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APPLICATION OF TFL/LTR ROBUST CONTROL TECHNIQUES TO FAILURE ACCOMMODATION

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Abstract. *Robust control deals with differences between the real system and the mathematic model explicitly during the design stage. Such differences can be caused, for example, by modeling simplifications and faults in the system. In the case of plants with uncertainties, on the assumption of a nominal model plant and a controller, it can be defined the concepts of nominal stability and robust stability. In the nominal stability case, the controller only stabilizes the nominal model, on the other hand, in the robust stability case, it stabilizes all possible process realizations within an uncertainty region. The term "fault" designates any impairment of system components that may result in performance degradation or even a complete stop of system functions. System faults can be classified as sudden (abrupt faults) or incipient faults (when the system suffers a slow degradation). Implementing a fault tolerant system, that keeps its dynamic response inside acceptable limits even under fault occurrence, is not trivial. In such case, the system can have its performance degraded but must continue to be operational. There have been several proposal solutions to the fault tolerant control problem. The proposal of this paper is to apply the TFL/LTR (Target Feedback Loop/Loop Transfer Recovery) robust control design to a 3DOF hover didactic system with uncertainty model and abrupt faults caused, for example, by power loss. TFL/LTR is a two step design procedure. In the first step, desired dynamics are defined for the controlled system (TFL). In this paper a linear quadratic regulator technique that considers a control signal gain will be used to ensure stability margins. These margins shall be defined so as to make the system robust to the abrupt faults. In the second step, a loop transfer recovery method (LTR) is used so that the controlled real system has characteristics close to those of the target feedback loop. Simulation results show that the control system under study has the intended robustness and fault tolerance.*

Keywords: *Robust control, Fault tolerant control, Model uncertainty*

1. INTRODUCTION

The robust control problem has several proposed solutions, it consists of defining a control strategy that is capable to tolerate mismatches between the nominal model and the real plant caused by, for example, modeling simplifications and faults in the system. Then, the concepts of nominal stability and robust stability can be define as: in first case, the controller only stabilizes the nominal model, and in second case, it stabilizes all possible process realizations within an uncertainty region. Faults may introduce differences between the nominal design model and the real plant, such faults can be sudden, abrupt, or incipient, in which the system suffers a slow degradation. A possible solution for the fault tolerant control problem, it keeps its dynamic response inside acceptable limits even under fault occurrence, can be the incorporation of robust stability.

There are various applications of robust control design available in the literature, it can be cited, for example, spacecraft attitude control (Lahdhiri and Alouani, 1993), VSTOL flight control system (Zarei, 2006) and direct current motor control (Gargouri et al., 2002). The objective of this paper is to apply TFL/LTR (Target Feedback Loop/Loop Transfer Recovery) strategy presented in Prakash (1990), that consists of the combination of Amplified Linear Quadratic Regulator (ALQR) and Kalman Filter to recovery the system, witch has favorably guaranteed stability margins.

This paper is organized as follow. Section 2 contains the system description. Section 3 presents the robust control methodology. Section 4 describes problem formulation and control design. Section 5 shows the results. Finally, section 6 contains conclusion and proposed future work.

2. 3DOF HOVER DIDACTIC PLANT MODEL DESCRIPTION

The 3DOF Hover didactic plant, presented in Fig. 1, is formed by a frame with four propellers. Such system is assembled on a pivot joint that enables rotations about the yaw, roll and pitch axes. The plant base is fixed to the workbench, having slip rings which allow the free movement on the yaw axis with low friction. Each propeller generates a lift force which is used to control the roll and pitch angles. Consequently, the torque resulting from the propellers rotation causes the movement of the structure around the yaw axis. In the case of a controlled environment, with the four forces balanced, the torque total is matched. In this study will be considered a simplified model of this system, as presented in Quanser manual. When a positive voltage is applied to any motor, a lift force is generated causing a lifting of the whole propellant system. The group formed by front and back motors (supply voltages given by v_f and v_b) causes the movement on the pitch and yaw axes, while the lateral motors (analogously v_r and v_l) move the roll and yaw axes. The system has three encoders which can measure the angular displacement in the three freedom axes of the plant from an initial position.

ABCM Symposium Series in Mechatronics - Vol. 4 - pp.189-197

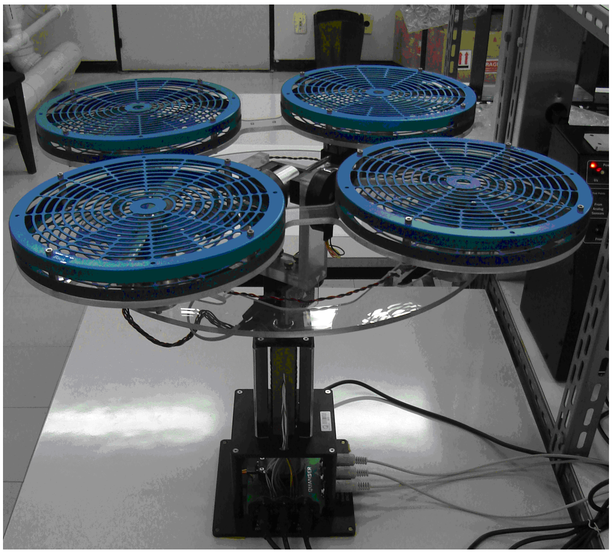

Figure 1. 3DOF Hover didactic plant.

Assuming the system decoupled and linear (with the equilibrium point in which the propellants are aligned with the axes X, Y and Z, Fig. 2), the pitch movement can be described as:

$$J_p \frac{\partial^2 p}{\partial t^2} = l K_f (v_f - v_b) \tag{1}$$

where, J_p is the equivalent moment of inertia about the pitch axis, p is the pitch angle, l is the distance between pivot to each motor and K_f is the propeller force-thrust constant.

In a similar manner, for the roll movement:

$$J_r \frac{\partial^2 r}{\partial t^2} = l K_f (v_r - v_l) \tag{2}$$

in which, J_r is equivalent moment of inertia about the roll axis and r is the roll angle.

The torque generated by the front and back propellants is called τ_f and τ_b, and similarly, the torque generated by right and left propellants are τ_r and τ_l. As shown in Fig. 2, the torque generated by lateral propellants has a reverse direction in relation to the torque generated by front and back propellants. The yaw movement is given by:

$$J_y \frac{\partial^2 y}{\partial t^2} = \tau_f + \tau_b + \tau_r + \tau_l \tag{3}$$

$$J_y \frac{\partial^2 y}{\partial t^2} = K_{t,c}(v_f + v_b) + K_{t,n}(v_r + v_l) \tag{4}$$

in which J_y is the equivalent moment of inertia about the yaw axis, y is the yaw angle, $K_{t,n}$ and $K_{t,c}$ are constants that relate the torque generated by propellant when a voltage is applied on the motor.

In short, the presented linear model is given by:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{5}$$

$$z(t) = Cx(t) + Du(t) \tag{6}$$

with

$$x(t)^T = [y(t), p(t), r(t), \dot{y}(t), \dot{p}(t), \dot{r}(t)] \tag{7}$$

and

$$u(t)^T = [v_f(t), v_b(t), v_r(t), v_l(t)] \tag{8}$$

Finally, the model matrices are:

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{9}$$

ABCM Symposium Series in Mechatronics - Vol. 4 - pp.189-197

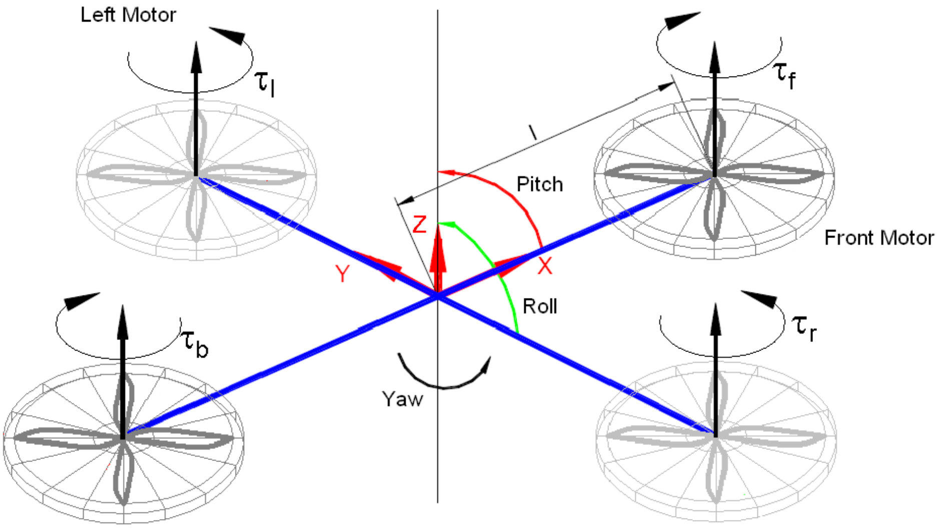

Figure 2. 3DOF Hover dynamic system (based in Matos (2008)).

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{K_{t,c}}{J_y} & \frac{K_{t,c}}{J_y} & \frac{K_{t,n}}{J_y} & \frac{K_{t,n}}{J_y} \\ l\frac{K_f}{J_p} & -l\frac{K_f}{J_p} & 0 & 0 \\ 0 & 0 & l\frac{K_f}{J_r} & -l\frac{K_f}{J_r} \end{bmatrix} \tag{10}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{11}$$

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{12}$$

The 3DOF Hover plant parameters are presented in Tab. 1 (Matos, 2008).

Table 1. 3DOF Hover parameters.

Symbol	**Value**	**Unit**
$K_{t,n}$	0.0036	$N.m/V$
$K_{t,c}$	-0.0036	$N.m/V$
K_f	0.1188	N/V
l	0.197	m
J_y	0.110	$kg.m^2$
J_p	0.0552	$kg.m^2$
J_r	0.0552	$kg.m^2$

3. CONTROL DESIGN METHODOLOGY: TFL/LTR TECHNIQUES

TFL/LTR ("Target Feedback Loop" and "Loop Transfer Recovery"') is a general class of robust control design procedures (Prakash, 1990). It consists of two steps: firstly, the design of a feedback closed loop system (TFL step), with the calculation of the constant TFL gain matrix K_r that specifies stability and robustness performance; and then, the LTR step, the determination of the constant LTR gain matrix, K_f, to recovery the loop transfer.

ABCM Symposium Series in Mechatronics - Vol. 4 - pp.189-197

This work applies the PRI-version (“perturbation-reflected-to-input") design procedure presented in Prakash (1990) that uses an amplified linear quadratic regulator (TFL step) and a Kalman filter (LTR step). The final control structure is given by:

$$K(s) \equiv C_k(sI - A_k)^{-1}B_k \tag{13}$$

with

$$A_k = A - K_f C - BK_r \tag{14}$$

$$B_k = -K_f \tag{15}$$

$$C_k = -K_r \tag{16}$$

3.1 Amplified Linear Quadratic Regulator (TFL step)

Considering a system described by:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{17}$$

$$z(t) = Cx(t) + Du(t) \tag{18}$$

For the case in which (A, B) is stabilizable and B is full rank, given a $Q_{TFL} = Q_{TFL}^T \geq 0$ matrix, with $(A, \sqrt{Q_{TFL}})$ detectable and $R_{TFL} = R_{TFL}^T > 0$, the regulator algebraic Riccati equation is given by:

$$PA + A^T P + Q_{TFL} - PBR_{TFL}^{-1}B^T P = 0 \tag{19}$$

this equation has only one solution $P = P^T \geq 0$. The square root of the matrix Q_{TFL}, a symmetric and positive semi definite (or positive definite) matrix, is obtained by:

$$Q_{TFL} = (\sqrt{Q_{TFL}})^T(\sqrt{Q_{TFL}}) \tag{20}$$

Finally, the amplified linear quadratic regulator (ALQR) gain, K_r, is:

$$K_r = (1/\beta)R_{TFL}^{-1}B^T P \tag{21}$$

with $0 < \beta < 1$. Therefore, the control signal is obtained by the amplification of the nominal controller LQR. The guaranteed multivariable margins of ALQR can be established, which correspond to the gain/phase perturbations that can be considered in all the inputs simultaneously or in an independent manner (Prakash, 1990).

For the particular case with $Q_{TFL} > 0$, Prakash (1990) demonstrates that the ALQR loop will be stable inside certain margins, even if the system has poles in imaginary axis. Moreover, for this case, the guaranteed multivariable margins are given by:

$$GGM = \beta/2, \infty \tag{22}$$

$$GPM = \pm cos^{-1}(\beta/2) \tag{23}$$

in which, GGM is the guaranteed gain margin and GPM is the guaranteed phase margin. This means that controlled system can tolerate a complex perturbation $diag(k_i e^{j\theta_i})$ providing $\beta/2 \leq k_i < \infty$ and $|\theta_i| \leq cos^{-1}(\beta/2)$ with $i = 1, ..., m$ and m the numbers of plant inputs (Prakash, 1990) (Skogestad and Postlethwaite, 1996).

3.2 Kalman Filter (LTR step)

In this study is used the LTR technique defined by Doyle and Stein (1981) *apud* Prakash (1990). In such case, the LTR gain (K_f) is given by the Kalman filter gain, with the Kalman filter matrices Q_{KF} and R_{KF} selected to recover the system output, using $Q_{KF} = q^2 BVB^T$ and V an arbitrary symmetric positive definite matrix. For $q \rightarrow \infty$ and if the system model is minimum phase, the recovery tends to be exact.

ABCM Symposium Series in Mechatronics - Vol. 4 - pp.189-197

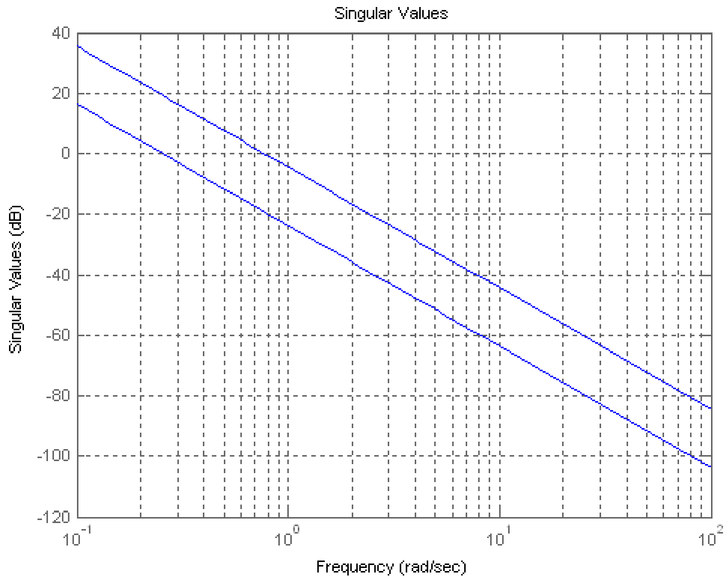

Figure 3. Singular values of the 3DOF Hover transfer function.

4. PROBLEM FORMULATION AND CONTROL DESIGN

The plant requirements asked by the ALQR method are considered: the pair (A, B) is stabilizable, the B matrix is full rank and the system model is minimum phase. The singular values of the transfer function are shown in Fig. 3.

Conjecturing that exists dynamics disconsidered in high frequency, it is defined the following characterization model for input uncertainties in the plant:

$$G(s) = G_{nom}(s)\frac{1}{1+T_1 s} \tag{24}$$

$$G(s) = G_{nom}(s)[I + E(s)] \tag{25}$$

$$E(s) = \frac{1}{1+T_1 s} - 1 = \frac{-T_1}{1+T_1 s} \tag{26}$$

$$G_{nom}(s) = C(sI - A)^{-1}B \tag{27}$$

For $T_1 = 0.01$, the error characterization is:

$$\sigma_{max}[E(jw)] < e_m(w) = \left|\frac{-0.01s}{1+0.01s}\right| \tag{28}$$

To agree with the requirement of robust stability:

$$\sigma_{max}[K(jw)G(jw)] < \frac{1}{e_m(w)} \tag{29}$$

The control objectives are: to obtain an offset free system and to guarantee the robust stability for the defined model uncertainty and abrupt faults caused by motor power loss. For the ALQR method it was defined the Q_{TFL} and R_{TFL} matrices:

$$Q_{TFL} = \begin{bmatrix} 1000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.01 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.01 \end{bmatrix} \tag{30}$$

$$R_{TFL} = 0.05 \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{31}$$

So that $(A, \sqrt{Q_{TFL}})$ is detectable. Considering, for example, a power loss up to 60%, must be imposed a gain multivariable margins of at least 0.4. So, it can arbitrate $\beta = 0.8$. The solution of the algebraic Riccati equation is:

$$P = \begin{bmatrix} 465.7728 & 0 & 0 & 108.4672 & 0 & 0 \\ 0 & 4.8697 & -0.0000 & 0 & 1.1807 & 0.0000 \\ 0 & -0.0000 & 4.8697 & 0 & -0.0000 & 1.1807 \\ 108.4672 & 0 & 0 & 50.5211 & 0 & 0 \\ 0 & 1.1807 & -0.0000 & 0 & 0.5750 & -0.0000 \\ 0 & 0.0000 & 1.1807 & 0 & -0.0000 & 0.5750 \end{bmatrix} \tag{32}$$

Therefore, the gain K_r is

$$K_r = (1/\beta)R_{TFL}^{-1}B^T P = \begin{bmatrix} -88.3883 & 12.5000 & -0.0000 & -41.1689 & 6.0871 & -0.0000 \\ -88.3883 & -12.5000 & 0.0000 & -41.1689 & -6.0871 & 0.0000 \\ 88.3883 & 0.0000 & 12.5000 & 41.1689 & -0.0000 & 6.0871 \\ 88.3883 & -0.0000 & -12.5000 & 41.1689 & 0.0000 & -6.0871 \end{bmatrix} \tag{33}$$

In LTR technique, it was selected $q = 500$, V as the identity matrix and $R_{KF} = 0.001I_3$ (I_3 is a 3×3 identity matrix), which results in the K_f matrix:

$$K_f = \begin{bmatrix} 2.4516 & 0 & 0 \\ 0 & 22.5221 & 0 \\ 0 & 0 & 22.5221 \\ 3.0051 & 0 & 0 \\ 0 & 253.6222 & 0 \\ 0 & 0 & 253.6222 \end{bmatrix} \tag{34}$$

The control strategy was tested by simulations using MatLab 6.5 software, with the only mismatch between the design model and the real plant is caused by power loss.

5. RESULTS AND DISCUSSIONS

The singular values of the $\frac{1}{e_m}$ constraint (Eq. (29)) and the controlled system are shown in Fig. 4.

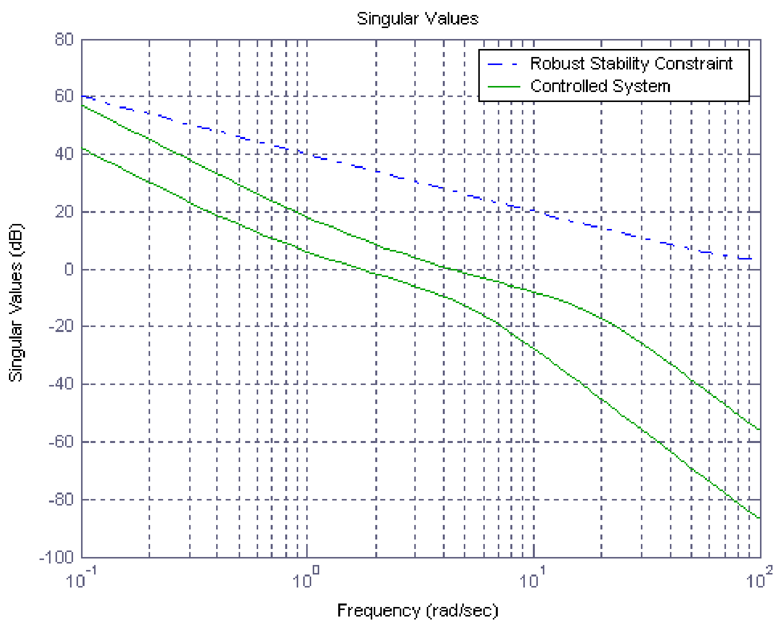

Figure 4. Robust stability requirement.

The performance of the system without and with a 60% power loss fault in all four motors, using the presented TFL/LTR method controller (ALQR and Kalman filter), is shown in Fig. 5 and Fig. 6. In both cases, the steady-state error converges to zero as required in the design procedures.

ABCM Symposium Series in Mechatronics - Vol. 4 - pp.189-197

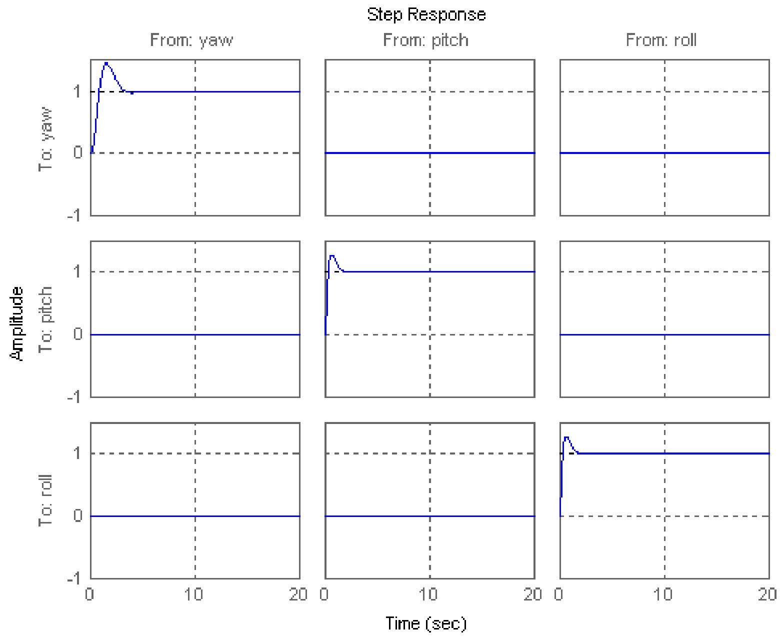

Figure 5. Step response without power loss fault.

Finally, in the Quanser manual is proposed an LQR control strategy with a second order filter to estimate $\dot{y}(t)$, $\dot{p}(t)$ and $\dot{r}(t)$. The vector control gain and the filter are given by:

$$K_{LQR} = \begin{bmatrix} -61.2 & 111.8 & 0 & -30.6 & 27.6 & 0 \\ -61.2 & -111.8 & 0 & -30.6 & -27.6 & 0 \\ 61.2 & 0 & 111.8034 & 30.6 & 0 & 27.6 \\ 61.2 & 0 & -111.8034 & 30.6 & 0 & -27.6 \end{bmatrix} \tag{35}$$

$$H(s) = \frac{\omega_c^2 s}{s^{2+2\zeta\omega_c s+\omega_c^2}} \tag{36}$$

with $\omega_c = 40\pi$ and $\zeta = 0.6$.

To compare the robust stability performance of both techniques, in the case of multiplicative input uncertainty, it is necessary to analyze their input complementary sensibility T_I given by (Skogestad and Postlethwaite. 1996):

$$T_I(s) = K_{controller}G_{nom}(I + K_{controller}G_{nom})^{-1} \tag{37}$$

Figure 7 shown that the proposed strategy has the input complementary sensitivity maximum value lower than Quanser LQR one, which implies a better robust stability performance.

6. CONCLUSIONS

The method ALQR with the recovery method based on Kalman filter were applied to 3DOF Hover didactic plant and fulfill the design requirements of robust stability performance. Moreover, this approach is favorably compared with the a LQR strategy using second order filters to estimate the velocity variables. A proposal for future work is to applies this control strategy in the real didactic plant.

7. ACKNOWLEDGEMENTS

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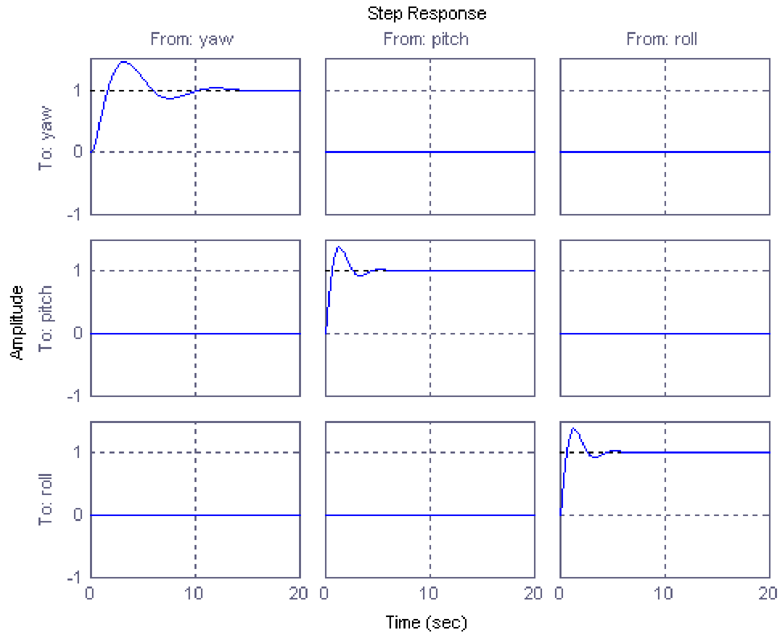

Figure 6. Step response with power loss fault .

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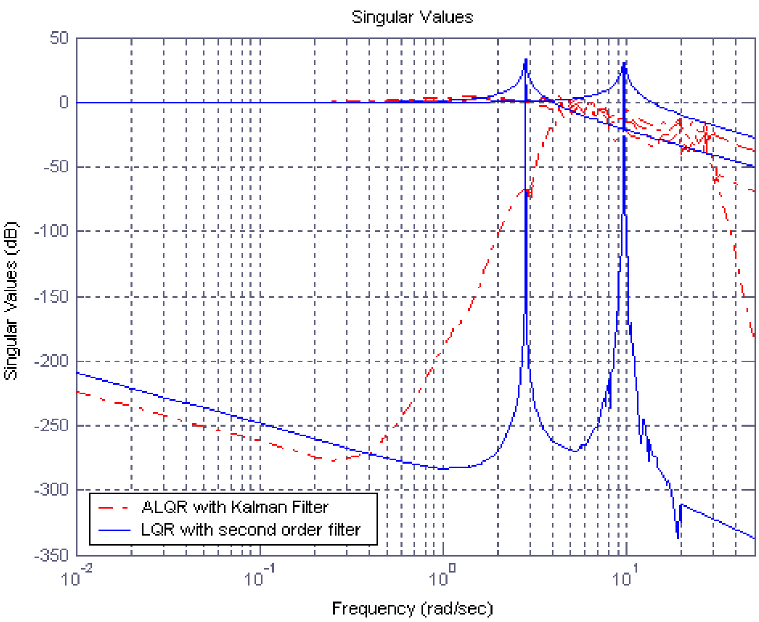

Figure 7. Singular value diagram for robustness analysis.