A DIRECT MINIMIZATION METHOD TO REDUCED ORDER H-INFINITY CONTROLLER DESIGN: A GENETIC ALGORITHM APPROACH

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Abstract. *The design and implementation of full-order controllers often requires advanced hardware and high computational processing effort mainly for problems that involves large models, such as the vibration control of structures. To avoid this, it is recommended to use reduced order controllers. Approaches using Linear Matrix Inequalities (LMI) are widely employed in the design of reduced or fixed order controllers. This approach presented good results of performance and minimization of the H-infinity norm. However, the computational cost to obtain the controller can be high for large systems. The aim of this work is to present a direct minimization method for designing reduced order H-infinity controllers with a low computational cost. For this goal, it is formulated an optimization problem, so that minimizing the H-infinity norm guaranteeing the stability of the closed loop system. The solution of the optimization problem is obtained using the genetic algorithms, exploiting the advantage of this point of view where there are numerical difficulties and a complex search space. This formulation is verified in the design of a wind gust disturbance controller for the linearized model of the F-8 aircraft often used with other methods of reduced order H-infinity controller design. A comparison of the proposed formulation and the combination of the LMI and the Augmented Lagrangian method are presented in this work. The proposed approach can be applied to the vibration control problem of large structures. The optimization problem is solved using MATLAB software and some numerical aspects of the problem are also discussed.*

Keywords: *Reduced-order controller, H-infinity, genetic algorithm, vibration control.*

1. INTRODUCTION

In the controller design problem for active vibrations, one usual criterion is the minimization of the H-infinity norm, which consists of peak frequency response attenuation of the controlled system, while maintaining stability. Particularly in the case of problems of vibration control, the interest of the reduction of the peak value is associated to reduce the resonance peaks.

The H-infinity control is particularly interesting to increase the ability of the system to reject disturbances, as well as to increase its robustness.

One way to solve the problem of H-infinity control is through its formulation as a problem of linear matrix inequalities (LMI) (Boyd *et al.*, 1994; Scherer *et al.*, 1997). The usual H-infinity formulation leads to a full order controller (same order of the plant). This can represent difficulties for practical controller implementation. To avoid this difficulty, reduced order controllers can be designed, looking for reasonable performance and stability.

The design of reduced order H-infinity controllers characterizes a non-convex optimization problem. In this context, some authors have succeeded to treat the non-convexity using a combination of the Augmented Lagrangian method and linear matrix inequalities (LMI) (Apkarian *et al.*, 2003; Sarracini, 2006). This approach can be an efficient method but can lead to high numerical cost for the solution.

In this work an optimization problem to design reduced order controller is presented. The synthesis of reduced order controllers can be viewed as the minimization of the H-infinity norm of the closed loop system subject to the stability conditions. In this nonlinear optimization problem, each element of the controller matrices is an optimization variable to be found. To reduce the computational cost, the controller matrices will be obtained in its state-space canonical form. For the case of multiple-input and multiple output (MIMO), the controller is adapted to maintain the canonical form in one of its subsystems single-input and single-output (SISO).

The major difficulty of this problem is related to the search process (non-linear/non-convex optimization problem). The genetic algorithms are search stochastic algorithms based on natural selection where the central focus of research is the robustness and the balance between efficiency and efficacy necessary. The genetic algorithm are theoretically and empirically proven to provide robust search in complex space (Goldberg, 1989) and its potential for the reduced order controller design is investigated in this work. In this work it is used the MATLAB genetic algorithm toolbox. This allows to work easily with an algorithm already structured with a diversity of options to define the search process.

2. H-INFINITY CONTROLLER DESIGN

The H-infinity controller design represents an approach that can be treated as optimization problem in the frequency domain. It is used when levels of performance and stability against external disturbances needs to be ensured.

The H-infinity controller design looks for the reduction of the effects of external inputs and minimizes the frequency response peak of the system.

Methods to solve the H-infinity controller design problem have been developed in the last decades, for examples, the methods based on linear matrix inequalities, which make the full order H-infinity design as a convex optimization problem (Boyd *et al.*, 1994).

2.1 H-infinity norm

The H-infinity design consist in reducing the infinity norm of the transfer function of the output performance with respect to exogenous inputs, which represents the minimization of the frequency response peak of the closed loop system. In case of multivariable systems, the frequency response diagram refers to the singular values diagram, and in the case of single-input single-output systems it refers to the Bode diagram (Skogestad and Postlethwaite, 1996; Zhou and Doyle, 1998).

The H-infinity norm of the transfer function G(s) (SISO) is (Zhou *et al.*, 1996)

$$\|G(s)\|_\infty = sup_w |G(jw)| \tag{1}$$

and the H-infinity norm of the transfer matrix $\mathbf{G}(s)$ (MIMO) is (Zhou *et al.*, 1996)

$$\|\mathbf{G}(s)\|_\infty = sup_w \sigma_{max}(\mathbf{G}(jw)) \tag{2}$$

where $\sigma(\mathbf{G}(jw))$ represents the singular values of the transfer function $\mathbf{G}(s)$. $\sigma_{max}(\mathbf{G}(jw))$ is defined as (Skogestad and Postlethwaite, 1996)

$$\sigma_{max}(\mathbf{G}(jw)) = \sqrt{\lambda_{max}\left(\mathbf{G}^*(jw)\mathbf{G}(jw)\right)} \tag{3}$$

where $\lambda_{max}(\ )$ represents the largest eigenvalue and $\mathbf{G}^*$ denotes transpose and conjugate of the matrix $\mathbf{G}$.

2.2 Output feedback H-infinity Controller

Consider the linear plant of order n given by

$$\mathbf{\dot{x}} = \mathbf{Ax} + \mathbf{B_1w} + \mathbf{B_2u} \tag{4}$$

$$\mathbf{z} = \mathbf{C_1x} + \mathbf{D_{11}w} + \mathbf{D_{12}u} \tag{5}$$

$$\mathbf{y} = \mathbf{C_2x} + \mathbf{D_{21}w} + \mathbf{D_{22}u} \tag{6}$$

with $\mathbf{D_{22}} = \mathbf{0}$ without loss of generality since the result can be extended to the general case (Sánchez and Sznaier, 1998). $\mathbf{w}$ is the vector of exogenous inputs, $\mathbf{z}$ is the vector of performance outputs, $\mathbf{u}$ is the vector of control inputs, $\mathbf{y}$ is the vector of output measures and $\mathbf{x}$ is the state vector.

The transfer matrix of the plant given by Eq. (4), (5) and (6) from $\mathbf{w}$ and $\mathbf{z}$ is defined as:

$$\mathbf{P_{zw}}(s) = \mathbf{C_1}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B_1} + \mathbf{D_{11}} \tag{7}$$

Consider the linear output-feedback controller $\mathbf{K}(s)$ given by

$$\mathbf{\dot{x}_c} = \mathbf{A_c x_c} + \mathbf{B_c y} \tag{8}$$

$$\mathbf{u} = \mathbf{C_c x_c} + \mathbf{D_c y} \tag{9}$$

Thus, the control signal is given by

$$\mathbf{u} = \mathbf{K}(s)\mathbf{y}, \ \ \mathbf{K}(s) = \mathbf{C_c}(s\mathbf{I} - \mathbf{A_c})^{-1}\mathbf{B_c} + \mathbf{D_c}. \tag{10}$$

From the state space model of the plant, Eq. (4), (5) and (6), and the controller, Eq. (8) and (9), the closed loop system can be written as

$$\mathbf{\dot{x}_{cl}} = \mathbf{A_{cl}x_{cl}} + \mathbf{B_{cl}w}, \tag{11}$$

$$\mathbf{z} = \mathbf{C_{cl}}\mathbf{x_{cl}} + \mathbf{D_{cl}}\mathbf{w}, \tag{12}$$

where:

$$\dot{\mathbf{x}}_{\mathbf{cl}} = \begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{x}}_{\mathbf{c}} \end{bmatrix}, \mathbf{A_{cl}} = \begin{bmatrix} \mathbf{A} + \mathbf{B_2}\mathbf{D_c}\mathbf{C_2} & \mathbf{B_2}\mathbf{C_c} \\ \mathbf{B_c}\mathbf{C_2} & \mathbf{A_c} \end{bmatrix}, \mathbf{B_{cl}} = \begin{bmatrix} \mathbf{B_1} + \mathbf{B_2}\mathbf{D_c}\mathbf{D_{21}} \\ \mathbf{B_c}\mathbf{D_{21}} \end{bmatrix}, \tag{13}$$

$$\mathbf{C_{cl}} = [\mathbf{C_1} + \mathbf{D_{12}}\mathbf{D_c}\mathbf{C_2} \;\; \mathbf{D_{12}}\mathbf{C_c}],\; \mathbf{D_{cl}} = [\mathbf{D_{11}} + \mathbf{D_{12}}\mathbf{D_c}\mathbf{D_{21}}] \tag{14}$$

Thus, the transfer matrix of closed loop from $\mathbf{w}$ and $\mathbf{z}$ is defined as:

$$\mathbf{T_{zw}}(s) = \mathbf{C_{cl}}(s\mathbf{I} - \mathbf{A_{cl}})^{-1}\mathbf{B_{cl}} + \mathbf{D_{cl}} \tag{15}$$

The H-infinity optimal control seeks to find the admissible controllers $\mathbf{K}(s)$ (that internally stabilizes the system) so that $\|\mathbf{T_{zw}}\|_\infty$ is minimized (Zhou and Doyle, 1998), i.e. looks for to minimize, as much as possible, the frequency response peak between exogenous input and performance output.

The H-infinity sub-optimal control design (Zhou and Doyle, 1998) looks for to find the admissible controller $\mathbf{K}(s)$, if it exists, so that, for $\gamma > 0$,

$$\|\mathbf{T_{zw}}\|_\infty < \gamma \tag{16}$$

In search of the optimal admissible controller $\mathbf{K}(s)$, γ is minimized until the condition of Eq. (16) is not satisfied, or an unstable closed loop $\mathbf{T_{zw}}$ is found. In other words, the controller $\mathbf{K}(s)$ is optimum if solves the following optimization problem (Zhou and Doyle, 1998):

$$\begin{array}{ll} \text{min} & \gamma \\ \text{s.a} & \|\mathbf{T_{zw}}\|_\infty < \gamma \end{array} \tag{17}$$

with $\|\mathbf{T_{zw}}\|_\infty$ stable.

This optimization problem is often used to obtain the full order controller (controller with the same order of the plant). This can be solved using the formulations based on Riccati equations or LMI approaches (Boyd *et al.*, 1994; Skogestad and Postlethwaite, 1996).

3. DIRECT MINIMIZATION METHOD FOR REDUCED-ORDER H-INFINITY CONTROLLER

The reduced-order control design can be based on the solution of the optimization problem presented in Eq. (17). The linear controller shown in Eq. (8) and (9) can be of order $k \leq n$ where each element of the controller matrices is an optimization variable to be found.

To reduce the computational cost it is considered the following coordinate transformation:

$$\mathbf{x_c} = \mathbf{T}\mathbf{x_t} \tag{18}$$

Using the transformation Eq. (18) in the Eq. (19) and (20), the controller can be rewritten as

$$\dot{\mathbf{x}}_{\mathbf{t}} = \mathbf{T^{-1}}\mathbf{A_c}\mathbf{T}\mathbf{x_t} + \mathbf{T^{-1}}\mathbf{B_c}\mathbf{y} \tag{19}$$

$$\mathbf{u} = \mathbf{C_c}\mathbf{T}\mathbf{x_t} + \mathbf{D_c}\mathbf{y} \tag{20}$$

Considering the first controller input signal (first column of $\mathbf{B_c}$) as reference for the transformation, as presented in (Kailath, 1980; Ogata, 2003), it is obtained:

$$\mathbf{T} = [\mathbf{B_{c1}} \quad \mathbf{A_c}\mathbf{B_{c1}} \quad \mathbf{A_c^2}\mathbf{B_{c1}} \quad \ldots \quad \mathbf{A_c^{k-1}}\mathbf{B_{c1}}] \tag{21}$$

where $\mathbf{B_{c1}}$ is the vector component of the input matrix $\mathbf{B_c}$ with respect to the first input.

This transformation presents the controller in its controllable canonical form with respect to its first inputs as shown in Eq. (22) and (23),

$$\mathbf{A_c} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & \alpha_1 \\ 1 & 0 & 0 & \cdots & 0 & 0 & \alpha_2 \\ 0 & 1 & 0 & \cdots & 0 & 0 & \alpha_3 \\ \vdots & \vdots & \vdots & \ldots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 & \alpha_{k-1} \\ 0 & 0 & 0 & \cdots & 0 & 1 & \alpha_k \end{bmatrix}, \; \mathbf{B_c} = \begin{bmatrix} 1 & \beta_{1,1} & \beta_{1,2} & \cdots & \beta_{1,j} \\ 0 & \beta_{2,1} & \beta_{2,2} & \cdots & \beta_{2,j} \\ 0 & \beta_{3,1} & \beta_{3,2} & \cdots & \beta_{3,j} \\ \vdots & \vdots & \vdots & \ldots & \vdots \\ 0 & \beta_{k-1,1} & \beta_{k-1,2} & \cdots & \beta_{k-1,j} \\ 0 & \beta_{k,1} & \beta_{k,2} & \cdots & \beta_{k,j} \end{bmatrix} \tag{22}$$

$$\mathbf{C_c} = \begin{bmatrix} c_{1,1} & c_{1,2} & c_{1,3} & c_{1,4} & c_{1,5} & \cdots & c_{1,k} \\ c_{2,1} & c_{2,2} & c_{2,3} & c_{2,4} & c_{2,5} & \cdots & c_{2,k} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ c_{i,1} & c_{i,2} & c_{i,3} & c_{i,4} & c_{i,5} & \cdots & c_{i,k} \end{bmatrix}, \mathbf{D_c} = \begin{bmatrix} d_{1,1} & d_{1,2} & \cdots & d_{1,j} \\ d_{2,1} & d_{2,2} & \cdots & d_{2,j} \\ \vdots & \vdots & \cdots & \vdots \\ d_{i,1} & d_{i,2} & \cdots & d_{i,j} \end{bmatrix} \tag{23}$$

where j and i are the number of inputs signal and output signal of the controller respectively.

This controller state space model structure makes possible reducing the number of unknowns variables in the reduced-order controller design. The optimization problem can be rewritten to ensure stability of both controlled system and fixed order controller as follows:

$$\begin{array}{cc} \min_{\alpha_k,\beta_{k,j},c_{i,k},d_{i,j}} & \|\mathbf{T_{zw}}\|_\infty \\ \text{s.a} & \max(\text{real}(\lambda(\mathbf{A_k})))< \epsilon_1 \\ & \max(\text{real}(\lambda(\mathbf{A_{cl}})))< \epsilon_2, \end{array} \tag{24}$$

where $\lambda(\cdot)$ represent the corresponding eigenvalues and ϵ_1 and ϵ_2 are negative values close to zero to ensure stability.

To improve the results one can include the following constraint to the optimization problem Eq. (24):

$$\|\mathbf{T_{zw}}\|_\infty < \|\mathbf{P_{zw}}\|_\infty \tag{25}$$

which means that the closed loop system should present a better H-infinity norm compared to the non-controlled system.

The optimization problem given by Eq. (24) and (25) is a nonlinear optimization problem that presents a complex search space. For this solution the genetic algorithms are employed in this work exploiting its robustness in the search process.

4. GENETIC ALGORITHM APPROACH

The genetic algorithms (GA) are a class of search method that use forms of biological processes such as selection, inheritance, mutation and crossover. The GA can be used to solving optimization problems presenting the following advantages with respect to others conventional search methods (Goldberg, 1989):

1. They work with a code to the parameter set, not with parameters values.
2. They are not limited by restrictive assumptions such as concerning to continuity, existence of derivaties, unimodality and other matters.
3. They search from a population of points and not of a single point.
4. They use probabilistic transition rules and not deterministic rules.

These features make GA a potentially useful approach in the controller design of this work.

GA work simulating the behavior of the nature of the individuals. They work with populations of individuals that representing a possible solution to the optimization problem. The individuals compete among themselves to produce the next population where the natural selection depends on how good is the adaptation of the individual to the problem.

A simple GA use three operators to create the next generation from the current population (Goldberg, 1989):

1. Selection: Select the parents that contribute to the creation of the population of the next generation.
2. Crossover: Cross two parents to form the children of the next generation.
3. Mutation: Apply random changes to the parents to form the children.

The power of GA is their robustness, and it can be used successfully in a wide range of problem areas. GA are not guaranteed to find the global optimum solution to a problem, but they can find an acceptably good solutions in a reasonable time.

This work uses the MATLAB genetic algorithm toolbox (MATLAB, 2005). The command line ga is used allowing to work easily on an algorithm already structured.

To set the features of the GA, it is defined the fitness functions (objective function) shown in the optimization problem of Eq. (24). The constraints presented in Eq. (24) and (25) are also implemented and included in the function ga.

The default setting options of the function ga where the relevant information characterizing the algorithm with 20 individuals in each generation are used in this work. Maximum number of generations is 100. The tolerance in the changes of the fitness function value before stopping is 10^{-6}. The selection process is made in the stochastic uniform form (options *@selectionstochunif*). The crossover function, creates a random binary vector and selects the genes where the vector is a 1 from the first parent, and the genes where the vector is a 0 from the second parent, and combines the genes to form the child (options *@crossoverscattered*). A crossover fraction is 0.8. The mutation function used to create the mutation children adds a random number taken from a Gaussian distribution (options *@mutationgaussian*).

With these considerations the optimization problem can be solved and a reduced-order H-infinity controller can be designed.

5. APPLICATION EXAMPLE

The following linearized model of the F-8 aircraft presented in (Lublin *et al.*, 1996) and often used with other methods of reduced order H-infinity controller design (Apkarian *et al.*, 2003; Calafiore *et al.*, 2000) is used as an example here. The plant includes the aircraft model and weighting functions to the treatment of wind disturbance rejection and unmodeled dynamics.

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1.5 & -1.5 & 0 & 0.0057 & 1.5 & 0 & 0 & 0 \\ -12 & 12 & -0.6 & -0.0344 & -12 & 0 & 0 & 0 \\ -0.852 & 0.29 & 0 & -0.014 & -0.29 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.73 & 2.8289 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1.25 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1000 \end{bmatrix}, \mathbf{B_1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.1146 & 0 & 0 \\ 4 & 0 & 0 \\ 0 & 1024 & 0 \\ 0 & 0 & 1024 \end{bmatrix}$$

$$\mathbf{B_2} = \begin{bmatrix} 0 & 0 \\ 0.16 & 0.8 \\ -19 & -3 \\ -0.0115 & -0.0087 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \mathbf{C_1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \mathbf{D_{11}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{D_{12}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}, \mathbf{C_2} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -139.0206 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -139.0206 \end{bmatrix}$$

$$\mathbf{D_{21}} = \begin{bmatrix} 0 & 142.8571 & 0 \\ 0 & 0 & 142.8571 \end{bmatrix}, \mathbf{D_{22}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The H-infinity norm of the non-controlled system is 32.2424 dB. Using genetic algorithms and the plant model for the solution of the optimization problem Eq. (24) and (25), reduced order controllers of order 2 and 1 are designed as presented in the following sections.

5.1 Controller of order 2×2

The controller obtained, using genetic algorithms, of order 2×2 is

$$\mathbf{A_c} = \begin{bmatrix} 0 & -0.6333 \\ 1 & -2.14 \end{bmatrix}, \mathbf{B_c} = \begin{bmatrix} 1 & -2.557 \\ 0 & -0.3711 \end{bmatrix}, \mathbf{C_c} = \begin{bmatrix} -0.3144 & 1.814 \\ 1.703 & 3.679 \end{bmatrix}, \mathbf{D_c} = \begin{bmatrix} 0.5834 & -0.2051 \\ 0.1708 & -0.9859 \end{bmatrix}$$

The H-infinity norm of the controlled system is 2.9820 dB as shown in Fig. 1.

Implementing and using the LMI and Augmented Lagrangian method (LMI - AL method) (Apkarian *et al.*, 2003), the controlled system presented a minimization of the H-infinity norm of 17.2419 dB as shown in Fig. 1. The comparison of both method is shown in the Tab. 1.

The parameters used for the LMI and AL method were: update factor of the penalty parameters $\rho = 1.7$, initial matrix of the penalty parameters $\mathbf{C^0} = 10^{-8}\mathbf{I}$, where $\mathbf{I}$ is the identity matrix, initial matrix of the Lagrange multipliers $\mathbf{\Lambda^0} = \mathbf{0}$ and convergence value of the non-linear constraint $\epsilon = 10^{-5}$.

5.2 Controller of order 1×1

The controller obtained, using genetic algorithms, of order 1×1 is

$$\mathbf{A_c} = -1.054,\ \mathbf{B_c} = \begin{bmatrix} 1 & -4.454 \end{bmatrix}, \mathbf{C_c} = \begin{bmatrix} 0.3908 \\ 2.206 \end{bmatrix}, \mathbf{D_c} = \begin{bmatrix} 0.4708 & -0.1175 \\ 0.6139 & -0.8949 \end{bmatrix}$$

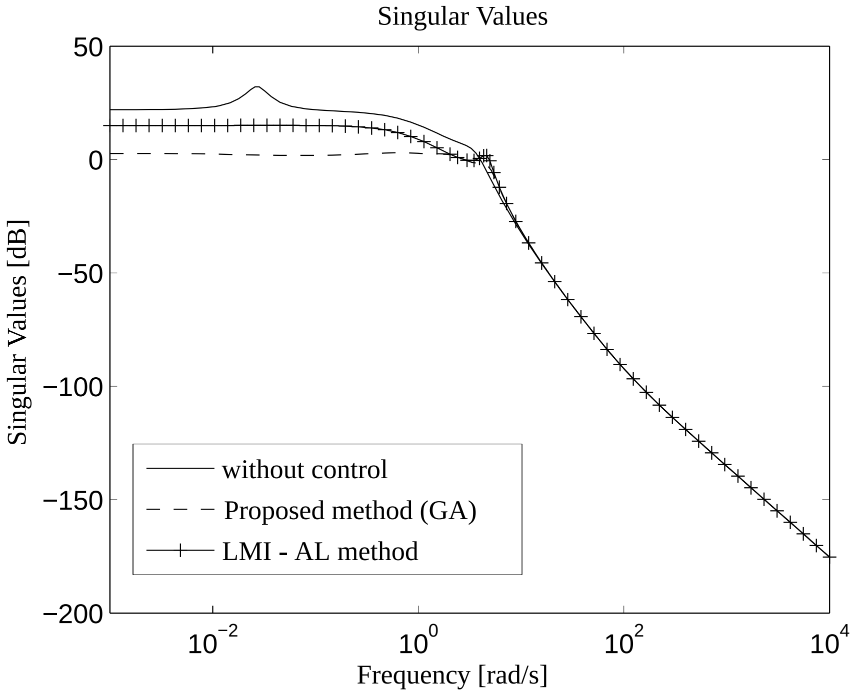

Figure 1. Frequency response of the controlled system (Proposed method and LMI and Augmented Lagrangian method) and the non-controlled system: controller of order 2×2

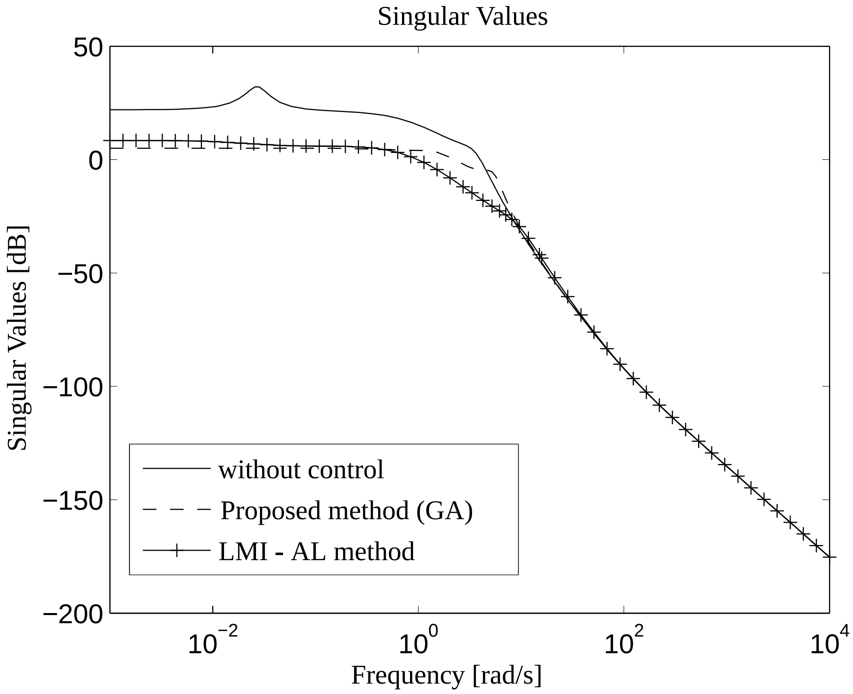

Figure 2. Frequency response of the controlled system (Proposed method and LMI and Augmented Lagrangian method) and the non-controlled system: controller of order 1×1

The H-infinity norm of the controlled system is 4.9967 dB as shown in Fig. 2.

Using LMI and Augmented Lagrangian method, the controlled system presented a minimization of the H-infinity norm of 23.7834 dB as shown in Fig. 2.

The results were obtained on a laptop with intel core 2 duo processor (1 and 2: P7450 - 213 GHz) with 3 GB of memory. The operating system used was Ubuntu 10.04 (Linux).

Table 1. Main results of both methods: proposed method using genetic algorithm and LMI - AL method

Method	Order of the controller	Minimization of the H-infinity norm [dB]	Computational time [s]
Proposed method (GA)	2×2	29.2604	178.2218
	1×1	27.2457	186.6126
LMI - AL method	2×2	17.2419	1591.6606
	1×1	23.7834	793.1117

6. CONCLUSION

The main idea of this work is to present a simple and intuitive method to the design of reduced order H-infinity controllers. The method is based on the solution of a non-linear optimization problem to guarantee stability and good performance of the system. The method exploited the advantages offered by genetic algorithms in the solution of complex optimization problems.

The method was employed in the linearized model of a F-8 aircraft to the wind disturbance control and compared with the LMI and Augmented Lagrangian method. The controllers designed presented a good minimization of the H-infinity norm, close to the LMI and Augmented Lagrangian method, in a shorter processing time for this specific problem.

The proposed method and the LMI and Augmented Lagrangian method were implemented considering a particular choice of parameters: selection function, crossover function, mutation function, tolerance and others for the proposed method and penalty parameters, Lagrange multipliers, values of convergence and others for the LMI and Augmented Lagrangian method. The appropriate choice of this parameters can lead to better results than those shown in this work.

The focus of this work was to present an alternative solution to the problem of reduced order controllers. The comparison of the proposed approach with the method presented in (Apkarian *et al.*, 2003) had the main purpose of validating the results. To show superiority between the methods, further studies should be made on the appropriate choice of search parameters.

Genetic algorithms have shown a good potential in the solution of the reduced order controller design. In the area of control design based on the solution of optimization problems the study and use of genetic algorithms can be an attractive alternative solution.

The solution approach has proved to be a good and simple design alternative which can be exploited to some classical formulations based on Riccati and LMI approaches.

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