)s_{rz_1})$.

By geometric inspection in the kinematic structure of Fig. (2) it can obtained the following equalities in Eq. (25), which help to simplify the equations.

$$p_x = L_a P_a + L_2 c_2 - L_r c_{rz_1} \qquad p_y = L_a Q_a + L_2 s_2 - L_r s_{rz_1} \tag{25}$$

The extended Jacobian matrix is then obtained by the Eq. (19) as shows the Eq. (26).

$$J = \begin{bmatrix} 0 & 1 & 1 & 1 \\ P_a & -L_2 s_2 - L_3 s_{23} - L_4 s_{234} & -L_3 s_{23} - L_4 s_{234} & -L_4 s_{234} \\ Q_a & L_2 c_2 + L_3 c_{23} + L_4 c_{234} & L_3 c_{23} + L_4 c_{234} & L_4 c_{234} \\ P_a c_{rz_1} + Q_a s_{rz_1} & L_1 s_{rz_1 - 2} & 0 & 0 \end{bmatrix} \tag{26}$$

and the vectors v and $\dot{q}$ are given by:

$$v = \begin{bmatrix} \dot{q}_{rz} & \dot{q}_{px} & \dot{q}_{py} & \dot{q}_{pr} \end{bmatrix}^T \qquad \dot{q} = \begin{bmatrix} \dot{q}_A & \dot{q}_B & \dot{q}_C & \dot{q}_D \end{bmatrix}^T \tag{27}$$

Comparing the Jacobian of Eq. (26) with the Jacobian obtained by classical methods, it can be observed that its last line has the additional line that characterizes it as an extended Jacobian. This line relates the vector of joint velocities $\dot{q}$ with the velocity of the actuated joint virtual p_r.

5.1 Evaluation of the singularity

The extended Jacobian matrix obtained in the previous section, allows to study some properties of the $P3R$ kinematic structure. This section will discuss the singularities introduced by the virtual chain, using the determinant of the new extended Jacobian.

Initially the singularities of $P3R$ robot are discussed using a methodology to study the singularities for redundant robots (Nokleby and Podhorodeski, 2001).

5.1.1 Singularity for $P3R$ redundant robot

Nokleby and Podhorodeski (2001) proposes a method to analysis the singularities in redundant robots. The method is based on analysis of sub-matrices of the Jacobian. So, it is observed that the singularity condition to $P3R$ redundant robot is achieved when:

- θ_3=$\pm k_1\pi$ $(k_1 = 0, 1, 2, \cdots)$ and;
- the angle θ_a is equal to $-\theta_2$, thus the angle between the prismatic joint A and the link 2 has magnitude equal to $\pm(2k_2 + 1)\frac{\pi}{2}$; $k_2 = 0, 1, 2, \cdots$.

Making $k_1 = 0$ e $k_2 = 0$ it has the configuration shown in Fig. (3).

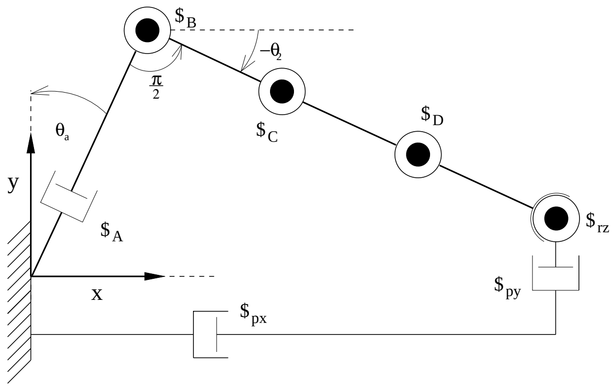

Figure 3. Singular condition for $P3R$ redundant robot.

It can be observed that trajectories commanded in the perpendicular direction to the prismatic joint A can not be performed. This restriction of movement indicates a singular condition.

Now based on the determinant of the extended Jacobian matrix (Eq. (26)), are analyzed the singularities imposed by the collision avoidance virtual chain.

5.2 Singularities of the $P3R$ robot with kinematic restrictions

Computing the determinant of the extended Jacobian D_J, it has as result Eq. (28).

$$D_J = -L_2L_3(P_ac_2 + Q_as_2)s_{rz_1-2-3} \tag{28}$$

where s_{rz_1-2-3} is the $sin(\theta_{rz_1} - \theta_2 - \theta_3)$.

Analyzing the D_J it can be observed that the singular condition is achieved under two different conditions:

- $P_ac_2 + Q_as_2 = 0$; and $s_{rz_1-2-3} = 0$

Considering that L_1 and L_2 are constants and different from zero, D_J is zero only if one of the above conditions are achieved.

From the first condition it has the Eq. (29).

$$\frac{P_a}{Q_a} = -\frac{s_2}{c_2} \tag{29}$$

in other words, geometrically, the angle between the vector direction of the prismatic joint A of the $P3R$ robot and the link 2 is equal to $\pm(2k + 1)\frac{\pi}{2}$ with $k = 0, 1, 2, \cdots$. This condition is shown in Fig. (4).

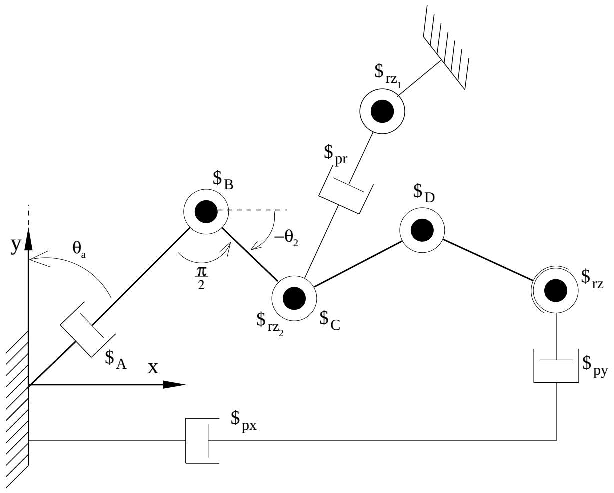

Figure 4. 1^{st} singularity condition

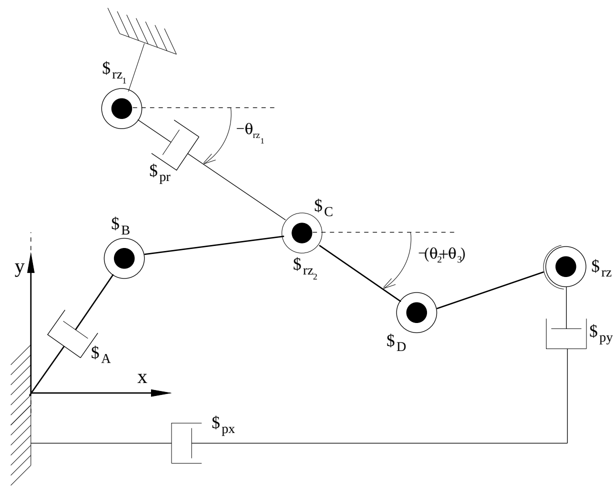

Figure 5. 2^{nd} singularity condition

The angle formed by the prismatic joint A has as complement to the angle $\frac{\pi}{2}$ the angle θ_a, which, by Eq. (29), must be equal to $-\theta_2$. By this equality, the singularity exists if the direction of the joint A is perpendicular to link 2.

It can be observed that keeping the prismatic pair $\$_{pr}$ fixed at a certain position, avoiding a collision for example, the $P3R$ redundant robot can not move because any movement that results on movement of the virtual joint rz_1 do not causes movements in the joints A and B. This singularity is a generalization of the case of the singularity of redundant robot shown in Fig. (3), since this singularity condition applies to any value for the angle θ_3.

Applying the singularity condition to extended Jacobian matrix it has the following result in Eq. (30).

$$J = \begin{bmatrix} 0 & 1 & 1 & 1 \\ -s_2 & -L_2s_2 - L_3s_{23} - L_4s_{234} & -L_3s_{23} - L_4s_{234} & -L_4s_{234} \\ c_2 & L_2c_2 + L_3c_{23} + L_4c_{234} & L_3c_{23} + L_4c_{234} & L_4c_{234} \\ s_{rz_1-2} & L_2s_{rz_1-2} & 0 & 0 \end{bmatrix} \qquad (30)$$

Note that the second column of J can be obtained by the sum of first column multiplied by L_2, with the third column. From the second condition it has:

$$q_{rz_1} = q_2 + q_3 + k\pi, \qquad k = 0, 1, 2, \cdots \qquad (31)$$

Considering the condition that $k = 0$, it has the kinematic configuration resulting in Fig. (5).

This configuration determines a condition of parallelism between the link 3 and prismatic joint p_r. It is observed from Fig. (5) that there is no possibility to impose trajectories in the direction of joint p_r if this joint is being actuated, as when under collision avoidance. In this condition, the Jacobian matrix has the following configuration in Eq. (32).

$$J = \begin{bmatrix} 0 & 1 & 1 & 1 \\ P_a & -L_2s_2 - L_3s_{23} - L_4s_{234} & -L_3s_{23} - L_4s_{234} & -L_4s_{234} \\ Q_a & L_2c_2 + L_3c_{23} + L_4c_{234} & L_3c_{23} + L_4c_{234} & L_4c_{234} \\ P_ac_{23} + Q_as_{23} & L_2s_3 & 0 & 0 \end{bmatrix} \qquad (32)$$

The Jacobian of the Eq. (32) presents a condition of dependence by the fact that, the fourth line is equal to the sum of the products of the first line by L_4s_4, second line by c_{23} and third line by s_{23}.

These results showed that the introduction of kinematic constraints to avoid collisions, introduces two additional singularities to the kinematics model. Thus, the method of extended Jacobian from kinematic constraints also introduce algorithmic singularities.

These algorithmic singularities are caused by restrictions on movement and occur in the configuration of the kinematic chain in which there are incompatibilities between the movements imposed to the end-effector and the other restrictions.

In the example, this incompatibility occurs in situations in which the trajectory imposed to the end-effector can not be performed without collision.

As a practical results it should be emphasized that, in this method the singularity has a clear and physical significance and can be detected and avoided.

6. Conclusion

This paper presented a systematic method to obtaining an extended Jacobian matrix. The methodology aimed to show that there is a representation for the extended Jacobian obtained by the method of kinematic constraints.

The proposal extended Jacobian is demonstrated mathematically and through a differential kinematic model to solving the redundancy in a $P3R$ robot by inclusion of a collision avoidance task of a joint robot to an obstacle inside its workspace.

In the analysis of the determinant of the obtained extended Jacobian matrix, it was shown that other singularities occur. However it was shown that unlike the results for the classical extended Jacobians found in the references, the new extended Jacobian has singularities that belong exclusively to the kinematic chains. These singularities reflect the conditions of incompatibility between the task imposed for end-effector and the collision avoidance.

The main advantage of the method presented is the possibility of a complete study of the mechanism behavior, including detection and control of conflicts between movements imposed on the end-effector and secondary tasks.

Preliminary studies, applied to 2D models, have shown good results in applications, where the same is waited when applied for spatial redundant robots.

7. ACKNOWLEDGEMENTS

This optional section must be placed before the list of references.

8. REFERENCES

Antonelli, G. and Chiaverini, S., 1998. "Task-priority redundancy resolution for underwater vehicle-manipulator systems". In *Proceedings of the 1998 IEEE International Conference on Robotics and Automation*. Leuven, Belgium, pp. 768–773.

Artobolevskii, I.I., 1970-75. *Mechanisms in modern engineering design: a handbook for engineers, designers and inventors*. Mir.

Baillieul, J., 1985. "Kinematic programming alternatives for redundant manipulators". In *IEEE International Conference on Robotics and Automation*. pp. 722–728.

Campos, A.A., 2004. *Cinemática diferencial de manipuladores empregando cadeias virtuais*. Tese de doutorado, Universidade Federal de Santa Catarina.

Campos, A.A., Guenther, R. and Martins, D., 2009. "Differential kinematics of parallel manipulators using Assur virtual chains". *Proceedings of the Institution of Mechanical Engineers. Part C, Journal of Mechanical Engineering Science*, Vol. 223, pp. 1697–1711.

Chiaverini, S., 1997. "Singularity-robust task-priority redundancy resolution for real-time kinematic control of robot manipulators". *IEEE Transactions on Robotics and Automation*, Vol. 13, No. 3, pp. 398–410.

Chiaverini, S. and Siciliano, B., 1994. "Review of the damped least-squares inverse kinematics with experiments on an industrial robot manpulator". *IEEE Transactions on Control Systems Technology*, Vol. 2, No. 2, pp. 123–134.

Davies, T.H., 1981. "Kirchhoff's circulation law applied to multi-loop kinematic chain". *Mechanism and Machine Theory*, Vol. 16, pp. 171–173.

Gibson, C.G. and Hunt, K.H., 1990. "Geometry of screw systems: Part 2: Classification of screw systems". *Mechanism and Machine Theory*, Vol. 25, pp. 11–27.

Hunt, K.H., 2000. "Don't cross-thread the screw!" In *Proceeding of A Symposium Commemorating the Legacy, Works, and Life of Sir Robert Stawell Ball Upon the 100th Anniversary of A Treatise on the Theory of Screws*. University of Cambridge.

Martins, D., 2002. *Analise cinemática hierárquica de robôs manipuladores*. Tese de doutorado, PG-EMC- Universidade Federal de Santa Catarina.

Martins, D. and Carboni, A.p., 2007. "Variety and connectivity in kinematic chains". *Mechanism and Machine Theory*, Vol. 43, pp. 1236–1252.

Nokleby, S.B. and Podhorodeski, R.P., 2001. "Reciprocity-based resolution of velocity degeneracies singularities for redundant manipulators". *Mechanism and machine theory*, Vol. 36, pp. 397–409.

Siciliano, B., Sciavicco, L., Villani, L. and Oriolo, G., 2009. *Modeling and control of robot manipulator*. Springer.

Simas, H., Guenther, R., Cruz, D.F.M. and Martins, D., 2009. "A new method to solve robot inverse kinematics using assur virtual chains". *Robotica, (Cambridge)*. doi:10.1017/S0263574709005426.

Tsai, L.W., 1999. *Robot Analysis: the Mechanics of serial and parallel manipulators*. John Wiley & Sons, New York. ISBN 0-471-32593-7.

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