ACTIVE VIBRATION CONTROL FOR EULER-BERNOULLI CANTILEVER BEAM

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Abstract. *It is quite common in flexible structures analysis the needs of attenuating vibration to desired levels or even to eliminate them. To achieve this goal, there are three control techniques: the passive, the active and the semi-passive control. Active control uses external actuators, controlled by a loop in real time to eliminate or mitigate the forces responsible for these vibrations. However for this technique to provide satisfactory results, there are several factors to take into consideration, among them the type of actuator and sensor being used and the type of controller. There are many studies related to this issue and one of them is the use of a piezoelectric material, which acts both as sensors and actuators. These materials have some advantages like the little added weight to the structure, associated to good performance. Most flexible structures are distributed parameter systems and therefore, problems with infinite dimensions are not practical for control design, so some mathematical techniques are used to bring such systems to finite dimensions, and one of the most used is the Finite Element Method. This paper proposes to model an Euler-Bernoulli cantilever beam and incorporate the piezoelectric sensors and actuators dynamics, using the Finite Element Method. It will be developed a control design, both in time and in frequency domains in order to compare the results obtained by both approaches.*

Keywords*: Active Vibration Control, Finite Element Method, Modal Analysis*

1. INTRODUCTION

Currently, due to the demands of quality and performance on the market, it becomes necessary the use of efficient equipments and structures as well low-cost manufacturing, maintenance and operation. Accordingly, various researches are focused on developing techniques for active vibration control. This technique use external actuators, in real time control loop, which act to eliminate or to reduce the forces responsible for the undesirable vibrations. The active control main idea is presented in Figure 1

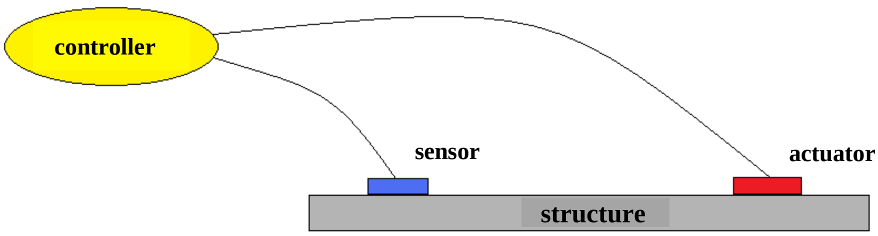


Figure 1: Active control scheme.

However, for this technique to show satisfactory results, there are several factors to be considered, among them the type of actuator and sensor used as well the type of controller. Thus there are many researches related to this issues and one of them concerns piezoelectric material, which acts both as sensor or actuator. Some advantages of these materials are the low weight added to the structure, combined to the fast response.

Most flexible structures are distributed parameters systems and these infinite-dimension problems are not practical for control design, so some mathematical techniques are used to bring these systems to finite dimensions and one of the most used is the Finite Element Method.

This paper proposes to model an Euler-Bernoulli cantilever beam as well to incorporate the piezoelectric sensors and actuators dynamics using the Finite Element Method. Afterwards it will be developed a control design, both in the time and in the frequency domains in order to compare the results obtained by them.

2. DYNAMICS OF EULER-BERNOULLI BEAM AND MODAL ANALISYS

The Finite Element Method is a numerical procedure for solving physical problems governed by a differential equation or an energy theorem. It has two characteristics that distinguish it from the other numerical procedures: a) utilizes an integral formulation to generate a system of algebraic equations and b) uses continuous piecewise smooth functions for approximating the unknown quantity or quantities (SEGERLIND, 1984).

The Euler-Bernoulli beam equation is represented by;

$$\rho\frac{\partial^2 v}{\partial t^2}+\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 v}{\partial x^2}\right)=q(x,t) \tag{1}$$

Where $v(x,t)$ is the beam transversal displacement, ρ is the mass density per volume, E is the Young Modulus, I is the Inertia Moment and $q(x,t)$ is the external applied load.

It was considered the typical beam finite element with two nodes and two degrees of freedom per node, as shown in Figure 2 below.

Figure 2: Beam finite element

The spring-mass system motion equation without damping can be written as:

$$[M]\{\ddot{d}\}+[K]\{d\}=\{F\} \tag{2}$$

Where,

$[M]$ = mass matrix
$[K]$ = stiffness matrix
$\{F\}$ = load vector
$\{d\}$ = displacement vector

To determinate the system eigenfrequencies, the applied load shall be done equal to zero.

$$[M]\{\ddot{d}\}+[K]\{d\}=0 \tag{3}$$

Whereas the free vibration movement is a simple harmonic, the solution can be of the type;

$$d(t)=D\sin(\omega t) \tag{4}$$

And, finally, substituting (4) in (3), it is possible to write;

$$[[K]-\omega^2[M]]\{d\}=0 \tag{5}$$

Last equation represents a problem of eigenvalue or characteristic value. The amount of ω^2 are eigenvalues or characteristic values indicating the square root of the natural frequencies, while the corresponding values of vector $\hat{d}$ indicates the vibrating system shapes (eigenvectors).

3. PIEZOELECTRIC TRANSDUCERS DYNAMICS

The piezoelectric effect was studied in 1880 in quartz crystals, by Pierre and Jacques Curie. This effect consists basically on materials geometrical deformation when they are subjected to an electric field and conversely, they produce electric polarization in response to mechanical stresses. Some of the most used materials that exhibit such property are the PZT ceramics (Piezoelectric Transducer) and plastic films PVDF (Polyvinylidene Fluoride).

The PZTs are made basically by lead oxide, zirconium and titanium and by having a large stiffness they are indicated to be used as actuators. On the other hand PVDFs are robust and flexible polymers which can be produced in complex geometries; they are flexible and have little weight. For such characteristics, they are suited for distributed sensoring.

The piezoelectric effect has a linear dependence between induced strain and applied electric field. So if the electric field direction is reversed, the deformation direction will be also reversed.

These materials generally show a good linearity in the ratio between the applied electric field and induced strain, but when they are subjected to an electric field with high intensity the polarization saturation phenomenon occurs and causes the electric dipoles inversion leading to a significant hysteresis and nonlinear ratios between electric field and induced strain. Another behavior that should be observed in the use of piezoelectric materials is that the temperature should not exceed a threshold value, named Curie temperature, from which there is a spontaneous material depolarization and loss of piezoelectric characteristics. But at temperatures below the Curie temperature, these materials show a relative insensitivity with respect to temperature variation.

3.1. Euler-Bernoulli beam element electromechanically coupled

In the electromechanical coupling, the structural element has three degrees of freedom per node, two mechanical (one linear v_i and one angular Θ_i displacements) and one related to electric potential φ_i.

Figure 3: Beam finite element electromechanically coupled.

3.2. Coupled electromechanical structure model

The finite element method basic idea is to use the variables as parameters for a nodal finite number of points previously chosen. Performing this procedure, the displacements d can be written as elements function using the nodal interpolation functions. This relation is expressed as follows below;

$$d = N_d \, d_i \tag{6}$$

where,

N_d = interpolation function (shape function)

d_i = displacement at node *i*

Besides the displacements it is also necessary to consider as nodal variable the electrical potential, φ. Analogously we can write it as;

$$\varphi = N_{\varphi}\varphi_i \tag{7}$$

where,

N_{φ} = interpolation function (shape function)

φ_i = electrical potential at node *i*

To find the piezostructure motion equation, it was used Lagrange's equation. This formulation considers mechanical degrees of freedom, which describe the movement in each structural element defined by d and the electrical degrees of freedom φ defined by the electrical potential. Thus Lagrange's equation is defined by;

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial d}\right) - \frac{\partial L}{\partial d} = F \tag{8}$$

and

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \varphi}\right) - \frac{\partial L}{\partial \varphi} = Q \tag{9}$$

with

F = external forces applied
Q = induced electric charge
L = Lagrangian defined as

$$L = T - U + W_e \tag{10}$$

where,

T = kinetics energy
U = potential energy
W_e = work of electrical potential

After mathematical manipulation, the final overall motion equation for a coupled electromechanical structure is defined by:

$$M\ddot{d} + D\dot{d} + Kd = F - K_{d\varphi}K_{\varphi\varphi}^{-1}Q \tag{11}$$

where,
M = mass matrix
D = damping matrix
K = stiffness matrix
$K_{d\varphi}$ = electromechanical coupling matrix

$K_{\varphi\varphi}$ = piezoelectrical capacitance matrix

4. PIEZO-STRUCTURE CONTROL

The control theory has two basic strands: the frequency domain (classical control) and time domain (modern control). In modern control theory, dynamic models are represented in the frequency domain by transfer function that characterizes the ratio between system input and output, i.e. the system transfer function is represented by time invariant linear differential equations defined as the ratio between the output and input signals Laplace transform.

The modern control theory is based on the dynamic systems description by means of ***n*** first order differential equations that can be combined in a vector-matrix differential equation of first order (PALMA, 2007). The modern control techniques allow the systems design with single input and single output (SISO) and multivariable systems, multiple input and multiple output (MIMO) as easily, i.e. increasing the number of state variables does not increase the equations complexity.

4.1. State observer

A control system with state feedback can not be done when some system variable is not available for measurement, unless this variable is estimated. When a computer or other device estimates this variable, it is named the state observer or simply an observer. The state observer is a mathematical model used to construct a physical system based on the sensor (OGATA, 2003). For a control system with state feedback, which was used in this work, we can use the estimated states as the feedback system.

The procedure is described as follows: The observer compares the output value of the real system with the observer and the output is fed back to a gain L. The estimated states are used to provide the system with a real gain K. To determine the gains value there are several methods, in particular in this paper we use the Linear Quadratic Regulator, which was originated from the theory of optimal control. This theory basic idea is to obtain a performance function and design a control law to minimize the first.

5. RESULTS

In order to simulate the subjects above exposed it was chosen a beam according to the Figure 4 below.

Figure 4: Beam model used in the simulation

The material properties as well the beam geometry are presented in the Table 1. The damping matrix D is proportional to the mass and stiffness matrices, and the constants used were α = 0.001 and β = 0.0003 for the relation $D = \alpha M + \beta K$

Table 1: Beam data used in simulation

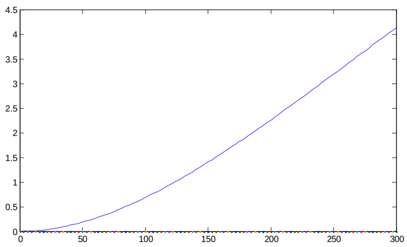
Young Modulus (MPa)	70000
Poisson ratio	0.3
Mass density (kg/ m³)	2710
Length (mm)	300
Width (mm)	20
Height (mm)	5

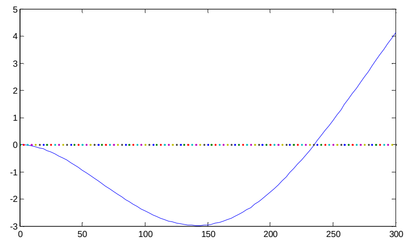
It was analyzed the first four eigenmodes using the Finite Element Method theory developed in this work whose results were compared to a commercial FEM software. The results are shown in the Table 2 as follow.

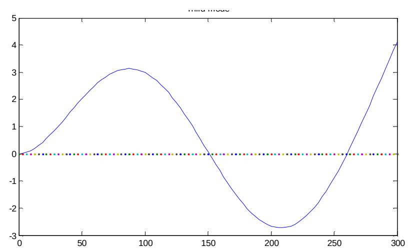
Table 2: First four eigenfrequencies

Eigenvalues	Eigenfrequencies (Hz)		
	FEM – this work	FEM – commercial software	Error (%)
1°	9.0626	9.0477	0.164683
2°	56.7946	56.7629	0.055846
3°	159.0337	158.4179	0.388719
4°	311.6973	309.4849	0.714865

In possession of these data it was plotted the first four eigenmodes based on the values calculated above. These plots were done both by the theory developed in this work, Figure 5, and also by a commercial FEM software, Figure 6.


First eigenmode


Second eigenmode


Third eigenmode

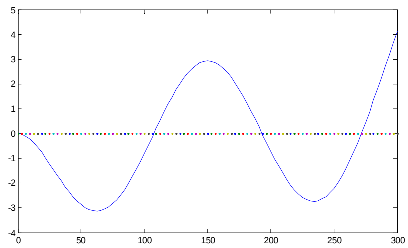
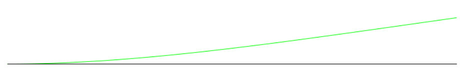
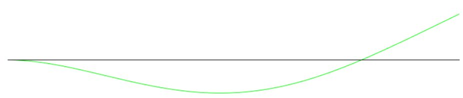
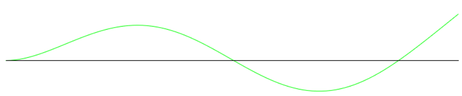

Fourth eigenmode

Figure 5: First four eigenmodes – this work


First eigenmode


Second eigenmode


Third eigenmode

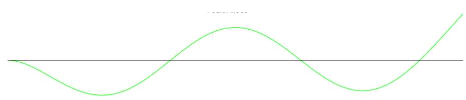

Fourth eigenmode

Figure 6: First four eigenmodes – commercial FEM software

As can be seen, for the structure eigenfrequencies and eigenmodes, the model developed in this work showed good results and can be considered to progress the analysis.

The next step was to model the piezo-structures using the geometrical and electrical physical properties described in Table 3

Table 3: Piezoelctric data used in the simulation

Young Modulus (MPa)	62000
Elasticity constant (MPa)	92.3e3
Piezoelectric voltage coefficient (C/mm²)	-1,63E-05
Dielectric Constant (F/mm)	3,36E-11
Mass density (kg/ m³)	7.5e-6
Length (mm)	20
Width (mm)	20
Height (mm)	0.26

This work used two actuators (one for control input and other for disturb) and one sensor. No special technique was used to define the sensor/actuators placement, but some candidate positions and finally chosen those with less interference to the eigenfrequencies as shown in Figure 7 (BUENO, 2007). The structure for the electromechanical coupling was divided into 15 elements and the PZTs length is equivalent to one element.

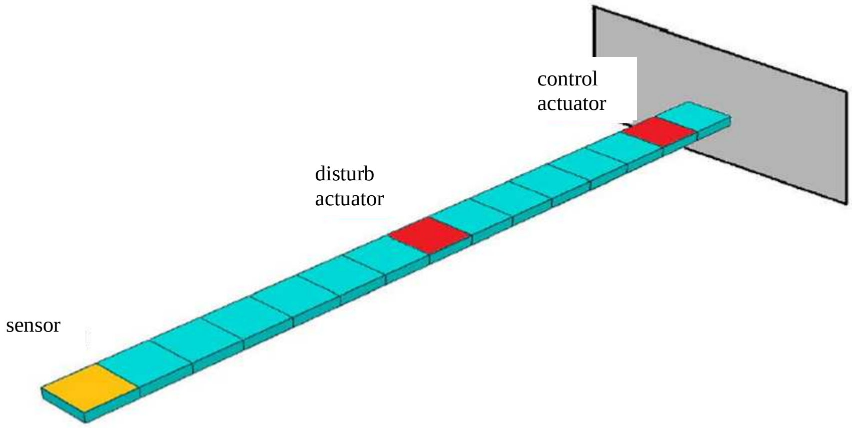


Figure 7: Sensor and actuators positions on the beam

Since the sensor and actuators positions have been defined the new eigenfrequencies were calculated and compared with those obtained previously without PZTs. These values are shown in the Table 4.

Table 4: First four eigenfrequencies with and without PZT

Eigenvalues	Eigenfrequencies (Hz)	
	without PZT	with PZT
1°	9,0626	9,3670
2°	56,7946	56,1671
3°	159,0337	158,0544
4°	311,6973	311,6873

An impulsive input was used to excite the system and the system frequency response function was obtained. Only the first five eigenmodes was considered in the simulated system. The frequency response function for the system with and without controller is presented below in Figure 8.

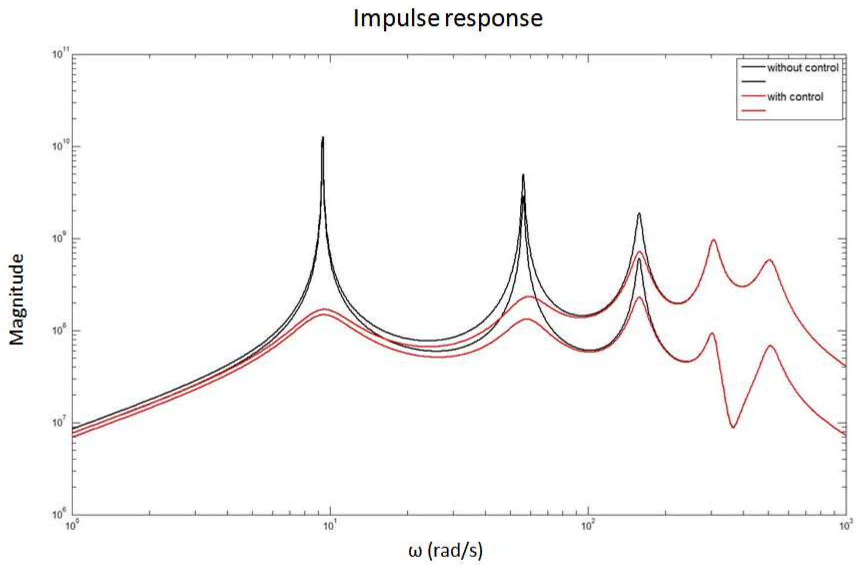


Figure 8: Frequency response function for the system with and without controller.

According to the figure above, greatest attenuation occurred in first three eigenmodes, which was the initial propose for the control design.

The system response with and without control for the same impulse input in the time domain can be seen in the Figure 9.

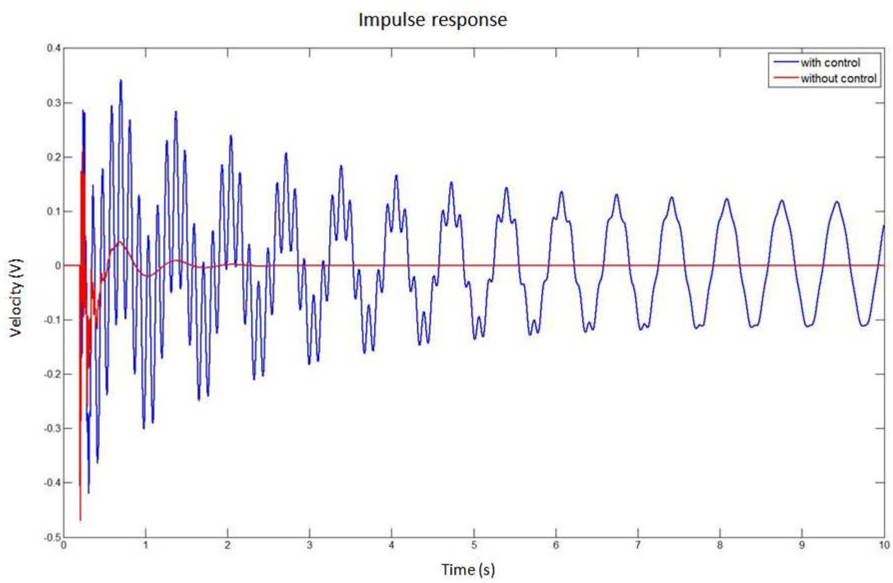
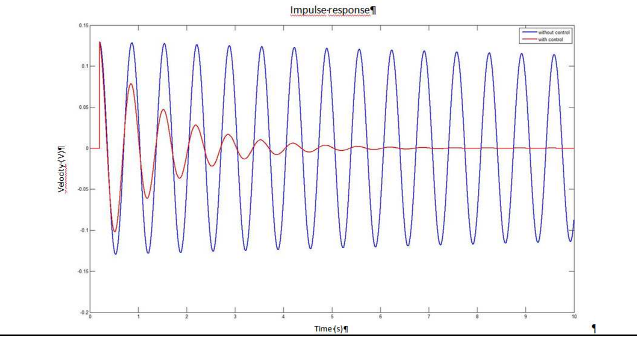
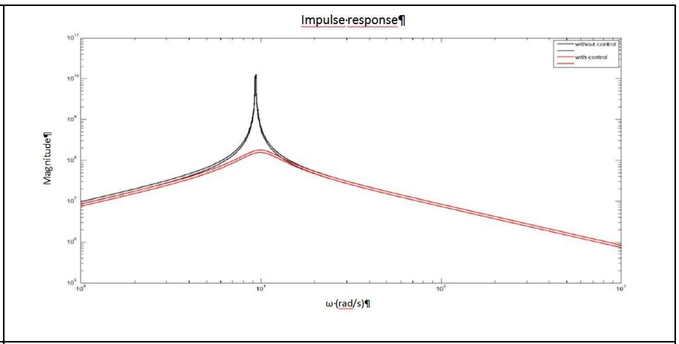


Figure 9: Time domain response for the system with and without controller.

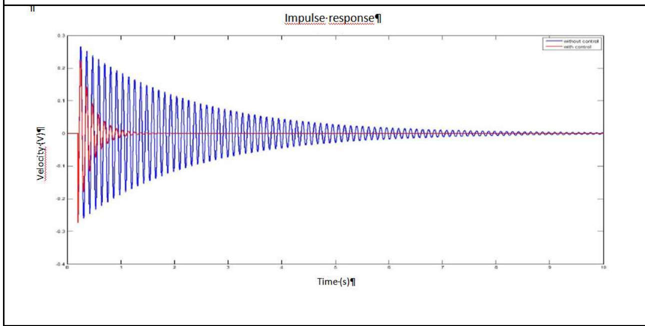
The first three eigenmodes particular behavior can be seen in the Figure 10, considering the impulsive input and comparing the controlled (red line) and non-controlled (blue or black line) system, both in time and frequency domain. This is possible because the system was modeled in modal coordinates and placed in canonical form after a linear transformation, where each diagonal element represents one eigenmode.



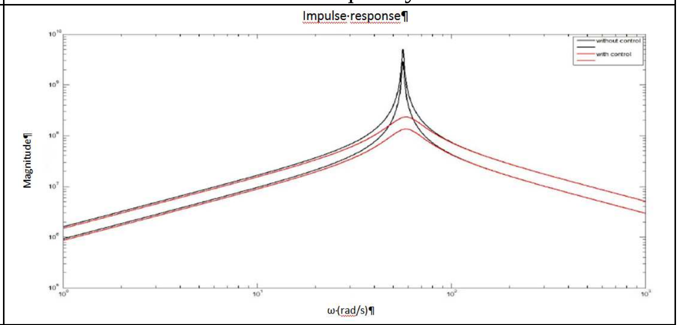
First mode - time domain



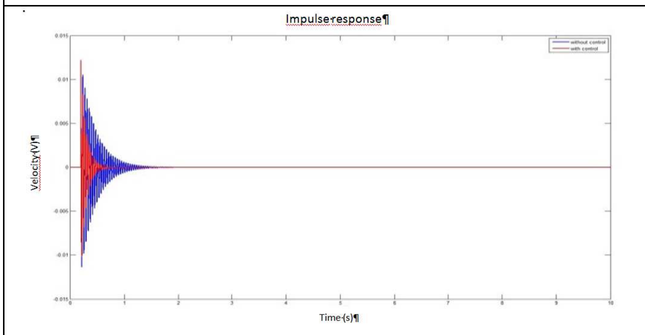
First mode - frequency domain



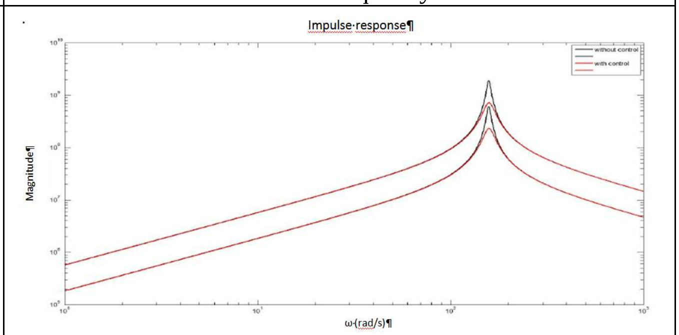
Second mode - time domain



Second mode - frequency domain



Third mode - time domain



Third mode - frequency domain

Figure 10: Eigenmodes in time and frequency domain with and without controller.

For examining the graphics in Figure 10 for the individual modes, it is possible to conclude that the greatest attenuation occurred in the first mode and the lowest occurred in the third. It has been proposed for control design, once the first mode is the one with higher amplitude.

5. CONCLUSIONS

This paper presented a flexible structure analytical modeling procedure and a vibration active control, considering sensors and actuators coupling. The system matrices were obtained by the finite element method and their validation were obtained using commercial FEM software. Sensors and actuators model have also been obtained by finite element method becoming easier their incorporation in the structure. It was applied a linear transformation to the electromechanical coupled structure model which allowed to work with the modes separately. This transformation became easier the control implementation since it was possible to reduce the system model, working only with the interested modes. It decreased the order system and reduced the computational cost.

It was used an optimal control technique, in particular the linear quadratic regulation to determine the controller gain, and an estimator based on Kalman filter, which is an excellent observer in the presence of white noise and has guaranteed stability margin. These methods advantage is related to robustness, since they can be used for more complex structures only performing some modifications. On the other hand, to determine these method parameters is not trivial, because it is commonly used trial and error. In general, the controller designed was efficient, with respect to the proposed objectives in the simulations with the analytical model. The problem related to the sensor and actuators proper positioning was neglected here but it is known that it interferes on the system stability as well its controllability.

6. ACKNOWLEDGEMENTS

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