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A CONTINUOUS APPROXIMATION OF THE LUGRE FRICTION MODEL

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Abstract. *This work proposes a modified version of the LuGre friction model, in which a discontinuous term is approximated by means of a continuous function. Such modification is necessary in order to allow the application of this model to a class of nonlinear controllers. Simulation results are employed to illustrate the similarity of behavior between the friction forces that are estimated by means of each model. Also, in order to illustrate its usefulness, the proposed model is employed as a friction compensator in a nonlinear control scheme applied to a pneumatic positioning system, and it is demonstrated that its use allows to obtain stability characteristics of the closed loop system that are stronger than those previously guaranteed.*

Keywords*: friction models, friction compensation, LuGre model, pneumatic systems, nonlinear control*

1. INTRODUCTION

In the field of servo actuators, there is an ever-growing demand for systems in which fast responses, high precision positioning and disturbance rejection abilities are required. Among the negative factors that affect the performance of such actuators, one of the most important is friction. Such effect depends not only on the physical characteristics of the contacting surfaces but also on the conditions of relative displacement between them. Thus, such behavior is strongly nonlinear and very difficult to compensate accurately. Some of the complex phenomena presented by friction effects are pre-sliding displacement, break-away force variations, and stick-slip motion, among others (see, for instance, Armstrong-Hélouvry et al., 1994). Because of such effects, in closed loop servo systems, friction may lead to significant steady-state tracking errors, limit cycle oscillations, and potential instability of the controlled plant.

Among the control strategies employed to overcome the difficulties posed by friction, the use of model-based compensation techniques is found to be an effective solution. In this context, the LuGre model (Canudas de Wit et al., 1995) is one of the most commonly used ones. It is based on the understanding of friction phenomena in a microscopic scale, by means of a nonlinear state observer. Because of its relatively high degree of complexity, the LuGre model is capable of modeling friction phenomena such as the Stribeck effect, hysteresis, pre-sliding displacements, and varying break-away forces. For this reason, this model has been used in several control algorithms that include some explicit friction compensation scheme, such as those given by Zeng and Sepehri (2006) or Guenther et al. (2006).

In its original form, the LuGre model employs a discontinuous term in the mathematical structure of the state observer upon which the model is based. Because of such discontinuous term, this model cannot be applied in its original form to some classes of control schemes, because the presence of such discontinuity makes it impossible to calculate the value of the necessary control signal to be applied to the plant under some circumstances. On the other hand, this model possesses some important properties from a theoretical standpoint, because they play an important role in the task of ensuring the stability of the controlled system. Therefore, in order to employ the LuGre model in the cases where its application in the original form is not possible, it was necessary to develop a continuous version that is able to retain the aforementioned analytical properties.

This work proposes a continuous approximation of the LuGre model that preserves the most important analytical properties possessed by its original form. The main features of the original LuGre model are outlined, and its proposed continuous version is defined. The similarities between the predictions made by means of each model are illustrated by means of simulation results, and it is verified that such similarities depend strongly on the value of one parameter of the newly proposed model. Finally, the usefulness of the proposed approximation is illustrated by applying the new model to the case of the nonlinear control of a pneumatic servo system. By means of the newly proposed model, it is seen that the controlled system can be proven to present stronger stability properties than those that were previously guaranteed.

This paper is organized as follows. Section 2 presents the original LuGre model, while, in Section 3, its continuous approximation is discussed. Simulation results comparing the proposed model to its discontinuous predecessor are presented in Section 4. In Section 5, the proposed model is applied to compensate friction in a pneumatic positioning system as an illustration of its usefulness. Finally, the main conclusions are outlined in Section 6.

2. THE LUGRE MODEL

The LuGre model represents friction effects by means of microscopic interactions between two contacting surfaces. In this scale, the surfaces are very rough, and the relationship between their asperities determines the complex nature of

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friction. When one of these surfaces is forced to move with respect to the other, such asperities tend to stick to one each other, thus causing a force contrary to the movement to arise. If the causes of the relative movement are strong enough, these asperities may break or bend, and the force that opposes to the movement is diminished. As the movement goes on, however, newer pairs of stuck asperities are formed, so that the value of friction force may rise once again. As these asperities are very irregular in size, shape and spatial distribution, this process of sticking and releasing generates a force that generally opposes to the relative movement but is very random in terms of amplitude, especially for low velocities. For greater velocities, the new pairs of stuck asperities are formed at approximately the same rate at which previously formed pairs become undone, thus causing the friction force to reach an almost steady value that depends on the velocity of the movement. Mathematically, the relationship between such asperities is modeled by means of the elastic deformations of microscopic elements. Because of their random nature, such deformations are represented by an average deflection, given by an internal state *z*, which is not directly measurable. Thus, such state must be observed by some suitable algorithm. By using this state, the friction force acting between the moving bodies can be written as:

$$F_a = \sigma_0 z + \sigma_1 \dot{z} + \sigma_2 \dot{y} \tag{1}$$

where σ_0 is a stiffness coefficient, σ_1 is a damping coefficient related to the time derivative of *z,* and σ_2 is the viscous friction coefficient due to the velocity term $\dot{y}$. The dynamics of *z* is observed by employing the following expression:

$$\dot{z} = \dot{y} - \frac{|\dot{y}|}{f_s(\dot{y})} z \tag{2}$$

where $f(\dot{y})$ is a positive function that describes the characteristics of friction for constant velocities that are depicted in Fig. 1. This function is defined as:

$$f_s(\dot{y}) = \frac{1}{\sigma_0}\left[F_c + (F_s - F_c)e^{-(\dot{y}/\dot{y}_s)^2}\right] \tag{3}$$

where F_c is the Coulomb friction force, F_s is the static friction force, and $\dot{y}_s$ is the Stribeck velocity.

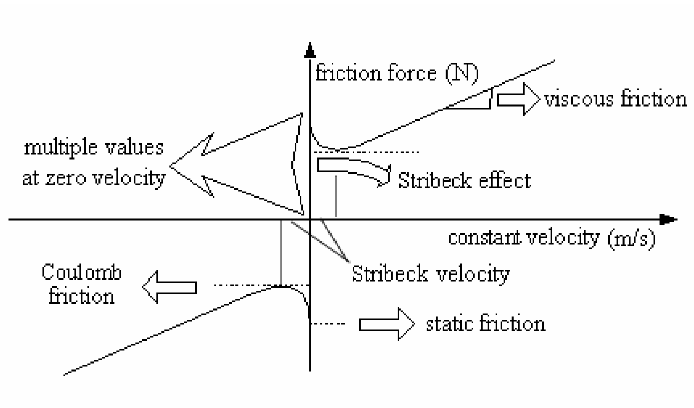

Figure 1. Static relationship between velocity and friction force

According to Canudas de Wit et al. (1995) and Olsson (1996), the LuGre model is capable of representing most of those nonlinear effects inherent to friction, such as varying break-away forces, Stribeck effects and hysteresis. Also, it predicts undesirable effects that could arise in closed-loop systems due to friction, such as limit cycles. Detailed descriptions of the properties of this model can be found in the two aforementioned works as well as in Barabanov and Ortega (2000). Among all of such properties, three are of special importance in the context of control systems: **(i)** the solution of the differential equation expressed in Eq. (2) exists and is unique for all $t \geq 0$ (Olsson, 1996); **(ii)** the internal state $z(t)$ is limited (Canudas de Wit et al., 1995); **(iii)** with a proper choice of the values of its parameters, the model is passive with respect to an input in terms of velocity and an output in the form of friction force (Barabanov and Ortega, 2000). These properties are much relevant to control algorithms that include friction compensation schemes because they are vital to ensuring the stability of the closed-loop plant. For this reason, the LuGre model has been widely employed in such friction-compensating control schemes. Examples of its application can be found in the cases

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of many different types of systems, such as hydraulic manipulators (Zeng and Sepehri, 2006), pneumatic actuators (Guenther et al., 2006) and electric servomotors (Wenjing and Qinghai, 2008), among others.

3. LUGRE MODEL – CONTINUOUS APPROXIMATIONS

Despite its efficiency in representing many aspects of friction, the use of the LuGre model is problematic in some cases of control algorithms, when the value of the time derivative of the estimated friction force is required. As it can be seen in expressions (1) and (2), the estimated friction force depends directly on $\dot{z}(t)$, and this value depends on the term $|\dot{y}|$, whose time derivative is discontinuous in $\dot{y} = 0$. Thus, $\ddot{z}(t)$ does not exist at this point, and the time derivative of the estimated friction force cannot be determined under such conditions. Therefore, in those cases when the employed controller takes into account the time derivative of the desired force to be applied to the plant in an explicit form, the corresponding control signal cannot be calculated at this point. This problem has been pointed out in works such as Makkar et al. (2005) and Guenther et al. (2006). For this reason, in such cases, continuous approximations of the LuGre model must be employed. In the following sections, two versions of such approximations are discussed. In the first case, a classical (current) approach for approximating the LuGre model is presented and some of its limitations are pointed out. Then, in order to avoid such limitations, a modified version of such approximation is proposed.

3.1 A Classical Approximation of the LuGre Model

In order to employ the LuGre model as a friction compensating element in control algorithms that depend on the time derivative of Eq. (2), it is necessary to approximate the behavior of the term $|\dot{y}|$ in a continuous way. One of the ways which it can be done is observing that $|\dot{y}| = \dot{y}\,\mathrm{sgn}(\dot{y})$. It can be noticed that the discontinuous term $\mathrm{sgn}(\dot{y})$ is employed only to inform the sign of $\dot{y}$, a task that can also be carried out by means of some continuous odd function. Thus, one possible candidate as a continuous approximation of the term $\mathrm{sgn}(\dot{y})$ is the function $S_0(\dot{y})$, defined as:

$$S_0(\dot{y}) = \frac{2}{\pi}\arctan(k_V\dot{y}) \tag{4}$$

with k_V being a positive constant. By employing this function, Guenther et al. (2006) proposed the following approximation function for $|\dot{y}|$:

$$m(\dot{y}) = \dot{y}S_0(\dot{y}) = \frac{2\dot{y}}{\pi}\arctan(k_V\dot{y}) \tag{5}$$

Thus, Eq.(2) becomes:

$$\dot{z} = \dot{y} - \frac{m(\dot{y})}{f_s(\dot{y})}z \tag{6}$$

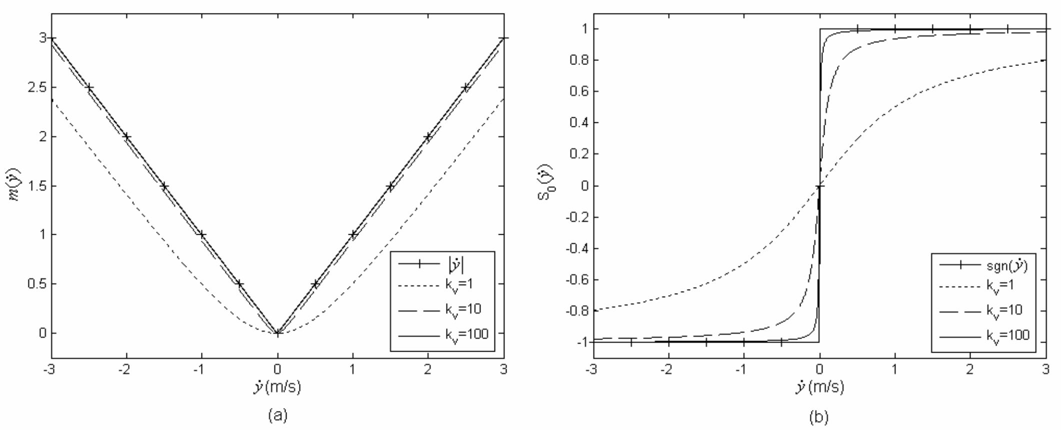

Figure 2. Behavior of $m(\dot{y})$ and $S_0(\dot{y})$ for different values of k_V.

The behaviors of the functions $m(\dot{y})$ and $S_0(\dot{y})$ are illustrated in Figs. 2(a) and 2(b), respectively. It is seen that, in qualitative terms, both of them present the same characteristics as their discontinuous counterparts. Quantitatively, however, it can be observed that good approximations may require large values of k_V .

As inferred from Fig. 2, this modification provides a reasonable approximation of the LuGre model from a numerical standpoint. Nevertheless, in order to safely use it as part of some control scheme, it is necessary to prove that the approximate model does preserve those three previously mentioned analytical properties of its predecessor. Here, a major difficulty arises: so far, to the best of the knowledge of the authors of this work, it has not been possible to produce a conclusive analytical proof showing that the internal state $z(t)$ estimated by means of Eq (6) is limited regardless of the value of $\dot{y}(t)$. Also, as the proof of the passivity properties of the LuGre model relies on the existence of such limit (Barabanov and Ortega, 2000), it follows that the altered model as defined by Eq. (6) has not been proved to be passive either. Thus, it is seen that such altered model could be deprived of two important analytical properties. As both of them have expressive physical meanings and play important roles in ensuring the stability of the controlled system, it would be risky to employ such a modified model as part of any control algorithm. If the model is actually not passive, for instance, it may cause the control signal applied to the plant to be excessively large, leading to equally large tracking errors or even operation accidents with hazardous consequences to the system and/or its surroundings. Therefore, as long as the existence of a definitive proof showing whether or not the approximate model defined in Eq. (6) possesses such important properties remains an open issue, it should be considered unsafe to employ such model as part of a control algorithm. This measure must be taken at least in those cases where there are no other characteristics of the proposed controller ensuring that the problems arising from the lack of such properties cannot occur. This topic is discussed with more detail in Sobczyk (2009).

3.2 A Modified Approximation of the LuGre Model

In order to employ a continuous version of the LuGre model and still keep the most important analytical properties it possesses in its original form, this work proposes that Eq. (2) be modified once again, becoming:

$$\dot{z} = S_1(\dot{y})\dot{y} - \frac{S_2(\dot{y})}{f_s(\dot{y})} z \tag{7}$$

where $S_1(\dot{y})$ and $S_2(\dot{y})$ are two approximation functions, respectively defined as:

$$S_1(\dot{y}) = \left(S_0(\dot{y})\right)^2 \tag{8}$$

$$S_2(\dot{y}) = m(\dot{y}) = \dot{y}S_0(\dot{y}) \tag{9}$$

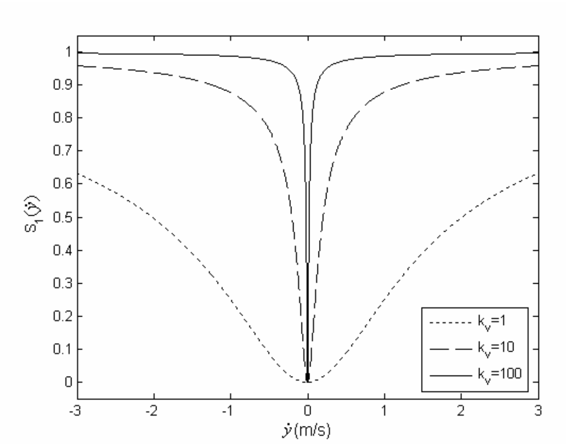

Figure 3. Behavior of $S_1(\dot{y})$ for different values of k_V .

Compared to the previous approximation given in Eq. (6), the only modification is the inclusion of the auxiliary term $S_1(\dot{y})$, whose behavior as a function of k_V is depicted in Fig. 3. Once $S_2(\dot{y})$ is equal to $m(\dot{y})$, its behavior is also illustrated by means of Fig. 2. Regarding $S_1(\dot{y})$, it can be observed that the value of k_v determines how closely this function can approximate a constant value of 1: far from the origin, its value is very close to unity; for nearly zero

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velocities, the region in which the difference $S_1(\dot{y})-1$ is expressive becomes narrower as k_v is increased. Thus, it is inferred that this newly modified version of the Lugre is also capable of presenting numerical predictions that are very close to those obtained by means of its discontinuous predecessor, provided that the value of k_v is sufficiently large.

Even though very similar to the first continuous version of the LuGre model, this newer approximate model does not suffer from the analytical restrictions mentioned in the previous section. Thus, it can be proven (Sobczyk, 2009) that this newly modified model also possesses the three most important properties of the original LuGre model: **(i)** existence and uniqueness of solution of Eq. (7); **(ii)** existence of a superior bond for the value of $|z(t)|$; **(iii)** passivity between a velocity input and a friction force output. In addition, as it will be observed in the next section, it is capable of predicting friction behavior in a form that is very similar to that of the original model. For these reasons, the new model can be regarded as an acceptable substitute for the LuGre model in those cases when the employed control algorithm does not support the discontinuous nature of its original form. From this point on, the model modified according to Eq. (7) shall be referred to as the *proposed* model, whereas the LuGre model in its original form will be called the *original* one.

Remark #1: it should be noticed that the proposed approximation is only able to provide a continuous approximation of the mathematical structure of the LuGre friction model in such a way that its most important analytical properties are preserved. It does *not* guarantee that the predictions of some phenomena related to friction effects are carried out in an improved way when compared to the original LuGre model, as in the case of the Elasto-Plastic friction model with respect to stiction (see Dupont et al., 2000). However, it should be pointed out that the proposed approximation can also be extended to the case of such Elasto-Plastic model because its mathematical structure is very similar to that of the original LuGre model and because the discontinuous term $|\dot{y}|$ is employed in this case also.

4. SIMULATION RESULTS

In this section, the results obtained from simulating the proposed friction model are presented. The simulations are designed so as to show that the proposed model is capable of representing satisfactorily most of the nonlinear phenomena that are associated with friction. The results obtained by means of the original form of the LuGre friction model are also presented as a reference, and the results obtained by means of each model are compared.

Table 1. Parameters employed in the simulations

Parameter	**Value**	**Units**	**Parameter**	**Value**	**Units**
σ_0	10^5	[N/m]	F_C	1,0	[N]
σ_1	$\sqrt{10^5}$	[Ns/m]	F_S	1,5	[N]
σ_2	0,4	[Ns/m]	v_S	0,001	[m/s]

In all simulations, most parameters are always the same as given in Tab. 1. These values are the same ones employed by Canudas De Wit et al. (1995) for illustrating the properties of the original LuGre model. The only parameter that is modified is the value of k_V. In this case, three values are employed: 10^6 [s/m], 10^7 [s/m] and 10^8 [s/m], and the results obtained in each simulation are presented in a grouped form so they can be compared.

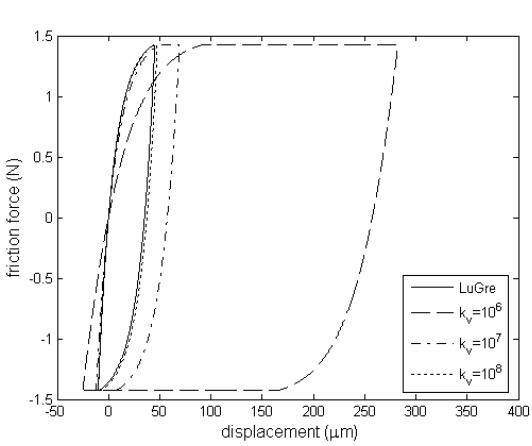

Figure 4. Simulation results for pre-sliding displacement.

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The first set of simulation results regard the *pre-sliding displacement* phenomenon, that is, the elastic displacements that occur when a force not large enough to overcome static friction is applied to a rigid body at rest. Under these circumstances, due to the elastic behavior of the asperities in the contact region with its supporting surface, the body presents very small displacements due to the applied force, but it does not acquire any translational movement in its true sense. Thus, if the applied force vanishes, the body returns to its former position as in a typical situation involving only elastic deformations. In order to simulate this behavior, a force was applied to a body with unit mass and subject to friction. The force was slowly ramped-up from zero to 1,425 [N], which is 95% of the static friction value. After being kept at that value for a while, the force was ramped-down until it reached -1,425 [N]. Then, it was kept constant for some time, after which it was ramped-up to zero once again. The resulting friction force is plotted as a function of the position of the body in Fig. 4. It is seen that, in a qualitative sense, the predictions made by means of the proposed model agree with those obtained when the original model is employed. Quantitatively, however, it can be noticed that a satisfactory approximation is reached only for very large values of k_V (at a magnitude order of 10^7 [s/m] or larger).

It is now illustrated the capability of the proposed model of representing *stick-slip motion*, a phenomenon that occurs at low velocities. This effect can be understood in an intuitive way by imagining a spring-mass system as the one depicted in Fig. 5. The input is the displacement x of the right extremity of the spring. As the spring is slowly pulled to the right, the magnitude of the elastic force it applies to the mass m is increased. When the elastic force becomes large enough to overcome static friction, the object starts to move. From the behavior of the friction force as a function of velocity depicted in Fig. 1, it is seen that, at this moment, the absolute value of such force decreases sharply. Thus, the object acquires a large acceleration at the initial stages of its movement, the deformation $x - y$ of the spring is abruptly reduced, and the same occurs to the elastic force applied by the spring. Thus, such force becomes not large enough to keep overcoming friction, and the block stops. As the value of x keeps increasing, the whole process is repeated, and the block acquires a motion pattern that alternates velocity "bursts" with periods of standing still.

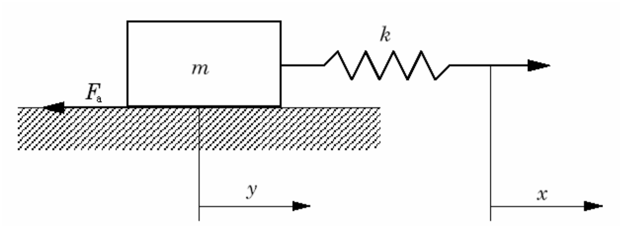

Figure 5. Physical setup for illustrating stick-slip motion

The situation described above was simulated with a block with mass of 1 [kg], and a spring with a stiffness constant of 2 [N/m]. The obtained results are given in Fig. 6. It is observed that the response of the proposed model is much closer to that of the original model than in the case of pre-sliding displacement. In the detailed view (Fig. 6(b)), it is verified that the difference between the models is visually distinguishable only for the case when $k_V = 10^6$ [s/m].

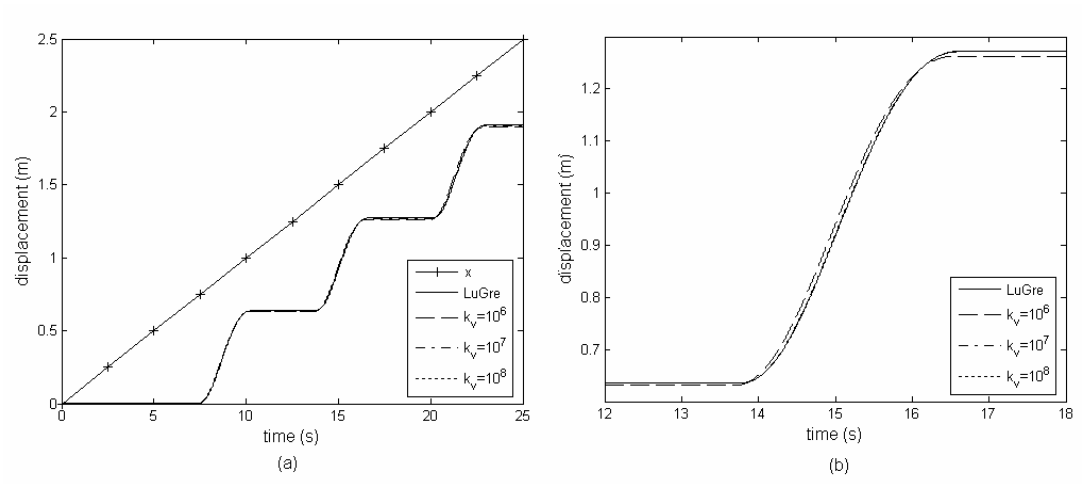

Figure 6. Simulation results for stick-slip motion (first scenario): (a) complete simulation (b) detailed view

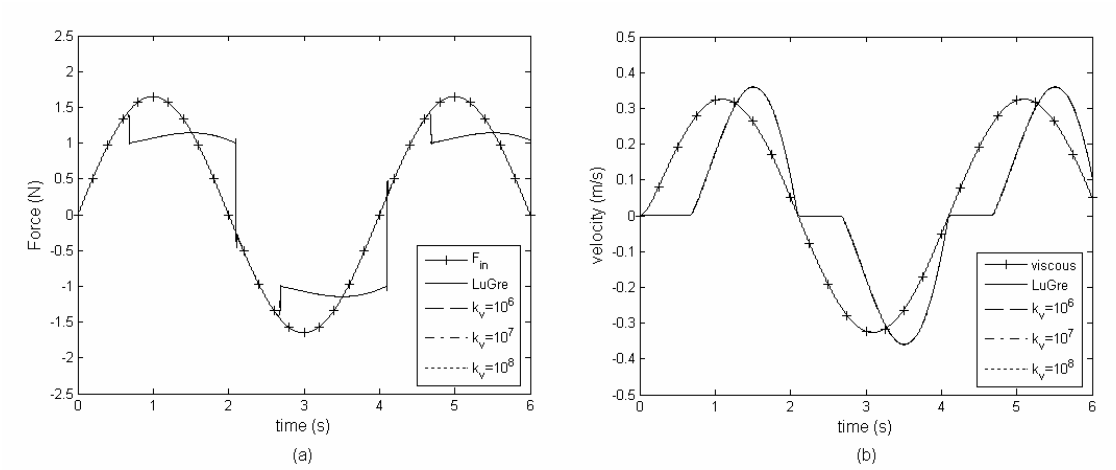

Figure 7. Simulation results for stick-slip motion (second scenario): (a) force results (b) velocity results

Stick-slip behavior can be also verified by applying a slowly varying force to unit mass subject to friction. In this case, while the absolute value of the applied force is kept below the static friction limit F_S, the body must remain immobile. When this limit is reached, the value of the friction force must fall rapidly to F_C, and the mass starts to move in the direction of the applied force. This behavior is illustrated in Fig. 7. It represents the results obtained when a sinusoidal force F_{in} with frequency 0.25 [Hz] and amplitude $1.1F_S$ is applied to a unit mass subject to friction. In Fig. 7(a), the friction forces calculated by means of each model are presented. The sharp reduction in the friction force when the absolute value of F_S is reached can be clearly observed in all cases. In Fig. 7(b), the velocities acquired by the unit mass in each case are shown. For comparison, it is also presented the behavior of the velocity of the unit mass when only viscous friction is present. It can be noticed that both the original version of the LuGre model and its approximation that is proposed in this work are capable of predicting the velocity of the unit mass in a way that is more consistent with the expected behavior of a real system than in the case when only viscous friction is considered. Finally, it should be noticed that the differences in the predicted behavior of the system for the original LuGre model and for each value of k_V used in the proposed approximation are hardly distinguishable for this simulation.

The next set of simulations deal with the prediction of *limit cycles* in a closed–loop system. This phenomenon can arise in positioning tasks subject to severe dry-friction conditions, especially when very small positioning errors are required, and it is directly related to stick-slip motion. Due to such effect, the employed controller might act in an analogous way to that of the elastic force applied by the spring in Fig. 5: when the break-away force is reached, the friction force is suddenly reduced. Thus, the control action employed to move the body becomes suddenly excessive, causing the manipulated object to stop beyond its desired position. Then, as the controller reverses its action in order to move the object to its desired position, the same effect takes place in the opposite direction, and the body is incorrectly positioned once again. Under some circumstances, this process can continue indefinitely, with the manipulated body moving continuously around its desired position but never stopping at it as intended.

In order to simulate this phenomenon, it was considered a rigid body with unit mass and subject to friction. The employed controller was of the PID-type, as illustrated in Fig. 8. As in the example given in Canudas de Wit et al. (1995), the gains of the controller are $K_P = 3$, $K_I = 4$ and $K_D = 6$. The output y of the system was required to track a desired input step y_d, starting at $t = 10$ s. As it can be observed in Fig. 8, both models clearly predict limit cycles. Also, as in the other performed simulations - with exception to the case of the pre-sliding displacement – the differences between the responses of both models are hardly distinguishable for $k_V = 10^7$ [s/m] or greater.

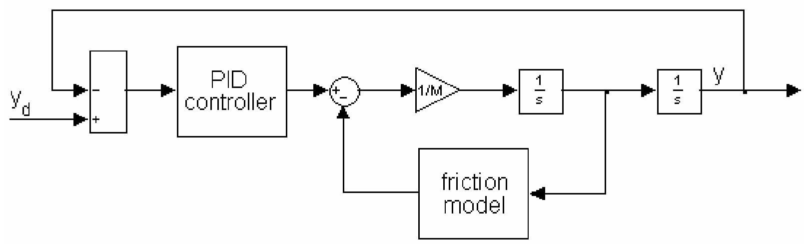

Figure 8. Block diagram for representing limit cycles

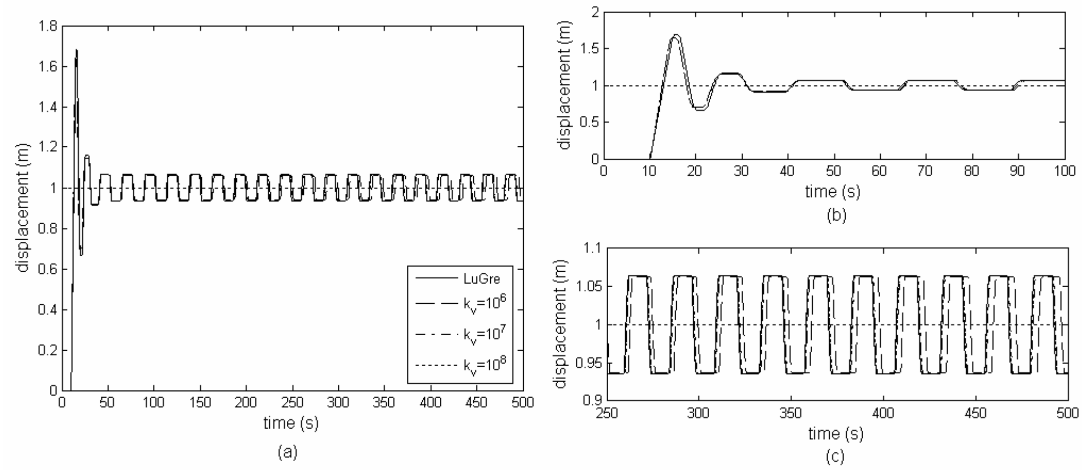

Figure 9. Simulation results for limit cycles: (a) complete simulation; (b) detailed view – first 100 s; (c) detailed view – after initial transients.

Another important phenomenon related to friction is the variation of the break-away force, that is, the force at which a body presents a sharp variation in its velocity, indicating that a true phase of translational movement was reached. The value of such force depends on the rate at which the external force applied to the body is varied: in general terms, the break-away force becomes smaller as such variation rate is increased. In the simulations, a ramped up force was applied to a unit mass at different rates, and the value of the friction force when the body presented an abrupt variation in its velocity was registered. The results are shown in Fig. 10. Once again, it is noticed that for values of k_V about 10^7 [s/m] or larger, the proposed model behaves in a way that resembles very closely the original one.

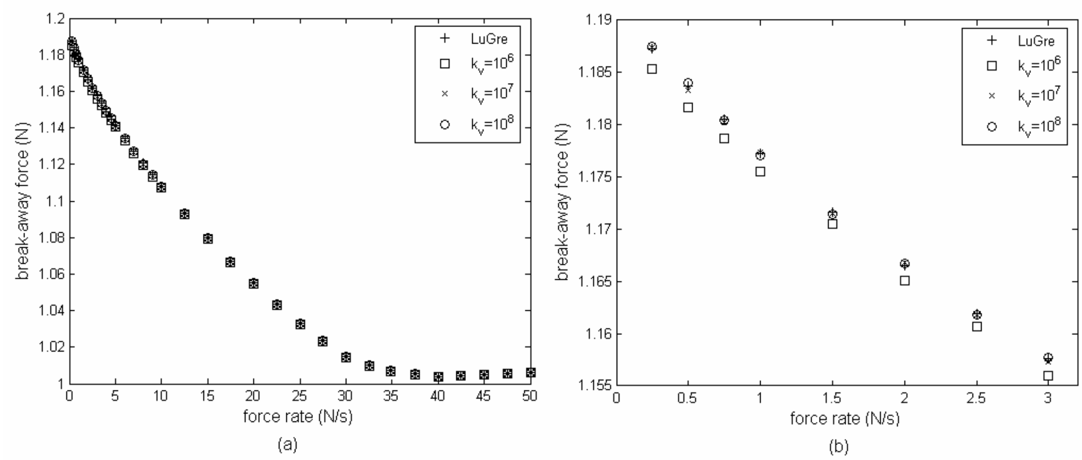

Figure 10. Simulation results for varying break-away force: (a) complete simulation; (b) detailed view

5. APPLICATION EXAMPLE OF THE PROPOSED MODEL

In this section, the proposed version of the LuGre friction model is applied to compensate friction in a pneumatic servodrive, as described in Guenther et al. (2006). In that work, due to the discontinuous nature of the LuGre model, the employed friction observer was developed according to its first continuous version, as described in Section 3.1 of the present work. In the open loop model, however, friction was still considered according to the original LuGre model. Due to such difference between the two employed models, the stability analysis of the controlled system was limited to guaranteeing that, even with a perfect knowledge of all necessary parameters, the tracking errors of the controlled system would converge to a limited region around the origin of the state space. By means of that approach, it was not possible to show that, without parametric uncertainties, such errors would converge asymptotically to zero, because of the difference between the two models employed at different parts of the system.

When the proposed continuous version of the LuGre model is employed, such difficulty does not arise. This model does not suffer from the analytical limitations of the first continuous version employed by Guenther et al. (2006), and, as seen in the simulations section, it can provide numerical predictions that are very close to those obtained by means of the original LuGre model. Therefore, there is no practical reason for employing two different friction models in the stability analysis of the controlled system. As a result, when the proposed friction model is employed to the case studied by Guenther et al. (2006), it becomes possible to prove that the controlled system is asymptotically stable.

In order to prove such statement, the mathematical model of the pneumatic actuator is now presented. The complete development of this model can be found in Perondi and Guenther (2002) and in Guenther et al. (2006). According to such works, the pneumatic positioning system can de conveniently modeled as:

$$M\ddot{y} = Ap_{\Delta} - F_a = Ap_{\Delta d} + A\tilde{p}_{\Delta} - F_a \tag{10}$$

$$\dot{p}_{\Delta} = \hat{h}(p_1, p_2, y, \dot{y}) + \hat{u}(p_1, p_2, y, u) \tag{11}$$

where M is the mass of the piston-load assembly, F_a is the friction force, A is the cylinder cross-sectional area, p_{Δ} is the differential pressure applied to the piston of the actuator, $p_{\Delta d}$ is the desired value for this pressure so as to cause the piston to track its desired trajectory, $\tilde{p}_{\Delta} = p_{\Delta} - p_{\Delta d}$ is the tracking error of the differential pressure, and $\hat{u} = \hat{u}(p_1, p_2, y, u)$ and $\hat{h} = \hat{h}(p_1, p_2, y, \dot{y})$ are nonlinear functions that accumulate all terms affected by the servovalve control voltage u in the first function, and the terms that are dependent only on the states of the system (chamber pressures p_1 and p_2, position y and velocity $\dot{y}$ of the piston) in the second one. In the two previously mentioned works, the friction force is modeled according to the original LuGre model (Eqs. (1) and (2)). In this work, it is employed the newly proposed model, in which Eq. (2) is replaced by its continuous approximation given by Eq. (7).

For controlling this system, Guenther et al. (2006) propose an algorithm based on the cascade control methodology, which consists in interpreting the whole system as two connected subsystems (mechanical and pneumatic), so that two control laws can be defined independently: one for each subsystem. The control law applied to the mechanical subsystem calculates $p_{\Delta d}$ so that the system tracks its desired trajectory y_d. Then, with that information, the control law applied to the pneumatic subsystem acts upon the servovalve control voltage u so as to cause the actual pressure difference p_{Δ} to track its desired value $p_{\Delta d}$ as closely as possible. The control laws applied to each subsystem are:

$$Ap_{\Delta d} = -K_D s + M\left(\ddot{y}_d - \lambda\dot{\tilde{y}}\right) + \hat{F}_a \tag{12}$$

$$\hat{u} = -\hat{h}(p_1, p_2, y, \dot{y}) - As - K_P \tilde{p}_{\Delta} + \dot{p}_{\Delta d} \tag{13}$$

where K_D and K_P are positive constants; $s = \dot{\tilde{y}} + \lambda\tilde{y}$ is a weighted sum of the trajectory tracking errors $\dot{\tilde{y}}$ and $\tilde{y}$ (in terms of piston velocity and position, respectively), defined in the same way as $\tilde{p}_{\Delta}$; $\ddot{y}_d$ is the desired acceleration so that the desired trajectory is correctly tracked; $\dot{p}_{\Delta d}$ is the time derivative of the desired differential pressure; $\hat{h}(p_1, p_2, y, \dot{y})$ is the part of the pneumatic subsystem that is independent of the control signal $u(t)$ in Eq. (11), and $\hat{F}_a$ is the observed value of the friction force. Basically, these two laws represent a version of the well-known control law of Slotine and Li (Slotine and Li, 1988) augmented with a friction compensator in the mechanical subsystem, combined to a feedback linearization controller in the pneumatic subsystem. The friction compensator $\hat{F}_a$ is developed by employing the observed version $\hat{z}$ of the internal state z defined in the LuGre friction model. In Guenther et al. (2006), such observer is defined according to the first continuous version of the LuGre model presented in this work. Redefining this observer in terms of the proposed model, the algorithm for estimating $\hat{z}$ becomes:

$$\hat{F}_a = \sigma_0 \hat{z} + \sigma_1 \dot{\hat{z}} + \sigma_2 \dot{y} \tag{14}$$

$$\dot{\hat{z}} = S_1(\dot{y})\dot{y} - \frac{S_2(\dot{y})}{f_s(\dot{y})}\hat{z} - K\sigma_0 s \tag{15}$$

where K is a positive constant. Defining the estimation error of the state z as $\tilde{z} = \hat{z} - z$ and subtracting Eq. (7) from Eq. (15), the time derivative $\dot{\tilde{z}}$ of such estimating error becomes:

$$\dot{\tilde{z}} = -\frac{S_2(\dot{y})}{f_s(\dot{y})}\tilde{z} - K\sigma_0 s \tag{16}$$

Once the closed-loop system is completely defined, the actual proof of the asymptotic stability of such system will be given. The proof is based on the following non-negative function candidate:

$$V(t)=\frac{1}{2}\left(Ms^2+\tilde{p}_\Delta^2+K^{-1}\tilde{z}^2\right) \tag{17}$$

whose time derivative is given by:

$$\dot{V}(t)=Ms\dot{s}+\tilde{p}_\Delta\dot{\tilde{p}}_\Delta+K^{-1}\tilde{z}\dot{\tilde{z}} \tag{18}$$

By substituting Eq. (12) in Eq. (10) and making some routine calculations, the term $M\dot{s}$ is determined. Acting likewise with Eqs. (13) and (11), $\dot{\tilde{p}}_\Delta$ is obtained, whereas $\dot{\tilde{z}}$ is readily given in Eq. (16). Thus, substituting these three terms in Eq. (18), it is possible to show that $\dot{V}(t)$ becomes:

$$\dot{V}(t)=-(K_D+\sigma_1\sigma_0)s^2-K_D\tilde{p}_\Delta^2-K^{-1}\frac{S_2(\dot{y})}{f(\dot{y})}\tilde{z}^2-\sigma_1\frac{S_2(\dot{y})}{f(\dot{y})}\tilde{z}s \tag{19}$$

from which, employing the inequality $ab\le\frac{1}{2}\left(a^2+b^2\right)$ in the term $\tilde{z}s$, it can be shown that:

$$\dot{V}(t)\le-\left(K_D+\sigma_1\sigma_0-\sigma_1\frac{S_2(\dot{y})}{2f(\dot{y})}\right)s^2-K_D\tilde{p}_\Delta^2-\frac{S_2(\dot{y})}{f(\dot{y})}\left(K^{-1}-\frac{\sigma_1}{2}\right)\tilde{z}^2 \tag{20}$$

Thus, $\dot{V}(t)$ is negative semi-definite if K and K_D are chosen so that the following relations are respected:

$$\begin{cases} K\le\dfrac{\sigma_1}{2} \\ K_D\ge\sigma_1\left(\dfrac{S_2(\dot{y})}{2f(\dot{y})}-\sigma_0\right) \end{cases} \tag{21}$$

Because both $S_2(\dot{y})$ and $f(\dot{y})$ are limited, such values of K_D and K exist. Thus, $V(\infty)\le V(0)$, implying that s, $\tilde{p}_\Delta$ and $\tilde{z}$ are limited quantities. Also, integrating Eq. (20) with respect to time from zero to infinity, and multiplying the resulting function by -1, one obtains:

$$V(0)-V(\infty)\ge\int_0^\infty\left(K_D+\sigma_1\sigma_0-\sigma_1\frac{S_2(\dot{y})}{2f(\dot{y})}\right)s^2dt+\int_0^\infty K_D\tilde{p}_\Delta^2dt+\int_0^\infty\frac{S_2(\dot{y})}{f(\dot{y})}\left(K^{-1}-\frac{\sigma_1}{2}\right)\tilde{z}^2dt \tag{22}$$

It is already known that all terms in the right side of Eq. (22) are limited. Additionally, the term $V(0)-V(\infty)$ is a limited and non-negative quantity, implying that the results of all integrals to the right of the inequality sign are also limited once they are non-negative quantities too. As these integrals are evaluated from zero to infinity, it follows that their arguments must tend to zero as time progresses. Thus it must be concluded that both s and $\tilde{p}_\Delta$ converge asymptotically to zero, for these are the only quantities that can become null in the first two integrals in the right side of Eq. (22). The same conclusion cannot be drawn for the case of $\tilde{z}$, however, because the third integral also depends on the value of $S_2(\dot{y})$, which is nil when $\dot{y}=0$. Thus, the third integral term can become zero even if $\tilde{z}$ is nonzero.

As $s=\dot{\tilde{y}}+\lambda\tilde{y}$ can be interpreted as a first order filter applied to $\tilde{y}$, and as s converges asymptotically to zero, it follows that both $\dot{\tilde{y}}$ and $\tilde{y}$ must also converge asymptotically to zero. Thus, when the proposed friction model is employed as part of the control scheme described by Guenther et al. (2006), it is concluded that all three trajectory tracking errors of the controlled system ($\tilde{y}$, $\dot{\tilde{y}}$ and $\tilde{p}_\Delta$) converge to zero as $t\to\infty$, which completes the proof.

Remark #2: it should be noticed that $\tilde{z}$ is *not* a trajectory tracking error of the system in the true sense, because it refers to a state pertaining only to the LuGre model approach for representing friction. Thus, the fact that $\tilde{z}$ may not converge to zero does not mean that asymptotic stability is not proven. Furthermore, if the controlled system is persistently excited (in the sense that the system is continuously required to stay in movement), then Eq. (22) ensures that even $\tilde{z}$ must converge to zero as time goes to infinity.

6. CONCLUSIONS

In this paper, a continuous version of the LuGre model was proposed. The main characteristics of the proposed model were outlined, with emphasis given to the fact that the most important analytical properties of the original LuGre model are retained in its newly proposed version. Simulation results were employed to show that the friction behavior predicted by means of the proposed model can be made very similar to that of its discontinuous predecessor, provided that the value of the parameter k_V is chosen to be sufficiently high. Also, the usefulness of the newly proposed model was illustrated by its application as part of a nonlinear control scheme for a pneumatic positioning system. In particular, it was observed that the employment of the proposed model makes it possible to ensure stronger stability properties for the controlled system than those that could be previously guaranteed.

Future works will include the proofs of the analytical properties that are necessary for the stability analysis of the closed-loop systems when the proposed model is part of a friction-compensating control algorithm. Also, the development of an adaptive version of the control algorithm discussed in Section 5 is currently under research.

7. REFERENCES

Armstrong-Hélouvry, B., Dupont, P., Canudas de Wit, C., 1994. “A Survey of Models, Analysis Tools and Compensation Methods for the Control of Machines with Friction”, Automatica, Vol 30, n^o 7, pp1083 – 1138.

Barabanov, N., Ortega, R., 2000. "Necessary And Sufficient Conditions for Passivity of the LuGre Friction Model", IEEE Transactions on Automatic Control, Vol. 45, pp. 830-832.

Canudas de Wit, C., Olsson, H., Astrom, K.J., Lischinsky, P., 1995, “A New Model for Control Systems with Friction”, IEEE Transactions on Automatic Control, Vol. 40, n. 3, pp.419-425.

Dupont, P., Armstrong, B., Hayward, V., 2000, "Elasto-plastic friction model: contact compliance and stiction", Proceedings of the American Control Conference, Illinois, United States of America, pp. 1072-1077.

Guenther, R., Perondi, E. A., DePieri, E. R., Valdiero, A. C., 2006. “Cascade Controlled Pneumatic Positioning System with Lugre Model Based Friction Compensation”. Journal of the Brazilian Society of Mechanical Sciences & Engineering, Vol 28, n^o 1, pp.48-57.

Makkar, C., Dixon, W. E., Sawyer, W.G., Hu, G., 2005. “A New Continuously Differentiable Friction Model for Control Systems Design”, Proceedings of the 2005 IEEE/ASME International Conference on Advanced Intelligent Mechatronics, pp. 600 - 605.

Olsson, H., 1996. “Control systems with friction”, PhD Thesis, Lund Institute of Technology, Lund, Sweden.

Perondi E. Guenther, R., 2002, “Pneumatic Positioning System Cascade Control With Friction Compensation”. Proceedings of XIV CBA. Natal – RN, Sepetember 2002.

Slotine, J.-J. E., Li, W., “Adaptive Manipulator control: a Case Study”. IEEE Transaction on Automatic Control, Vol. 33, No. 11, pp. 995-1003, November 1988.

Sobczyk, M. R., 2009 “Variable Structure Cascade Control with Parameter Adaption and Friction Compensation Applied to a Pneumatic Positioning System”, PhD Thesis, Federal University of Rio Grande do Sul. In Portuguese.

Wenjing, Z., Qinghai, F., 2008. “Adaptive Compensation for Friction and Force Ripple in Ship-Borne Gun Servo System”, Proceedings of the 7th World Congress on Intelligent Control and Automation, 2008, pp. 3434 – 3438.

Zeng, H., Sepehri, N., 2006. “Adaptive Backstepping Control of Hydraulic Manipulators with Friction Compensation Using Lugre Model”, Proceedings of the American Control Conference – 2006.

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