OPTIMAL LINEAR CONTROL DESIGN APPLIED IN A MAGNETICALLY LEVITATED BODY

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Abstract. *In this paper, a simplified model of a magnetically levitated body is considered. The origin O of an inertial Cartesian reference frame is set at the pivot point of the pendulum on the levitated body in its static equilibrium state (the gap between the magnet on the base and the magnet on the body in this state). The levitated body, is restrained to move freely only in the z-direction. The motion is expressed by the displacement of the pivot from O in the vertical direction. The repulsive nonlinear magnetic force between the magnet on the body and the magnet on the base, for finite small variations of the gap between the magnets, can be well approximated by a polynomial function with quadratic and cubic terms. A pendulum, whose length is r and mass m, is attached to the body as an active vibration absorber and is subjected to a time-varying torque t at its pivot point. The motion of the pendulum is nonlinearly coupled with the main system. Therefore, the absorber addition does not increase the number of linear vibrational modes. The governing equations of motion were derived and the characteristic feature of the strategy is the exploitation of the nonlinear effect of the inertial force associated with the motion of a pendulum-type vibration absorber driven by an appropriate control torque. The problem was analyzed and also an optimal linear control design to stabilize the problem was developed. The simulations results showed the effective of the linear optimal control design.*

Keywords*: Magnetically Levitated Body, Non-Ideal Dynamics, Optimal Linear Control Design*

1. INTRODUCTION

The suspension of objects and people with no visible means of support is fascinating to most people. To deprive objects of the effects of gravity is a dream common to generations of thinkers from Benjamin Franklin to Robert Goddard, and even to mystics of the East. This modern fascination with magnetic levitation stems from two singular technical and scientific achievements: (i) the creation of high-speed vehicles to carry people at 500 km/h and (ii) the discovery of new superconducting materials.

The modern development of magnetic levitation transportation systems, known as Mag-Lev, started in the late 1960s as a natural consequence of the development of low-temperature superconducting wire and the transistor and chip-based electronic control technology. In the 1980s, Mag-Lev had matured to the point where Japanese and German technologists were ready to market these new high-speed levitated machines (Moon, 2004).

In this paper, a pendulum-type vibration absorber is applied to a magnetically levitated system (Yabuno *et al,* 1989; Zheng *et al*, 2000) which is subjected to an unsymmetrical restoring force exciting the principal parametric resonance. An active control strategy for the stabilization of parametric resonance in a magnetically levitated body was proposed (Yabuno *et al,* 2004) and the characteristic feature of the strategy was the exploitation of the nonlinear effect of the inertial force associated with the motion of a pendulum-type vibration absorber driven by an appropriate control torque.

In the last years, a significant interest in control of the nonlinear systems, exhibiting unstable behavior, has been observed and many of the techniques discussed in the literature (Ott *et al*, 1990; Sinha *et al,* 2000; Rafikov, Balthazar, 2008; Coultier *et al*, 1996; Mracek *et al*, 1996; Banks *et al*, 2007; Shawky *et al*, 2007). Among strategies of control with feedback, the most popular is the OGY (Ott-Grebogi-York) method (Ott *et al*, 1990). This method uses the Poincaré map of the system. Recently, a methodology based on the application of the Lyapunov-Floquet transformation, was proposed by Sinha et al. (Sinha *et al,* 2000; Peruzzi *et al*, 2007; Dávid, Sinha, 2000) in order to solve this kind of problem. This method allows directing the chaotic motion to any desired periodic orbit or to a fixed point. It is based on linearization of the equations, which described the error between the actual and desired trajectories. Recently, a technique was proposed by Rafikov and Balthazar in (Rafikov, Balthazar, 2008): The linear feedback control problem for nonlinear systems has been formulated, under optimal control theory viewpoint. Asymptotic stability of the closed-loop nonlinear system is guaranteed by means of a Lyapunov function, which can clearly be seen to be the solution of the Hamilton-Jacobi-Bellman equation, thus guaranteeing both stability and optimality. The formulated theorem (Rafikov, Balthazar, 2008), expresses explicitly the form of minimized functional and gave the sufficient conditions, which allow using the linear feedback control for nonlinear system. The aim of this paper is to propose the application of the optimal linear control (Rafikov, Balthazar, 2008) to control the unstable movement of the Magnetically Levitated Body.

We organized the paper as follows: in Section 2, we presented the used mathematical model, analyzed the dynamics and stability of the non-linear dynamics of the Magnetically Levitated Body model. In Section 3, we discussed an optimal control design problem for the Magnetically Levitated Body. In Section 4, we made some concluding remarks of this paper. In Section 5, we made some acknowledgements. Following, we list out the bibliographic references.

2. MAGNETICALLY LEVITATED BODY MODEL

Here, we consider a mechanical model and the derivation of governing equations done by (Yabuno, *et al* 2004) for the magnetically levitated body with the active vibration absorber, shown in Fig. 1.

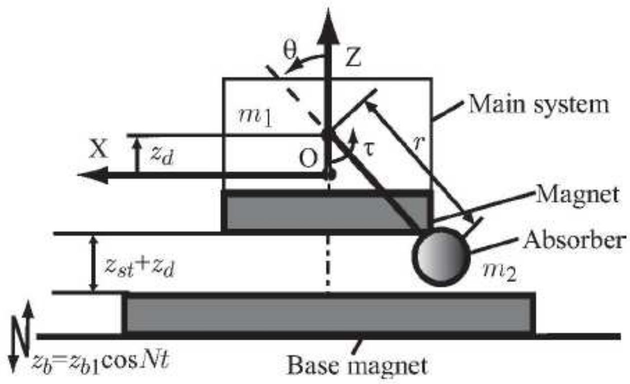


Figure 1. Model of the magnetically levitated body with the active vibration absorber (Yabuno, *et al* 2004).

The origin *O* of an inertial Cartesian reference frame is set at the point of the pendulum on the levitated body in its equilibrium point state (the gap between the magnet on the base and the magnet on the body in this state is denoted by z_{st}). The levitated body, whose mass is m_1, is restrained to move freely only in the *z*-direction. The motion is expressed by the displacement of the pivot from *O* in the vertical direction and is denoted by z_d. The repulsive magnetic force between the magnet on the body and the magnet on the base, for finite but small variations of the gap between the magnets, can be well approximated by a polynomial function with quadratic and cubic terms. (Yabuno *et al*, 1989; Yabuno *et al,* 1991).

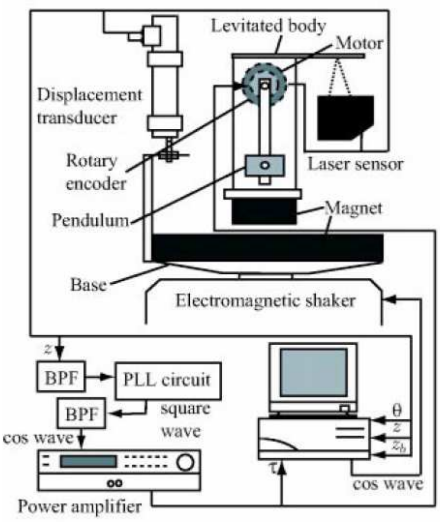
A pendulum, whose length is *r* with a tip mass m_2, is attached to the body as an active vibration absorber and is subjected to a time-varying torque τ at its pivot point. The motion of the pendulum is nonlinearly coupled with the main system. Therefore, as mentioned, the absorber addition does not increase the number of linear vibrational modes. The angle θ, denoting the current pendulum configuration, is measured from the positive *z*-axis. The base is sinusoid ally excited in the vertical direction with a prescribed displacement, $z_b=z_{b1} \cos \Omega t$, where z_{b1} and Ω are the amplitude and frequency of the base excitation, respectively.

The natural frequency of the body, when the pendulum is locked, is denoted by Ω_z. The dimensionless variables t^* and z^*_d as $t=(1/\Omega_z)t^*$ and $z_d=z_{st}z^*$, respectively. The following dimensionless parameters are: $r=z_{st}r^*$, $m_2=(m_1+m_2)m^*$, $w_\theta^2=(g/r)\Omega_z^2$, $\tau=mrg\,\tau^*$, $\epsilon=z_{b1}/z_{st}$, and $\nu=\Omega/\Omega_z$. Substituting the dimensionless variables and putting $\tau^*=b^*\cos(\nu_\theta t^*+\gamma)$ in the dimensional equations of motion yield the following no dimensional equations up to $O(z^3)$ and $O(\theta^3)$ (Yabuno *et al*, 1991):

$$
\begin{aligned}
\ddot{z}^* + z^* &= -\mu_z \dot{z}^* + \in \cos vt^* + 2\alpha_{zz} \in z^* \cos vt^* - \alpha_{zz} z^{*^2} - \alpha_{zzz} z^{*^3} + m^* \ddot{z}^* \theta^2 + m^* r^* w_\theta^2 \theta^2 - m^* r^* \dot{\theta}^2 \\
&\quad - m^* r^* w_\theta^2 b^* \theta \cos(v_\theta t^* + \gamma) + \frac{1}{6} m^* r^* w_\theta^2 b^* \theta^3 \cos(v_\theta t^* + \gamma), \\
\ddot{\theta} + \left(w_\theta^2 + \frac{\ddot{z}^*}{r^*} \right)\theta &= -\mu_\theta \dot{\theta} + w_\theta^2 b^* \cos(v_\theta t^* + \gamma) + \frac{1}{6} w_\theta^2 \theta^3,
\end{aligned}
\tag{1}
$$

where $\mu_z \dot{z}^*$ and $\mu_\theta \dot{\theta}$ express the linear viscous-type forces acting in the main system and the pendulum, respectively; b^*, v_θ, and γ the dimensionless amplitude, frequency, and phase of the torque; α_{zz}, and α_{zzz} are the coefficients of z^2 and z^3, in the Taylor series expansion of magnetic force (Yabuno *et al*, 1989). Henceforth, the star (*) is omitted for ease of notation and the ($\dot{}$) represents differentiation with respect the dimensionless time.

The Figure 2 illustrates the experiment setup the work done by Yabuno *et al* (Yabuno *et al*, 2004) and the Figure 3 illustrate the dynamics behavior of the adopted dynamics model, by using numerical values of experiment setup, for the chosen parameters $\epsilon=0.0629$, $r=7.28$, $b=4.56$, $\alpha_{zz}=-0.732$, $m_1=0.0795$, $m_2=0.0000588$, $\alpha_{zzz}=0.442$, $w_\theta^2=0.144$, $\mu_\theta=0.345$, $\mu_z=0.0281$, $\Omega_z/2\pi=3.95Hz$, and $\Omega_\theta/2\pi=1.50Hz$ is the natural frequency of the pendulum.



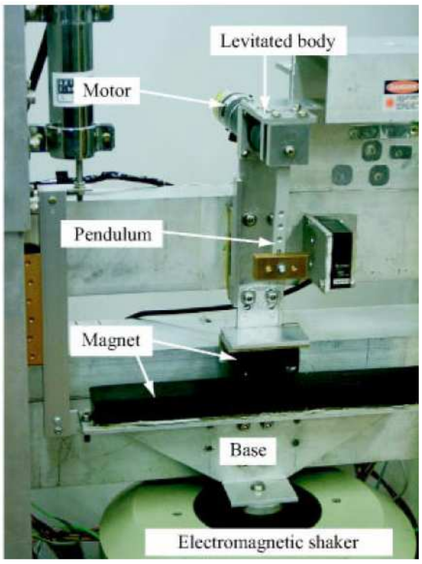


Figure 2. Experimental Setup (Yabuno *et al*, 2004).

Yabuno et al (Yabuno *et al*, 2003) verified theoretically and experimentally that an auto parametric vibration absorber can prevent the occurrence of 1/3-order sub harmonic resonance regardless of the initial conditions. The vibration absorber is a passive-type pendulum and the linear natural frequency is tuned to be in the neighborhood of one-half the linear natural frequency of the main system. In (Yabuno *et al*, 2004), was attached the same absorber to the magnetically levitated body under parametric excitation, and the pendulum-type vibration absorber, although effective in stabilizing the 1/3-order sub harmonic resonance, does not act as an effective absorber for parametric resonance. Furthermore, parametric excitation of the auto parametric resonance generates chaotic motions in the main system and absorber, the Figures 3-5 illustrated this behavior, when the system is coupled with the pendulum has chaotic behavior and when this behavior without the pendulum is stable.

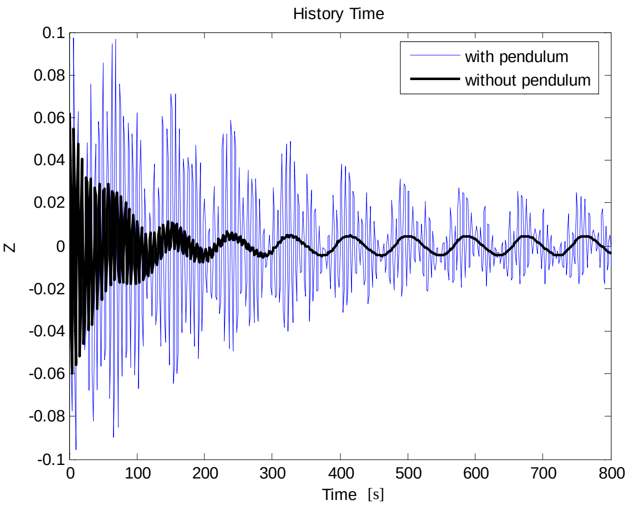


Figure 3. Time history for system with and without pendulum.

Figure 4 (a) presents the time history for *z*. (b) presents the time history for $\dot{z}$. (c) presents the time history for θ. (d) presents the time history for $\dot{\theta}$. (e) presents the phase portrait for *z* and $\dot{z}$; (f) shown the phase portrait for θ and $\dot{\theta}$. (g) illustrate the stability diagram for x_1 and (h) for x_3, of the adopted dynamics model, by using numerical values for the chosen parameters $\epsilon=0.0629$, $r=7.28$, $b=4.56$, $m_1=0.0795$, $m_2=0.0000588$, $\alpha_{zzz}=0.442$, $w_\theta^2=0.144$, $\mu_\theta=0.345$, $\mu_z=0.0281$ and α_{zz}=varied.

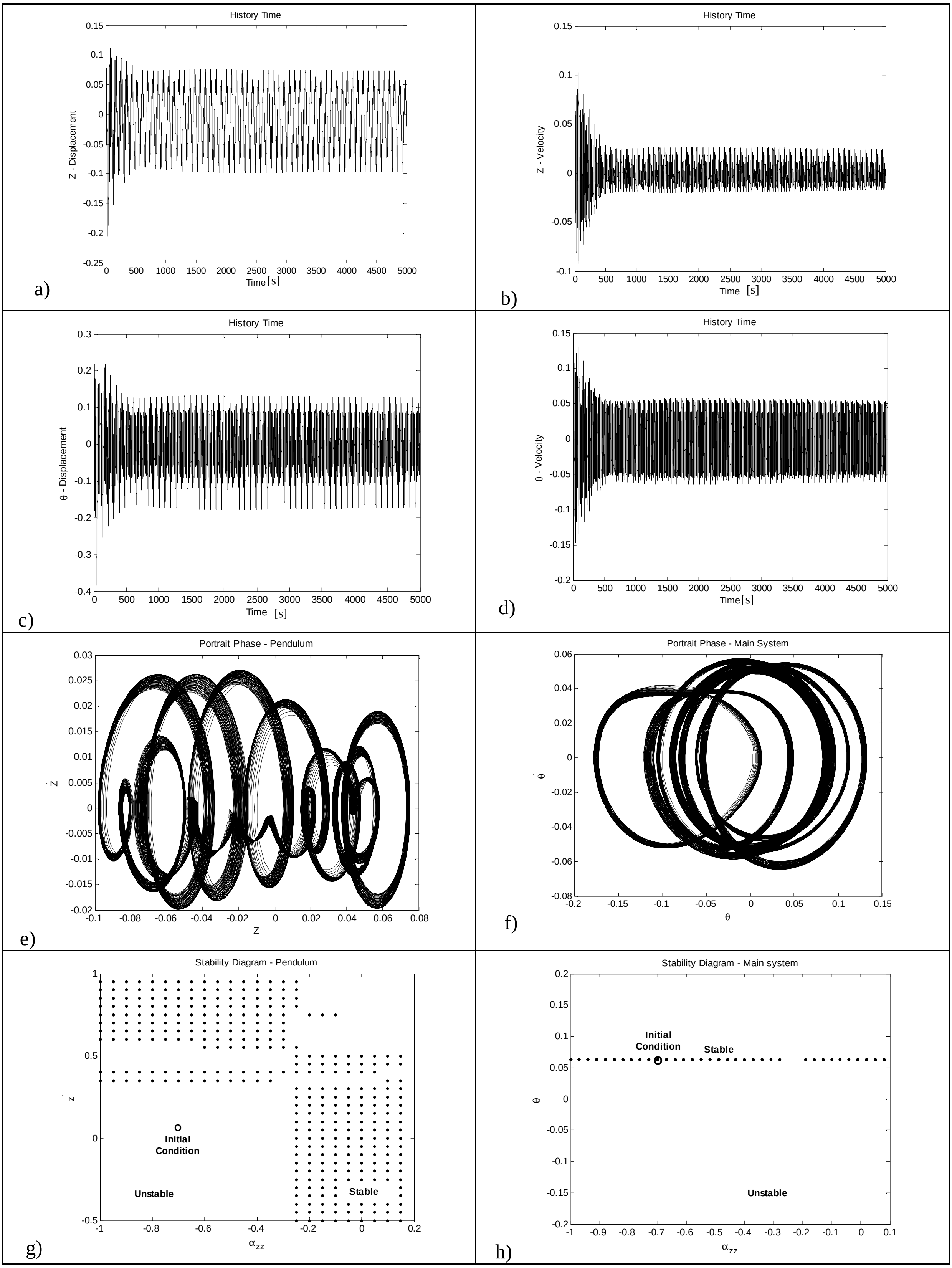


Figure 4. (a) Time history for *z*. (b) Time history for $\dot{z}$. (c) Time history for θ. (d) Time history for $\dot{\theta}$. (e) Portrait Phase for *z* and $\dot{z}$. (f) Portrait Phase for θ and $\dot{\theta}$. (g) Stability diagram for *z* and α_{zz}. (g) Stability diagram for θ and α_{zz}.

The eigenvalues are $\lambda_{1,2}$= 0,024122±1.6027i; λ_3= -0.9820; λ_4= 1.0246. The eigenvalues $\lambda_{1,2}$ indicates that the magnetically levitated body is unstable and the Figure 4 illustrates the chaotic dynamic (λ_4=0.46) of Lyapunov exponent for magnetically levitated body.

This chaotic behavior illustrated in Figure 5, motivated us to establish a stabilization control method aimed at direct cancellation of the parametric excitation without using the auto parametric energy transfer that requires special tuning of the natural frequencies of the main system and the absorber.

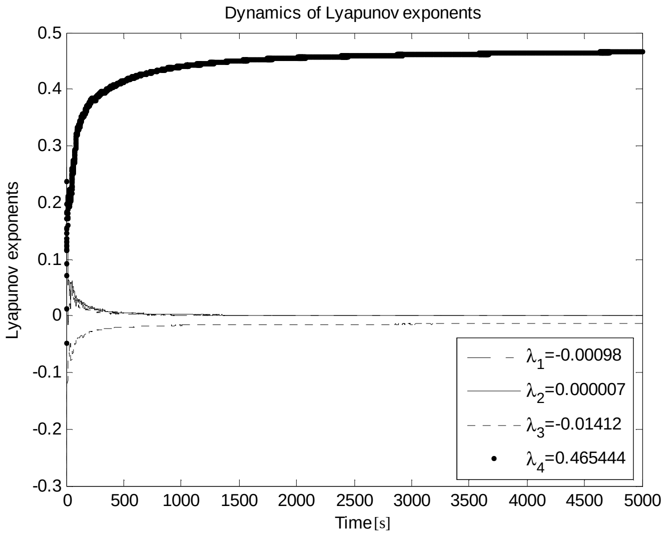


Figure 5. Dynamics of Lyapunov expoents for the magnetically levitated body.

3. CONTROL DESIGN

In this section, we applied optimal linear control design for the magnetically levitated body (figure 1), reducing the oscillatory movement to a small stable orbit. Next, we present the theory of the used methodology.

Due to the simplicity in configuration and implementation, the linear state feedback control, it is especially attractive (Rafikov, Balthazar, 2008; Chavarette *et al*, 2010a; 2010b; 2009; Fenili, Balthazar, 2010).

We remarked that this approach is analytical, and it may use without dropping any non-linear term.

Let the governing equations of motion (1), re-written in a state form

$$\dot{x} = Ax + g(x)\,. \tag{2}$$

If one considers a vector function $\tilde{x}$, that characterizes the desired trajectory, and taken the control *U* vector consisting of two parts: $\tilde{u}$ being the feed forward and u_f is a linear feedback, in such way that

$$u_t = Bu \tag{3}$$

where *B* is a constant matrix. Next, one taking the deviation of the trajectory of system (2) to the desired one $y = x - \tilde{x}$, may written as being

$$\dot{y} = Ay + g(x) - g(\tilde{x}) + Bu \tag{4}$$

where $G(y, \tilde{x})$ is limited matrix we proved the important result (Rafikov, Balthazar, 2008).

If there exit matrices *Q(t)* and *R(t)*, positive definite, being *Q* symmetric, such that the matrix $\tilde{Q} = Q - G^T(y, \tilde{x})P(t) - P(t)G(y, \tilde{x})$ is positive definite for the limited matrix *G*, then the linear feedback control is

$$u = -R^{-1}B^T P\, y\,. \tag{5}$$

It is optimal, in order to transfer the non-linear system (6) from any initial to final state $y(t_f)=0$, minimizing the functional $\tilde{J} = \int_0^{\infty} (y^T \tilde{Q}\, y + u^T R u)dt$, where the symmetric matrix *P(t)* is evaluated through the solution of the matrix Ricatti differential equation

$$PA + A^T P - PBR^{-1}B^T P + Q = 0 \tag{6}$$

satisfying the final condition $P(t_f)=0$.

In addition, with the feedback control (6), there exists a neighborhood $\Gamma_0 \subset \Gamma$, $\Gamma \subset \Re^n$, of the origin such that if $x_0 \in \Gamma_0$, the solution $x(t) = 0,\ \ t \geq 0,$ of the controlled system (4) is locally asymptotically stable, and $J_{\min} = x_0^T P(0)\, x_0$. Finally, if $\Gamma = \Re^n$ then the solution $y(t) = 0,\ t > 0,$ of the controlled system (4) is globally asymptotically stable.

Using the theorem by Rafikov and Balthazar the dynamic error **y** can be minimized ($y \to 0$) (Chavarette *et al*, 2010a; 2010b; 2009; Rafikov and Balthazar, 2008).

3.1 Theorem (Rafikov and Balthazar, 2008).

If there is matrixes Q and R, positive definite, Q symmetric, such that the matrix

$$\tilde{Q} = Q - G^T(x,\tilde{x})P - PG(x,\tilde{x}) \tag{7}$$

is positive definite for the limited matrix G , then the linear feedback control

$$u = -R^{-1}B^T P\, y \tag{8}$$

is optimal, in order to drive the non-linear system (6) of any initial state to the terminal state

$$y(\infty) = 0 \tag{9}$$

minimizing the functional

$$J = \int_0^\infty (y^T \tilde{Q}\, y + u^T R u)dt \tag{10}$$

where the symmetric matrix P is calculated from the nonlinear Riccati equation:

$$PA + A^T P - PBR^{-1}B^T P + Q = 0 \tag{11}$$

Next, we will apply this methodology in the magnetically levitated body (1).

3.2 Application of the Linear Optimal Control to the Non-linear System.

The equations (1) describing the magnetically levitated body controlled:

$$\begin{aligned} \ddot{z}^* + z^* &= -\mu_z \dot{z}^* + \in \cos vt^* + 2\alpha_{zz} \in z^* \cos vt^* - \alpha_{zz} z^{*2} - \alpha_{zzz} z^{*3} + m^* \ddot{z}^* \theta^2 + m^* r^* w_\theta^2 \theta^2 - m^* r^* \dot{\theta}^2 \\ &\quad - m^* r^* w_\theta^2 b^* \theta \cos(v_\theta t^* + \gamma) + \frac{1}{6} m^* r^* w_\theta^2 b^* \theta^3 \cos(v_\theta t^* + \gamma) + U, \\ \ddot{\theta} + \left(w_\theta^2 + \frac{\ddot{z}^*}{r^*} \right)\theta &= -\mu_\theta \dot{\theta} + w_\theta^2 b^* \cos(v_\theta t^* + \gamma) + \frac{1}{6} w_\theta^2 \theta^3, \end{aligned} \tag{12}$$

where the function of control U is defined in the equation (1).

We will obtain $B = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $y = \begin{bmatrix} x_1 - \tilde{x}_1 \\ x_2 - \tilde{x}_2 \\ x_3 - \tilde{x}_3 \\ x_{\backslash 4} - \tilde{x}_4 \end{bmatrix}$, $\tilde{x} = \begin{bmatrix} 0.06 \\ 0.06 \\ 0.06 \\ 0.06 \end{bmatrix}$, $Q = I_6$, $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1.092 & 0.057 & -0.072 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -0.008 & -0.353 & 0 \end{bmatrix}$,

where the controllability matrix R of the system to the pair [A,B] is obtained by $R = [B \,|\, AB \,|\, A^2B ... |\, A^{2n-1}B]$.

Thus, $R = \begin{bmatrix} 1 & 1 & -1.1069 & -1.2282 \\ 1 & -1.1069 & -1.2282 & 1.1644 \\ 1 & 1 & -0.3421 & -0.3439 \\ 1 & -0.3621 & -0.3439 & 0.1386 \end{bmatrix}$.

Then the Matrix *P(t)* is done by $P = \begin{bmatrix} 7.1068 & -0.8881 & -0.5870 & 2.0366 \\ -0.8881 & 6.3010 & -1.6019 & -3.4036 \\ -0.5870 & -1.6019 & 3.8756 & 1.1664 \\ 2.0366 & -3.4036 & 1.1664 & 8.3649 \end{bmatrix}$ and (an optimal control) $u = 0.3834x_1 + 0.0203x_2 + 0.1426x_3 + 0.4082x_4$. The trajectories of the system with control may be seen, through Figure 6. According to the optimal control verification (Rafikov, Balthazar, 2008), the function (4) is numerically calculated across $L(t) = y^T \tilde{Q} y$,where *L(t)* is defined positive and it is show in Fig. 6-g.

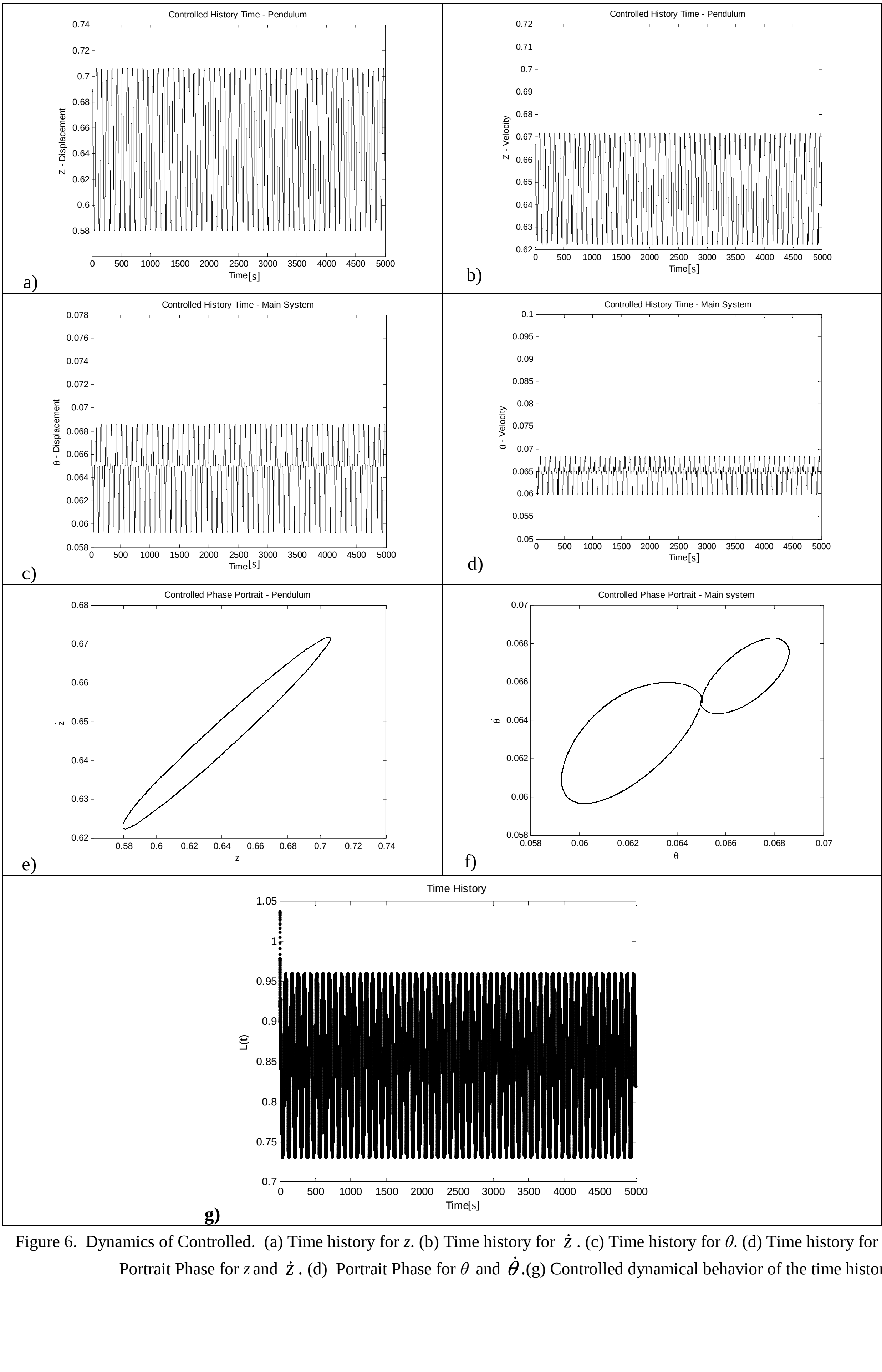


Figure 6. Dynamics of Controlled. (a) Time history for z. (b) Time history for $\dot{z}$. (c) Time history for θ. (d) Time history for $\dot{\theta}$. (e) Portrait Phase for z and $\dot{z}$. (d) Portrait Phase for θ and $\dot{\theta}$.(g) Controlled dynamical behavior of the time history

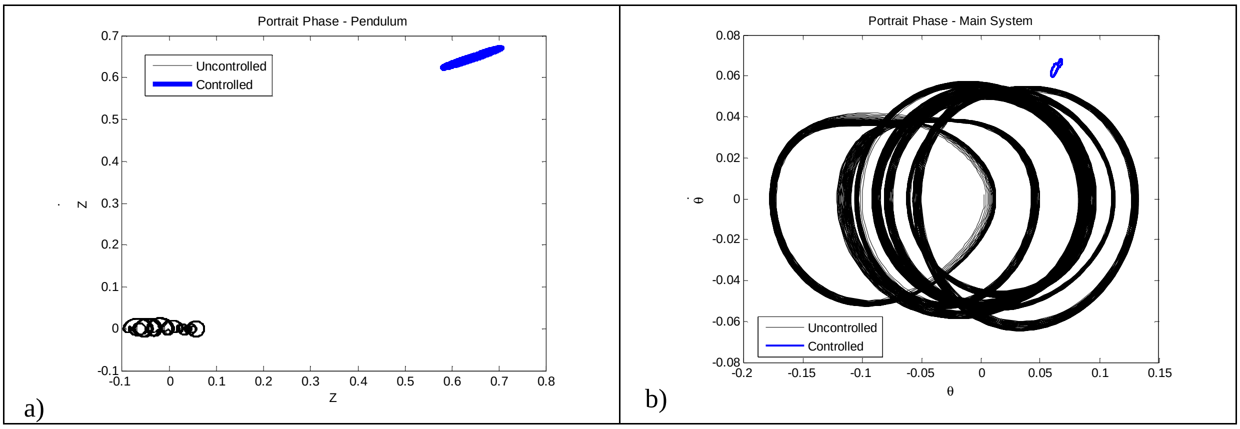


Figure 7. Dynamics of Controlled and Uncontrolled. (a) Portrait Phase for z and $\dot{z}$. (b) Portrait Phase for θ and $\dot{\theta}$.

We observe in Figures 6 and 7 that the performance of the proposed controller reduced the amplitude of oscillation of the system, eliminating the chaotic behavior shown in Figure 5. It is noted from Figure 7 that the orbits of the controlled system (blue) is smaller than the uncontrolled (black). It is noted also that the positive feature presented in Figures 6e and 6f and consequence of the proposed controller illustrating the efficiency of this.

4. CONCLUSION

In this work, a dynamics of the magnetically levitated body proposed (Yabuno *et al*, 2004) is investigated through numerical simulations using the software Matlab 7.0®. The model of the magnetically levitated body with the active vibration absorber is shown in Figure 1 and Figure 2 shown the experimental setup this work (Yabuno *et al*, 2004).

The Figure 4 illustrated the behavior dynamics proposed by (Yabuno *et al,* 2004) and Figure 5 illustrated the chaotic dynamic through of positive Lyapunov exponent (λ_4=0.46) for magnetically levitated body.

This chaotic behavior illustrated in Figure 5, motivated us to establish a stabilization control method aimed at direct cancellation of the parametric excitation without using the auto parametric energy transfer that requires special tuning of the natural frequencies of the main system and the absorber. We proposed the application of the optimal linear control (Rafikov, Balthazar, 2008) to control the unstable movement and this kind control strategy reduced the chaotic movement to a small stable orbit. Figures 6 and 7 illustrates the effectiveness of the control strategy. We see Figures. 4 and 6, that the control technique applied to the chaotic system has the amplitude of the oscillation decrease.

The data obtained here are in agreement with the experimental work by (Yabuno *et al*, 2004), but with the difference that in this work, we can see that the amplitude of oscillation of the controlled system is smaller than the result presented by (Yabuno *et al*, 2004) allowing a gain in performance of the controlled system.

5. ACKNOWLEDGMENTS

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