DYNAMIC SIMULATOR FOR CONTROL OF TANDEM COLD METAL ROLLING

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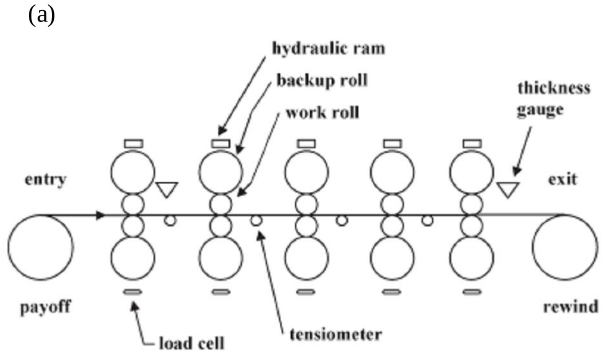
Abstract. *The control of tandem cold rolling of metal strip is still an important engineering challenge. The complexity of the interactions of the multiple process variables, the non-linearities and the time delays, both extremely dependents on operating conditions are mostly the reasons for accurate controllers and predictions of optimized process set up. Nowadays, this problem has been addressed by methods based on the division into various independent control sub-problems. This paper describes a dynamic simulator of the tandem cold metal rolling, implemented in MATLAB/Simulink, that is able to implement new control strategies and combinations of optimizations techniques. In this paper the tandem cold rolling process is described by a mathematical model based on algebraic equations developed for control purposes and empirical relations. Results in open loop are presented, a study of the process sensitivity with regard to the variations in process parameters is presented and the possibility to check the effect of parameters uncertainties in modeling and measurement is offered. Therefore, this work intent to be a contribution for development in new control strategies for tandem cold rolling process that offer the potential to reduce the efforts for design, commissioning and maintenance in rolling mills. The preliminary results obtained with this model have shown reasonable agreement with operational data presented at literature for industrial cold rolling process.*

Keywords*: rolling mill control, tandem cold rolling, process simulation, mathematical model*

1. INTRODUCTION

The tandem cold metal rolling process Fig. 1a is typically characterized by the passage of a metal strip through a continuous sequence of electrically driven rolls, called work rolls, each of which is supported by a back-up roll of larger diameter. The set of work roll, back-up roll and others associated equipments is known for rolling stand. In this process, the passage of the strip through the individual pairs of work rolls results in successive thickness reductions by compressive force applied to material in a small region, the roll bite shown in Fig. 1b. The metal is plastically deformed in roll bite and the strip slips as much in input as in stand output, this slip is caused by the fact of speed differences between the work roll surface and the strip surface. This compressive force is applied by hydraulic rams, or by screw arrangement driven by electric motor, that press the work rolls over the strip material.

(a)



(b)

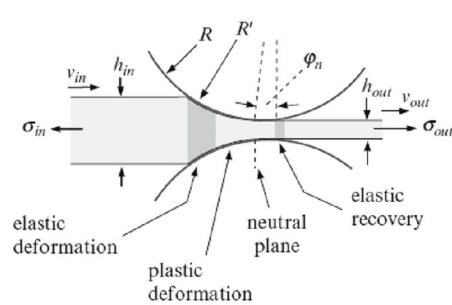


Figure 1. (a) Typical five-stand tandem cold mill. (Pittner and Simaan, 2006). (b) Schematic diagram for the material deformation in roll bite. (Pittner and Simaan, 2011)

According to Figure 1a, the process is monitored by the thickness sensors (gaugemeters), the interstand strip tension forces (tensiometers) and roll force (load cell). The actuators are the hydraulic rams or screws that make roll gap control and the electric drives for the roll speed control, promoting an adequate value of interstand tension according to schedules production. The tandem cold rolling of metal is a high complex engineering process, with interactions of the multiple process variables, non-linearities and time delays, both extremely dependents of the operating conditions and of the speed rolling mill. In general, the rolling mill automation technology to deal with the control problem by division into subproblems, where both the adjust thickness and the interstand tension are independents, according to Bryant (1973). This classical strategy is employed and the production results are acceptable, but fundamentals studies like Geddes and Postlethwaite (1998) and Hoshino et al. (1998), shown the possibility of important progresses in product quality by the use of control advanced strategies.

The process controllers design is essentially based in an adequate process model for control. A model for control, different from a model for prediction is not characterized for great complexity mathematic. The aim is to achieve a good variational similarity of the process variable in the operating point neighborhood, and to be easily implemented computationally. The dominant idea is the process linearization and the state space modeling practice, pretty recomended for MIMO systems - multiple input multiple output, according to Asano and Morari (1998), Lee and Lee (1999), Pittner et al. (2002) e Kim et al. (2004).

This work aims to show a dynamic simulator for the tandem cold metal rolling, that will be employed as basis for the research and development in new controllers which will be suggested how advances in search of better quality of rolled products.

2. METHODOLOGY

2.1. Introduction

The simulator was developed and implemented in MatLab/Simulink R2007b, which is a computational environment greatly known and by your specific resources in control systems area, which make it very common in similar works.

In rolling processes very attention is provided in strip deformation on roll bite. Many mathematic models were shown and implemented according to Alves and Hemerly (2007). The strip deformation model developed by Ford et al. (1951), showed good coherency with experimental data and was the basis of the Bryant's work, Bryant (1973). The mathematic model of process, in this work, is composed by a set of algebraic equations greatly applied in industrial areas, Bryant (1973).

In rolling processes, the strip deformation occurs by the movement of the work rolls, which is caused by position and speed actuators. Such actuators were treated by linear first order continuous dynamic systems with time constant adjusted with practice industrial and literature suggestions.

A tandem cold rolling mill with five stands was mathematically modeled, afterwards the model was linearized, their variables were normalized and finally a state space model was generated. A Matlab script was developed for the realization of the linearization process and the state space model generation. A Simulink diagram processed, in open loop, a rolling mill plant simulation and provided inputs and outputs for to connect controllers in future stages of this work, carrying out the close loop control. The linearization process was based in Pittner et al. (2002), however extended to five stands.

2.2. Description

2.2.1. Non-linear mathematical model

The deformation modeling occurred in roll bite is composed of the specific roll force calculation, specific roll force means force by strip width in kN/mm and the forward slip calculation too, that is the fractional increase in the speed of the strip exiting the roll bite area, this parameter is dimensionless. The strip speed and roll speed are equal only in one contact point strip-roll, called neutral point.

The specific roll force is approximated by Eq. (1), where weighting coefficients are employed in according to Bryant (1973). The compressive yield stress $k_{in(out)}$ were formulated depending of the thickness reduction $r_{in(out)}$ applied in strip in relation to H_a, strip annealed thickness, is implicit in this equation. The hypothesis of the circular arc contact and the flattening work roll occurrence, allowed the use of Hitchcock's formula for a deformed work roll radius in according to Alves and Hemerly (2007).

$$P = \left(\left(0{,}2k_{in} + 0{,}8k_{out}\right) - \left(\frac{2}{3}\sigma_{in} + \frac{1}{3}\sigma_{out}\right)\right) \cdot \sqrt{R'\delta}\left(1 + 4\left(\sqrt{\frac{h_{out}}{h_{in}}}\, exp\left(\frac{\mu\sqrt{R'\delta}}{0{,}28h_{in} + 0{,}72h_{out}}\right) - 1\right)\right)$$
$$k_{in(out)} = 0{,}819\left(0{,}002 + r_{in(out)}\right)^{0{,}27} \qquad r_{in(out)} = \frac{H_a - h_{in(out)}}{H_a} \tag{1}$$

where $\sigma_{in(out)}$ is the input (output) tension stress, *R'* is the deformed work roll radius, υ is the Poisson's ratio of the work roll, *E* is the Young's module of the work roll, δ is the drift (h_{in} - h_{out}), $h_{in(out)}$ is the input (output) strip thickness , μ is the coefficient of friction and the subscript *0* in some variables means the value in operating point.

The forward slip *f*, is approximated by Eq. (2).

$$f = \left(\frac{\delta}{h_{out}}\right)\left(\frac{\frac{1}{2}\frac{h_{out}}{\left(0{,}28h_{in} + 0{,}72h_{out}\right)}\sqrt{\frac{\delta}{R'}} - \frac{1}{4}\frac{h_{out}\delta}{\left(0{,}28h_{in} + 0{,}72h_{out}\right)\mu R'} + \frac{1}{4}\frac{h_{out}}{\mu R'}\left(\frac{\sigma_{out}}{k_{out}} - \frac{\sigma_{in}}{k_{in}}\right)}{\sqrt{\frac{\delta}{R'}}}\right)^2 \tag{2}$$

The Equation (3) for interstand tension is obtained by application of the Hookes's law to a length of strip between successive stands:

$$\frac{\partial\sigma_{i,i+1}}{\partial t} \equiv \dot{\sigma}_{i,i+1} = \frac{E\left(V_{in,i+1} - V_{out,i}\right)}{L_0}, \qquad \sigma_{i,i+1}(0) = \sigma_{0,i,i+1} \tag{3}$$

where *E* is the Young's module of strip, $V_{in(out)}$ is the input (output) strip speed in the rolling stand, L_0 is the distance between adjacent mill stands and the subscript *i* means the order of rolling stand, {1, 2, 3 e 4}.

A linear approximation for the mill stretch of the rolling stand is used for estimates the output thickness according to Eq. (4).

$$h_{out} = S + \frac{F}{M_m} \tag{4}$$

where *S* is the position of roll position actuator over work roll, *F = PW* is the total roll force, *W* is the strip width and M_m is the mill modulus.

The interstand time delay is approximated by Eq. (5).

$$\sigma_{d,i,i+1} = \frac{L_0}{V_{out,i}} \tag{5}$$

where $\tau_{d,i,i+1}$ is the time delay between stands *i* and *i*+1 and $V_{out,i}$ is the strip speed exiting the roll bite area *i*, with *i* = 1, 2, 3 e 4.

Considering the time delay between adjacent mill stands the input thickness is modeled by Eq. (6), with *i* = 2, 3, 4 and 5.

$$h_{in,i}(t) = h_{out,i-1}\left(t - \tau_{di,i-1}\right) \tag{6}$$

With the forward slip known and considering the hypothesis of material continuity between adjacent stands and through the roll bite, the Eq. (7) calculates all work roll peripheral speeds and strip input speeds on stands, since known the output strip speed of rolling mill.

$$V_{in} = V_{out}\left(\frac{h_{out}}{h_{in}}\right) \quad \text{com} \quad V_{out} = V\left(f+1\right) \tag{7}$$

2.2.2. Representation in state-space of the process linearized model

The fundamental idea of this work was the acquisition of a linear model of the system delimited by a tandem cold metal rolling, with five stands. This system have a multivariable nature, therefore the state space representation is used. This representation followed the format given by Eq.(8), state equation and Eq. (9), output equation.

$$\dot{\mathrm{X}} = \mathrm{AX} + \mathrm{BU} + \mathrm{D}_{in}\mathrm{d} \qquad \mathrm{X}(0) = 0 \tag{8}$$

$$\mathrm{Y} = \mathrm{CX} + \mathrm{D}_{out}\mathrm{d} \tag{9}$$

where **X** is the state vector, **Y** is the response vector, **U** is the input or control vector, **d** is the disturbance vector, **A** is the matrix of system, **B** is the input matrix, **C** is the output matrix, $\mathbf{D_{in}}$ is the input disturbance matrix and $\mathbf{D_{out}}$ the output disturbance matrix.

The Equations (10), (11), (12) e (13) describe how the vectors **X**, **Y**, **U** and **d** are constituted and the components of this vectors are explained in next section.

$$X = [\,\overline{\Delta\sigma}_{in2} \;\; \overline{\Delta\sigma}_{in3} \;\; \overline{\Delta\sigma}_{in4} \;\; \overline{\Delta\sigma}_{in5} \;\; \overline{\Delta h}_{in2} \;\; \overline{\Delta h}_{in3} \;\; \overline{\Delta h}_{in4} \;\; \overline{\Delta h}_{in5} \;\; \overline{\Delta q}_{1} \;\; \overline{\Delta q}_{2} \;\; \overline{\Delta q}_{3} \;\; \overline{\Delta r}_{1} \;\; \overline{\Delta r}_{2} \;\; \overline{\Delta r}_{3} \;\; \overline{\Delta z}_{1} \;\; \overline{\Delta z}_{2} \;\; \overline{\Delta z}_{3} \;\; \overline{\Delta a}_{1} \;\; \overline{\Delta a}_{2} \;\; \overline{\Delta a}_{3} \;\; \overline{\Delta V}_{1} \;\; \overline{\Delta V}_{2} \;\; \overline{\Delta V}_{3} \;\; \overline{\Delta V}_{4} \;\; \overline{\Delta V}_{5} \;\; \overline{\Delta S}_{1} \;\; \overline{\Delta S}_{2} \;\; \overline{\Delta S}_{3} \;\; \overline{\Delta S}_{4} \;\; \overline{\Delta S}_{5}\,]^{T} \tag{10}$$

$$\mathrm{Y} = [\,\overline{\Delta h}_{out1} \;\; \overline{\Delta h}_{out2} \;\; \overline{\Delta h}_{out3} \;\; \overline{\Delta h}_{out4} \;\; \overline{\Delta h}_{out5} \;\; \overline{\Delta\sigma}_{out1} \;\; \overline{\Delta\sigma}_{out2} \;\; \overline{\Delta\sigma}_{out3} \;\; \overline{\Delta\sigma}_{out4} \;\; \overline{\Delta P}_{1} \;\; \overline{\Delta P}_{2} \;\; \overline{\Delta P}_{3} \;\; \overline{\Delta P}_{4} \;\; \overline{\Delta P}_{5}\,]^{T} \tag{11}$$

$$\mathrm{U} = \left[\overline{\Delta U}_{V_1} \;\; \overline{\Delta U}_{V_2} \;\; \overline{\Delta U}_{V_3} \;\; \overline{\Delta U}_{V_4} \;\; \overline{\Delta U}_{V_5} \;\; \overline{\Delta U}_{S_1} \;\; \overline{\Delta U}_{S_2} \;\; \overline{\Delta U}_{S_3} \;\; \overline{\Delta U}_{S_4} \;\; \overline{\Delta U}_{S_5}\right]^{T} \tag{12}$$

$$\mathrm{d} = \left[\overline{\Delta h}_{in,1} \;\; \overline{\Delta\sigma}_{in,1} \;\; \overline{\Delta\sigma}_{out,5}\right]^{T} \tag{13}$$

The matrices **A**, **B**, **C**, $\mathbf{D_{in}}$ and $\mathbf{D_{out}}$ and the vectors **X**, **Y**, **U** and **d**, which compose the linear model in space state, can be obtained according to the next section.

2.2.3. Linearizations and normalizations of the process mathematical model equations

The mathematical model for cold rolling process, shown in section 2.2.1., is characterized as multivariable non-linear and greatly complex model. The development of controllers by the using of complex and non-linear models is very expensive computationally and the literature shows, in Asano and Morari (1998), Lee and Lee (1999), Pittner et al. (2002) and Kim et al. (2004), is the application of linearization techniques in the equations, around the operating conditions, or operating points. For small variations in process variables, around the operating points, the linear approximation represents very well the system in simulation.

In this work the equations were linearized as well as normalized. Therefore all the steps of this task, until to arrive to linear model in state space, will be shown.

The linearization was made with the variation of ±5% in the variable value. In the normalization of any variable the value in operating point was used, except in the case of the position of roll position actuator S, normalized by output thickness in operating point, $h_{out,i0}$. Considering a generic variable x your normalized variation is defined by Eq. (14) that is a fractional variation of referred variable x in relation to your value in operating point x_0.

$$\overline{\Delta x} = \frac{x - x_0}{x_0} \tag{14}$$

In sequence of this text will be showed initially the linearized and normalized equations for roll force, output thickness and interstand tension, after the the normalized equations for positions and speed actuators and interstand time delay.

The roll force depends of the input and output thickness, input and output tension stress, friction, material plastic characteristics, how shows the Eq. (1). In this work, possibilities of variation in friction and material do not will be considered. The Equation (15) shows the normalized and linearized variation of the specific roll force, where the coefficients are calculated by relations done in Tab. 1. The subscribes *j* e *i* take the values {1, 2, 3 e 4} e {1, 2, 3, 4 e 5}, respectively.

$$\overline{\Delta P}_i = e_{2i}\overline{\Delta h}_{in,i} + \left(e_{1i} - 1\right)\overline{\Delta S}_i + e_{3i}\overline{\Delta\sigma}_{in,i} + e_{4i}\overline{\Delta\sigma}_{out,i} \tag{15}$$

The Equation (4) shows that the output thickness depends of the position of roll position actuator and specific roll force, therefore depends of all variables that affects this last variable. In these conditions, the normalized and linearized variation in output thickness is done by Eq. (16). Their coefficients are done in Tab.1, too.

$$\overline{\Delta h}_{out,i} = e_{1i}\overline{\Delta S}_i + e_{2i}\overline{\Delta h}_{in,i} + e_{3i}\overline{\Delta\sigma}_{in,i} + e_{4i}\overline{\Delta\sigma}_{out,i} \tag{16}$$

The linearization of equations for interstand tension was made aidded by software MAPLE 8 searching assure reliable results, due to the size of the treated equations. Beginning from Equation (3) and working with variations the linearization was made.

Similar to roll force, the forward slip *f*, will be considered dependent only of the input and output thickness and input and output tension stress. Any coefficients used in the linearized equations, related to *f*, are defined by relations done in Tab. 1. An adaptation suggested by Bryant (1973), $f^* = f + 1$, was used as well.

Table 1. Necessary coefficients for linearized and normalized equations of specific roll force and output thickness

Coefficients α_{ji}	Coefficients e_{ji}	Coefficients β_{ji}
$\alpha_{1i} = \dfrac{\partial P_i}{\partial h_{in,i}}$	$e_{1i} = \dfrac{M_{mi}}{M_{mi} - \alpha_{2i}}$	$\beta_{1i} = \dfrac{\partial f_i}{\partial h_{in,i}}$
$\alpha_{2i} = \dfrac{\partial P_i}{\partial h_{out,i}}$	$e_{2i} = \dfrac{\alpha_{1i} h_{in,i_0}}{h_{out,i_0}\left(M_{mi} - \alpha_{2i}\right)}$	$\beta_{2i} = \dfrac{\partial f_i}{\partial h_{out,i}}$
$\alpha_{3i} = \dfrac{\partial P_i}{\partial \sigma_{in,i}}$	$e_{3i} = \dfrac{\alpha_{3i} \sigma_{in,i_0}}{h_{out,i_0}\left(M_{mi} - \alpha_{2i}\right)}$	$\beta_{3i} = \dfrac{\partial f_i}{\partial \sigma_{in,i}}$
$\alpha_{4i} = \dfrac{\partial P_i}{\partial \sigma_{out,i}}$	$e_{4i} = \dfrac{\alpha_{4i} \sigma_{out,i_0}}{h_{out,i_0}\left(M_{mi} - \alpha_{2i}\right)}$	$\beta_{4i} = \dfrac{\partial f_i}{\partial \sigma_{out,i}}$

For interstand tension between stand 1 and 2, the linearized and normalized equation is done by Eq. (17), where your coefficients are calculated by relations done in Tab. 2.

$$\begin{aligned}\frac{d}{dt}\overline{\Delta\sigma}_{in,2} = {} & M_1\overline{\Delta\sigma}_{in,2} + M_2\overline{\Delta\sigma}_{in,3} + M_3\overline{\Delta h}_{in,1} + M_4\overline{\Delta V}_1 + M_5\overline{\Delta S}_1 + M_6\overline{\Delta h}_{in,2} + M_7\overline{\Delta V}_2 + \\ & + M_8\overline{\Delta S}_2 + M_9\overline{\Delta\sigma}_{in,1}\end{aligned} \tag{17}$$

For interstand tension between stand 2 and 3, the linearized and normalized equation is done by Eq. (18), where your coefficients are calculated by relations done in Tab. 3.

$$\begin{aligned}\frac{d}{dt}\overline{\Delta\sigma}_{in,3} = {} & N_1\overline{\Delta\sigma}_{in,2} + N_2\overline{\Delta\sigma}_{in,3} + N_3\overline{\Delta h}_{in,2} + N_4\overline{\Delta V}_2 + N_5\overline{\Delta S}_2 + N_6\overline{\Delta h}_{in3} + N_7\overline{\Delta V}_3 + \\ & + N_8\overline{\Delta S}_3 + N_9\overline{\Delta\sigma}_{in,4}\end{aligned} \tag{18}$$

For interstand tension between stand 3 and 4, the linearized and normalized equation is done by Eq. (19), where your coefficients are calculated by relations done in Tab. 4.

$$\frac{d}{dt}\overline{\Delta\sigma}_{in,4} = Q_1\overline{\Delta\sigma}_{in,3} + Q_2\overline{\Delta\sigma}_{in,4} + Q_3\overline{\Delta\sigma}_{in,5} + Q_4\overline{\Delta h}_{in,3} + Q_5\overline{\Delta h}_{in,4} + Q_6\overline{\Delta V}_3 + Q_7\overline{\Delta V}_4 + \\ + Q_8\overline{\Delta S}_3 + Q_9\overline{\Delta S}_4 \tag{19}$$

Table 2. Coefficients M_i of the linearized and normalized equation for interstand tension between stand 1 and 2.

$M_1 = \left(\frac{E}{L}\right)[\frac{V_{20}h_{out,20}\beta_{32}}{h_{in,20}} + \frac{V_{20}f^*_{20}e_{32}h_{out,20} + V_{20}\beta_{22}e_{32}\left(h_{out,20}\right)^2}{h_{in,20}\sigma_{in,20}} - \frac{V_{10}\beta_{21}e_{41}h_{out,10}}{\sigma_{out,10}} - V_{10}\beta_{41} + \frac{V_{20}h_{out,20}\beta_{32}}{h_{in,20}}]$	
$M_2 = \left(\frac{E}{L}\right)\left[\frac{V_{20}h_{out,20}}{\sigma_{in,20}h_{in,20}}\right]\left[\beta_{42}\sigma_{out,20} + f^*_{20}e_{42} + e_{42}\beta_{22}h_{out,20}\right]$	$M_3 = \left(\frac{E}{L}\right)\left[\frac{V_{10}h_{in,10}}{\sigma_{in,20}}\right]\left[\frac{-\beta_{21}e_{21}h_{out,10}}{h_{in,10} - \beta_{11}}\right]$
$M_4 = \left(\frac{E}{L}\right)\left[\frac{-V_{10}f^*_{10}}{\sigma_{in,20}}\right]$	$M_5 = \left(\frac{E}{L}\right)\left[\frac{-V_{10}\beta_{21}e_{11}h_{out,10}}{\sigma_{in,20}}\right]$
$M_6 = \left(\frac{E}{L}\right)\left[\frac{V_{20}h_{out,20}}{\sigma_{in,20}}\right]\left[\beta_{12} + \frac{f^*_{20}e_{22}}{h_{in,20}} + \frac{\beta_{22}e_{22}h_{out,20}}{h_{in,20}} - \frac{f^*_{20}}{h_{in,20}}\right]$	$M_7 = \left(\frac{E}{L}\right)\left[\frac{V_{20}f^*_{20}h_{out,20}}{h_{in,20}\sigma_{in,20}}\right]$
$M_8 = \left(\frac{E}{L}\right)\left[\frac{V_{20}e_{12}h_{out,20}}{h_{in,20}\sigma_{in,20}}\right]\left[h_{out,20}\beta_{22} + f^*_{20}\right]$	$M_9 = \left(\frac{E}{L}\right)\left(\frac{V_{10}}{\sigma_{in,20}}\right)\left(-h_{in,20}\beta_{21}e_{31} - \beta_{31}\sigma_{in,10}\right)$

Table 3. Coefficients N_i of the linearized and normalized equation for interstand tension between stand 2 and 3.

$N_1 = \left(\frac{E}{L}\right)\left[\frac{V_{20}}{\sigma_{out,20}}\right]\left[-\beta_{22}e_{32}h_{out,20} - \sigma_{in,20}\beta_{32}\right]$	$N_3 = \left(\frac{E}{L}\right)\left[\frac{-V_{20}\beta_{22}e_{22}h_{out,20}}{\sigma_{out,20}} - \frac{V_{20}\beta_{12}h_{in,20}}{\sigma_{out,20}}\right]$
$N_2 = \left(\frac{E}{L}\right)\left[\left(\frac{V_{30}h_{out,30}}{h_{in,30}}\right)\left(\beta_{33} + \frac{f^*_{30}e_{33}}{\sigma_{in,30}} + \frac{\beta_{23}e_{33}h_{out,30}}{\sigma_{in,30}}\right) - \frac{V_{20}\beta_{22}e_{42}h_{out,20}}{\sigma_{out,20}} - V_{20}\beta_{42}\right]$	
$N_4 = \left(\frac{E}{L}\right)\left[\frac{-f^*_{20}V_{20}}{\sigma_{out,20}}\right]$	$N_5 = \left(\frac{E}{L}\right)\left[\frac{-V_{20}\beta_{22}e_{12}h_{out,20}}{\sigma_{out,20}}\right]$
$N_6 = \left(\frac{E}{L}\right)\left[\frac{V_{30}h_{out,30}}{\sigma_{in,30}}\right]\left[\beta_{13} + \frac{f^*_{30}e_{23}}{h_{in,30}} + \frac{\beta_{23}h_{out,30}e_{23}}{h_{in,30}} - \frac{f^*_{30}}{h_{in,30}}\right]$	$N_7 = \left(\frac{E}{L}\right)\left[\frac{V_{30}f^*_{30}h_{out,30}}{h_{in,30}\sigma_{in,30}}\right]$
$N_8 = \left(\frac{E}{L}\right)\left[\frac{V_{30}h_{out,30}}{h_{out,20}\sigma_{in,30}}\right]\left[h_{out,30}\beta_{23}e_{43} + \beta_{43}\sigma_{in,40} + f^*_{30}e_{43}\right]$	$N_9 = \left(\frac{E}{L}\right)\left[\frac{V_{30}h_{out,30}}{h_{in,30}\sigma_{in,30}}\right]\left[f^*_{30}e_{13} + \beta_{23}h_{out,30}e_{13}\right]$

For interstand tension between stand 4 and 5, the linearized and normalized equation is done by Eq. (20), where your coefficients are calculated by relations done in Tab. 5.

$$\frac{d}{dt}\overline{\Delta\sigma}_{in5} = R_1\overline{\Delta\sigma}_{in4} + R_2\overline{\Delta\sigma}_{in5} + R_3\overline{\Delta h}_{in4} + R_4\overline{\Delta h}_{in5} + R_5\overline{\Delta V}_4 + R_6\overline{\Delta V}_5 + R_7\overline{\Delta S}_4 + \\ + R_8\overline{\Delta S}_5 + R_9\overline{\Delta\sigma}_{out5} \tag{20}$$

Table 4. Coefficients Q_i of the linearized and normalized equation for interstand tension between stand 3 and 4.

$Q_1 = \left(\frac{E}{L}\right)\left(\frac{V_{30} h_{out,30}}{\sigma_{in,40}}\right)\left(-\beta_{23} e_{33} - \frac{\beta_{33}\sigma_{in,30}}{h_{out,30}}\right)$	$Q_3 = \left(\frac{E}{L}\right)\left(\frac{V_{40} h_{out,40}}{\sigma_{in,40} h_{out,30}}\right)\left(h_{out,40}\beta_{24} e_{44} + \beta_{44}\sigma_{in,50} + f^*_{40} e_{44}\right)$
$Q_2 = \left(\frac{E}{L}\right)\left\{\left[\left(\frac{V_{40} h_{out,40}}{h_{out,30}\sigma_{in,40}}\right)\left(h_{out,40}\beta_{24} e_{34} + \beta_{34}\sigma_{in,40} + f^*_{40} e_{34}\right)\right] - \left[\left(V_{30} h_{out,30}\right)\left(\frac{\beta_{23} e_{43}}{\sigma_{in,40}} + \frac{\beta_{43}}{h_{out,30}}\right)\right]\right\}$	
$Q_4 = \left(\frac{E}{L}\right)\left(\frac{V_{30}}{\sigma_{in,40}}\right)\left(-\beta_{13} h_{out,20} - h_{out,30}\beta_{23} e_{23}\right)$	$Q_6 = \left(\frac{E}{L}\right)\left(\frac{-f^*_{30} V_{30}}{\sigma_{in,40}}\right)$
$Q_5 = \left(\frac{E}{L}\right)\left(\frac{V_{40} h_{out,40}}{\sigma_{in,40} h_{out,30}}\right)\left[\beta_{14} h_{out,30} + h_{out,40}\beta_{24} e_{24} + f^*_{40}\left(e_{24} - 1\right)\right]$	$Q_7 = \left(\frac{E}{L}\right)\left(\frac{f^*_{40} h_{out,40} V_{40}}{h_{out,30}\sigma_{in,40}}\right)$
$Q_8 = \left(\frac{E}{L}\right)\left(\frac{-V_{30} h_{out,30}\beta_{23} e_{13}}{\sigma_{in,40}}\right)$	$Q_9 = \left(\frac{E}{L}\right)\left(\frac{V_{40} h_{out,40} e_{14}}{h_{out,30}\sigma_{in,40}}\right)\left(h_{out,40}\beta_{24} + f^*_{40}\right)$

Table 5. Coefficients R_i of the linearized and normalized equation for interstand tension between stand 4 and 5.

$R_1 = \left(\frac{E}{L}\right)\left(\frac{V_{40}}{\sigma_{in,50}}\right)\left(-h_{out,40}\beta_{24} e_{34} - \beta_{34}\sigma_{in,40}\right)$	$R_3 = \left(\frac{E}{L}\right)\left(\frac{V_{40}}{\sigma_{in,50}}\right)\left(-\beta_{14} h_{out,30} - h_{out,40}\beta_{24} e_{24}\right)$
$R_2 = \left(\frac{E}{L}\right)\left\{\left[\left(\frac{V_{50} h_{out,50}}{h_{out,40}\sigma_{in,50}}\right)\left(h_{out,50}\beta_{25} e_{35} + \beta_{35}\sigma_{in,50} + f^*_{50} e_{35}\right)\right] - \left[V_{40}\left(\frac{h_{out,40}\beta_{24} e_{44}}{\sigma_{in,50}} + \beta_{44}\right)\right]\right\}$	
$R_4 = \left(\frac{E}{L}\right)\left(\frac{V_{50} h_{out,50}}{h_{out,40}\sigma_{in,50}}\right)\left[\beta_{15} h_{out,40} + h_{out,50}\beta_{25} e_{25} + f^*_{50}\left(e_{25} + 1\right)\right]$	$R_5 = \left(\frac{E}{L}\right)\left(\frac{-\overline{f_{40}} V_{40}}{\sigma_{in,50}}\right)$
$R_6 = \left(\frac{E}{L}\right)\left(\frac{f^*_{50} h_{out,50} V_{50}}{h_{out,40}\sigma_{in,50}}\right)$	$R_7 = \left(\frac{E}{L}\right)\left(\frac{-V_{40} h_{out,40}\beta_{24} e_{14}}{\sigma_{in,50}}\right)$
$R_8 = \left(\frac{E}{L}\right)\left(\frac{V_{50} h_{out,50} e_{15}}{h_{out,40}\sigma_{in,50}}\right)\left(h_{out,50}\beta_{25} + f^*_{50}\right)$	$R_9 = \left(\frac{E}{L}\right)\left(\frac{V_{50} h_{out,50}}{h_{out,40}\sigma_{in,50}}\right)\left(h_{out,50}\beta_{25} e_{45} + \beta_{45}\sigma_{out,50} + f^*_{50} e_{45}\right)$

The position and speed control systems of the work roll were used a first order lag. The Equation (21) compose the normalized version of this model, with $X = S$, for position and $X = V$ for speed.

$$\frac{d\overline{\Delta X_i}}{dt} = \frac{\overline{\Delta U_{X_i}}}{\tau_X} - \frac{\overline{\Delta X_i}}{\tau_X} \qquad X_i(0) = 0,\ \ i = 1, 2, 3, 4 \text{ e } 5. \tag{21}$$

where U_S e U_V are the references and τ_S and τ_V are the time constants, both referents to the position and speed systems of work roll, respectively.

For to simulate the delay time between two successive stands in linear form, were used four first order lags in cascade. Where the time constant used in each lag is equal to quarter part of nominal delay time between referred stands.

For the input thickness in stand 2 was used the Eq. (22), with auxiliary relations shown in Tab. 6.

$$\frac{d}{dt}\overline{\Delta h}_{in,2} = \left(\overline{\Delta q}_3 - \overline{\Delta h}_{in,2}\right)\left(\frac{4}{\tau_{12}}\right) \tag{22}$$

For the input thickness in stand 3 was used the Eq. (23), with auxiliary relations shown in Tab. 6.

$$\frac{d}{dt}\overline{\Delta h}_{in,3} = \left(\overline{\Delta r}_3 - \overline{\Delta h}_{in,3}\right)\left(\frac{4}{\tau_{23}}\right) \tag{23}$$

Table 6. Auxiliary relations for interstand delay time (first stands)

Auxiliary relations for stand delay 1-2	Auxiliary relations for stand delay 2-3
$\frac{d}{dt}\overline{\Delta q}_1 = \left(\overline{\Delta h}_{out,1} - \overline{\Delta q}_1\right)\left(\frac{4}{\tau_{12}}\right)$	$\frac{d}{dt}\overline{\Delta r}_1 = \left(\overline{\Delta h}_{out,2} - \overline{\Delta r}_1\right)\left(\frac{4}{\tau_{23}}\right)$
$\frac{d}{dt}\overline{\Delta q}_2 = \left(\overline{\Delta q}_1 - \overline{\Delta q}_2\right)\left(\frac{4}{\tau_{12}}\right)$	$\frac{d}{dt}\overline{\Delta r}_2 = \left(\overline{\Delta r}_1 - \overline{\Delta r}_2\right)\left(\frac{4}{\tau_{23}}\right)$
$\frac{d}{dt}\overline{\Delta q}_3 = \left(\overline{\Delta q}_2 - \overline{\Delta q}_3\right)\left(\frac{4}{\tau_{12}}\right)$	$\frac{d}{dt}\overline{\Delta r}_3 = \left(\overline{\Delta r}_2 - \overline{\Delta r}_3\right)\left(\frac{4}{\tau_{23}}\right)$

For the input thickness in stand 4 was used the Eq. (24), with auxiliary relations shown in Tab. 7.

$$\frac{d}{dt}\overline{\Delta h}_{in,4} = \left(\overline{\Delta z}_3 - \overline{\Delta h}_{in,4}\right)\left(\frac{4}{\tau_{34}}\right) \tag{24}$$

For the input thickness in stand 5 was used the Eq. (25), with auxiliary relations shown in Tab. 7.

$$\frac{d}{dt}\overline{\Delta h}_{in,5} = \left(\overline{\Delta a}_3 - \overline{\Delta h}_{in,5}\right)\left(\frac{4}{\tau_{45}}\right) \tag{25}$$

Table 7. Auxiliary relations for interstand delay time (last stands)

Auxiliary relations for stand delay 3-4	Auxiliary relations for stand delay 4-5
$\frac{d}{dt}\overline{\Delta z}_1 = \left(\overline{\Delta h}_{out,3} - \overline{\Delta z}_1\right)\left(\frac{4}{\tau_{34}}\right)$	$\frac{d}{dt}\overline{\Delta a}_1 = \left(\overline{\Delta h}_{out,4} - \overline{\Delta a}_1\right)\left(\frac{4}{\tau_{45}}\right)$
$\frac{d}{dt}\overline{\Delta z}_2 = \left(\overline{\Delta z}_1 - \overline{\Delta z}_2\right)\left(\frac{4}{\tau_{34}}\right)$	$\frac{d}{dt}\overline{\Delta a}_2 = \left(\overline{\Delta a}_1 - \overline{\Delta a}_2\right)\left(\frac{4}{\tau_{45}}\right)$
$\frac{d}{dt}\overline{\Delta z}_3 = \left(\overline{\Delta z}_2 - \overline{\Delta z}_3\right)\left(\frac{4}{\tau_{34}}\right)$	$\frac{d}{dt}\overline{\Delta a}_3 = \left(\overline{\Delta a}_2 - \overline{\Delta a}_3\right)\left(\frac{4}{\tau_{45}}\right)$

3. RESULTS AND DISCUSSIONS

After the construction of the linearization program and simulation diagram of model with Matlab/Simulink, the next step was the choice of the process operating points to obtain the linearized model. For the validation of model developed, three conditions of typical operating of five-stand tandem mill were adopted. In Pittner and Simaan (2011) are described this conditions and results, as well as the results of Bryant and Geddes too. Comparisons of the results obtained by the authors cited with results of this work are presented. For each case simulated, a disturbance in input thickness of stand 1 was applied and their effects valued in output thicknesses, roll forces and interstand tensions.

3.1. Operating conditions and characteristics of the rolling mill/strip

The operating conditions of rolling mill are defined by choice of the operating point. In the Tab. 8a are defined the three cases considered and Tab. 8b are the characteristics of the mill and strip used in simulation.

Table 8. Operating points and characteristics of the rolling mill and strip.

(a)

Parameter	Operating Point n.º		
	1	2	3
h_{in1} (mm)	3.56	2.36	1.78
h_{out1} (mm)	2.95	2.01	1.22
h_{out2} (mm)	2.44	1.52	0.79
h_{out3} (mm)	2.01	1.22	0.56
h_{out4} (mm)	1.68	0.97	0.38
h_{out5} (mm)	1.58	0.91	0.36
σ_{in1} (kN/mm^2)	0.0	0.0	0.0
σ_{in2} (kN/mm^2)	0.080	0.103	0.111
σ_{in3} (kN/mm^2)	0.078	0.126	0.132
σ_{in4} (kN/mm^2)	0.057	0.096	0.132
σ_{in5} (kN/mm^2)	0.055	0.060	0.085
σ_{out5} (kN/mm^2)	0.028	0.028	0.028

(b)

Parameter	Value
R (mm)	292
M_m (kN/mm)	3921
L_0 (mm)	4318
W (mm)	914
H_a / h_{in1}	1.095
E (kN/mm^2)	207
υ	0.3
μ	0.04

3.2. Calculated results

The variations in output thicknesses, roll forces and interstand tensions were registered after the application of a step with amplitude two in input thickness of stand 1, what means to apply a variation of +2% in this parameter at any time. The results were colleted for each operating point, in separated simulations and the Tab. 9 shows the results obtained in this work, with the obtained by Bryant, Geddes and Pittner, all contained in Pittner e Simaan (2011). In Tab. 9 are the mean values of the results obtained by the three operating points for each author cited and the obtained by this work.

Table 9. +2% Step change in stand 1 input thickness.

Variable	Source	Fractional Change in Variable % (Steady-State)				
		Stand 1	Stand 2	Stand 3	Stand 4	Stand 5
Output thickness	Bryant	1.8	1.8	1.8	1.8	1.7
	Geddes	2.32	2.13	1.90	2.14	2.21
	Pittner / Simaan	2.40	2.38	2.39	2.31	2.42
	Model	1.702	1.47	1.448	1.132	1.122
Roll force	Bryant	2.0	1.3	0.9	0.5	0.8
	Geddes	3.23	2.27	1.30	1.23	1.68
	Pittner / Simaan	2.17	1.67	1.35	1.09	1.57
	Model	1.702	1.47	1.448	1.132	1.122
Output tension stress	Bryant	-trace	-trace	+trace	-trace	-
	Geddes	+trace	1.7	4.6	-2.4	-
	Pittner / Simaan	1.2	0.7	0.4	8.8	-
	Model	-5.15	-5.24	-4.59	-5.14	-

3.3. Analysis of the results

In general the results were satisfactory, how shows the Tab. 9. In output thickness, regarding the work of Bryant (1973), a reference in mill control, the results obtained by this model showed a good consistency relative in initial stands. With relation to the roll force, there is consistency in the stand 1 and the results of model are better when compared with the others authors to stand 5. Important differences were detected in output tension stress, fact occurred to all works.

Several aspects can be appointed as responsible for differences observed. (i) The utilization of models with nature algebraic and empirical in calculation of roll force and forward slip, in according to Bryant (1973), whose principal characteristic is simplicity. The Bryant' model is widely used in control systems of industrial mills. There are models of bigger precision for this calculates; in Alves and Hemerly (2007) an example can be observed. Some effects can be added to the model for to improve the results, like the material elasticity in roll bite and strain rate. (ii) The linearization of system. For variations in inputs smaller than ±5% around the operating point, case in this work, the model offers a good response. (iii) The friction modeling. In Pittner and Simaan (2011) is adopted a model dependent of draft-diameter rate in work roll, speed roll and frictional characteristics. In this work the friction is considered constant. Finally (iv) The time delay interstand was considered constant and non-dependent of output speed of the stand.

4. CONCLUSIONS

The model developed is a model for control, dedicated to studies and researches on new strategies applied in tandem mill control, soon the results obtained do not represent relevant problems. The variations due to the input disturbances have small amplitude and the linearized model seemed adequate.

With this work a tool was built, favoring the study of conventional controllers, actually used in rolling mills and making a good contributing for the industrial control process.

The state space modeling offered the correct treatment for a multivariable system, favoring the access to internal variables, to state variables, and not only to output variables.

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5. RESPONSIBILITY NOTICE

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