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H_∞ CONTROLLER WITH OUTPUT FEEDBACK USING LINEAR MATRIX INEQUALITIES FOR APPLICATION IN BUILDING MODEL

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Abstract. *Experimental verification of structural vibrations control strategies is essential for eventual full-scale implementations. However, few researchers have facilities readily available to them that are capable of even small-scale structural control experiments. Appropriately constructed bench-scale models can be used to study important aspects of full-scale structural vibrations control implementations, including: control-structure interaction, actuator and sensor dynamics, states feedback design, control spillover, etc. In this sense, the purpose of this article is the project of H∞ controller with output feedback using linear matrix inequalities (LMIs) for vibration attenuation in a flexible building like structure. The considered structure is manufactured by Quanser Consulting Inc., and it is controlled by an active mass driver (AMD). The structure consists of a steel frame with a controllable mass located at the top, which can be configured to have either 1 or 2 floors. In this paper, a 2-floor configuration is employed. The actual experiment considered herein is an effective way to deal with challenges associated with active control of flexible structures.*

Keywords*: Active Vibration Control, H_∞ controller, Linear Matrix Inequalities (LMIs) and Active Mass Driver (AMD)*

1. INTRODUCTION

Experimental investigations are essential to obtain a fundamental understanding of many phenomena. However, when physical systems are scaled down to a size appropriate for laboratory study, salient features of their behavior may be lost. This is particularly true for large civil engineering structures (Battaini, *et al*., 1998). In the area of control of civil structures, it is well-recognized that experimental verification of control strategies is necessary to focus research efforts in the most promising directions (Housner, *et al.* 1994a,b). However, few researchers have experimental facilities readily available to them that are capable of even small-scale structural control experiments. Consequently, the majority of control studies to date have been analytical in nature, a substantial number may have employed models that lacked important features of the physical problem. One such phenomena that has been neglected for many years is the control-structure interaction (CSI). Through a series of analytical and experimental studies, Dyke, *et al.* (1995) recognized that understanding CSI was key to developing acceleration feedback control strategies and showed that accounting for CSI is fundamental to achieving high performance controllers.

A variety of techniques have been intensively studied to reduce the structural vibration in order to achieve high-speed and high-precision motions (Singer and Seering, 1989). Recent development in microelectronics and smart structure techniques have provided new solutions for vibration control (Clark *et al*.,1998). “Smart structures” adopt microprocessors and distributed transducers to modify structural dynamics to actively suppress vibration in time-varying working environments. Use of smart structure techniques can lead to lightweight structures, leading to higher performance.

There are many robust techniques well know in the structural control literature to outline these problems. In this research work, w*e* have chosen a recent technique involving linear matrix inequalities (LMI). The LMI contributed to overcome many difficulties in control design. In the last decade, the LMI has been used to solve many problems that until then was unfeasible through others methodologies, (Boyd *et al*., 1994).

The major advantage of LMI design is to enable specifications as stability degree requirements, decay rate, input limitation on the actuators and output peak bounder. It is also possible to assume that the model parameters can involve uncertainties. The LMI is a very useful tool for problems with constraints, where the parameters are in a range of values.

Once formulated in terms of an LMI a problem can be solved efficiently by convex optimization algorithms, for example, using interior-point methods, (Gahinet *et al*., 1995). Only a few researchers explore the use of LMI in the structural control community. Sana and Rao (2000) utilized a cantilever beam with a distributed pieozoelectric actuator and sensor to design an output feedback controller to increase damping of some modes using LMI. However, the resulting matrix inequalities involved a bilinear matrix inequalities (BMI) in unknown variables and, hence, it became a non convex optimization problem. So, the BMI could not be solved directly using a standard convex optimization software package. In this case, it was necessary to use iterative methods, as for instance, the cone complementarily linearization algorithm (El Ghaoui, 1997), which is a high cost procedure.

In another way, Gonçalves *et al.* (2002) controlled a two degree-of-freedom (2DOF) mechanical system comparing the H_2 and H_∞ optimal control with state-feedback via LMI, in a procedure of solution proposed by Peres (1997). Gonçalves *et al*. (2003a) and (2003b) simulated a state-feedback synthesis using classical LMI, described in Boyd *et al*. (1994), considering norm-bound linear differential inclusions (LDI) for a 2DOF mechanical system and for a fixed-fixed aluminium beam, respectively.

The design of H_∞ output feedback control laws that meet desired performance and/or robustness specifications is an active research area of the control community for several decades. The cost functions used in H_∞ control are very general and can directly include performance specifications, disturbance rejection specifications, control input magnitude limitations and robustness requirements. The strength of this design methodology is the generality of the cost function (Carvalhal and Lopes Jr., 2005). In this work, the problem of H_∞ output feedback control is solved by convex optimization and is written through LMI for control vibration in a bench-scale structure. The model structure employed in the experiment represents a building controlled by an active mass driver (AMD) and consists of a steel frame with a controllable mass located at the top, as illustrated in Fig1.

2. EXPERIMENTAL SETUP

The equipment used for the active control experiment consisted of (see Figs.1 and 2):

Structure: The structural specimen, manufacutured by Quanser Consulting Inc., is a model of a flexible building, which can be configured to have either 1 or 2 floors. Herein, a 2-floor configuration of the model is employed. The interstorey height is 490 mm, with each column being steel with a section of 1.75 x 108 mm. The total mass of the structure is 3.5 kg, where the first floor mass is 1.16 kg, the second floor mass is 1.38 kg, and the mass of each column is 0.24 kg. The structure has the natural frequencies of 1.7 Hz and 5.1 Hz which have corresponding damping ratios of 0.013 and 0.062 respectively.

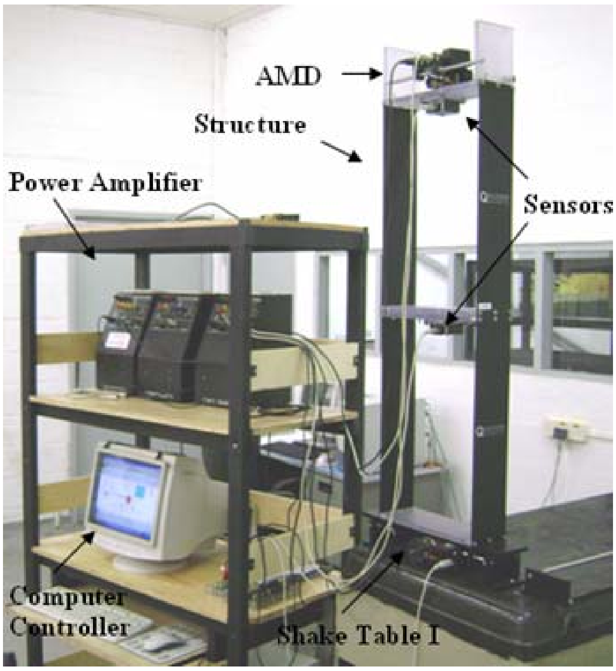


Figure 1: Bench-Scale Building Model

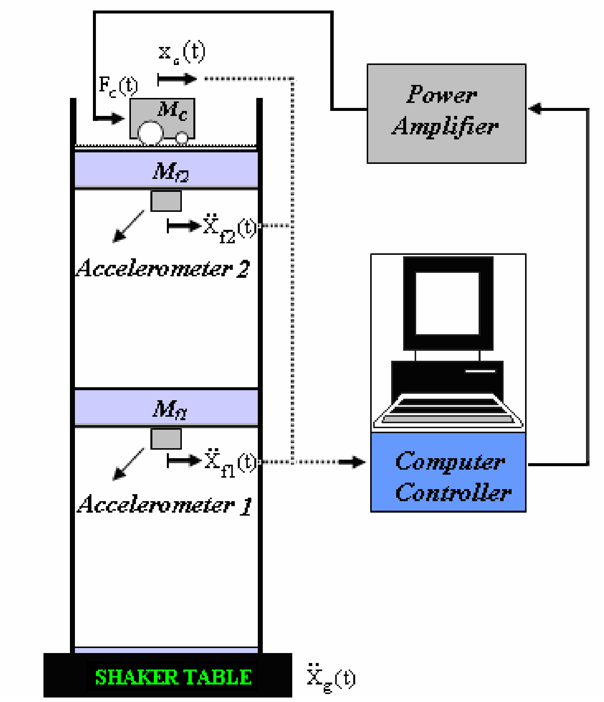


Figure 2: Experimental Set-Up

Active Mass Driver (AMD): The AMD provides the control force to the structure. As shown in Fig. 3, it consists of a moving cart with a DC motor that drives the cart along a geared rack. The maximum stroke is ± 95 mm and the total moving mass is 0.65 Kg.

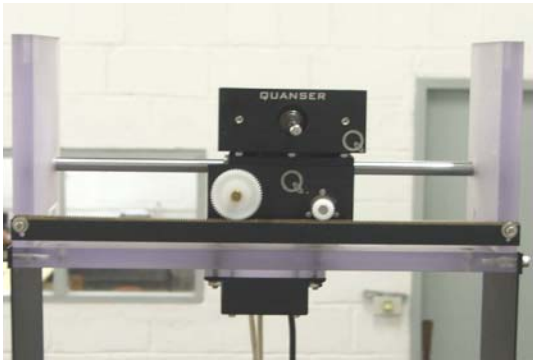


Figure 3: Active Mass Driver (AMD)

Sensors: Each floor of the building structure is equipped witch a capacitive DC accelerometer with full scale range of ± 5 g and sensitivity of 9.81 m/s2/V (i.e. 1g/V). It consists of a single-chip accelerometer with signal conditioning. The cart position is directly measured using an optical encoder whose shaft meshes with the track via an additional pinion. It offers a high resolution of 4096 counts per revolution and sensibility of 2.275E-5 m/count.

Digital Controller: Digital control is achieved by use of the MultiQ - PCI board with the WinCon real time controller. The controller is developed using SIMULINK (1997) and executed in real time using WinCon. This board has a 14-bit analog/digital (A/D) and 13-bit digital/analog (D/A) converters with four and sixteen outputs and inputs analog channels, respectively. The SIMULINK code is automatically converted to C code and interfaced through the Wincon software to run the control algorithm on the CPU of the PC.

Computer: The computer used is a Pentium IV 2.8 GHz configured with 1 GB of RAM and 40 MB of Hard Disc.

Shaker Table I: This system is used to excite the flexible modes of the structure and can be used to simulate earthquakes and evaluate the performance of active mass driver. The system consists of a high torque direct motor that can drive a 5 Kg mass at 1g (i.e. 9.81 m/s2). Maximum travel is ± 2 cm.

3. STATE-SPACE MODEL

For small floor deflection angles, both floors are modelled as standard linear spring-mass systems, as represented in Figure 2. The linear stiffness constants for both floors, K_{f1} and K_{f2}, for small angular structure oscillations, are 500 N/m. In the presented modelling approach, the structure viscous damping coefficients, B_{f1} and B_{f2}, are neglected. The Lagrange's method is used to obtain the dynamic model of the system. In this approach, the inputs to the system are considered to be the ground acceleration, $\ddot{x}_g(t)$, and the driving force of the linear motorized cart , $\mathbf{F_c}(t)$.

In order to design and implement a state-feedback controller for the system, a state-space representation needs to be derived. Moreover, it is reminded that state-space matrices, by definition, represent a set of linear differential equations that describe the system's dynamics. The following relationships represent the state space model:

$$\frac{d}{dt}\mathbf{x}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B_1}\mathbf{w}(t) + \mathbf{B_2}\mathbf{u}(t) \tag{1}$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) \tag{2}$$

where **A** is the dynamic matrix, $\mathbf{B_1}$ is the matrix of disturbance, $\mathbf{B_2}$ is the matrix of control input, **C** is the output matrix, **D** is the feed-through matrix, **w**(t) is the vector of disturbance input, **u**(t) is the vector of control input, **y**(t) is the output vector and **x**(t) is the system's state vector. In practice, **x**(t) is often chosen to include the generalized coordinates as well as their first-order time derivatives. In our case, **x**(t) is defined such that its transpose is as follows:

$$\mathbf{x}^T(t)=\left[x_c(t)\ \ x_{f1}(t)\ \ x_{f2}(t)\ \ \dot{x}_c(t)\ \ \dot{x}_{f1}(t)\ \ \dot{x}_{f2}(t)\right] \tag{3}$$

where $x_c(t)$ is the cart linear position relative to the second floor, $x_{f1}(t)$ is the first floor linear deflection relative to the ground and $x_{f2}(t)$ is the second floor linear deflection relative to the first floor.

Furthermore, it is reminded that the system's measured output vector is:

$$\mathbf{y}^T(t)=\left[x_c(t)\quad \ddot{x}_{f1}(t)\ \ \ddot{x}_{f2}(t)\right] \tag{4}$$

Also in Equation (1), the input **u**(t) is set in a first time to be $\mathbf{F_c}(t)$. Thus, we have:

$$\mathbf{u}(t) = \mathbf{F_c}(t) \tag{5}$$

where $\mathbf{F_c}(t)$ can be expressed by:

$$\mathbf{F_c}(t) = -\frac{K_g^2 K_t K_m \left(\frac{d}{dt}x_c(t)\right)}{R_m r_{mp}^2} + \frac{K_g K_t V_m(t)}{R_m r_{mp}} \tag{6}$$

where K_g is the cart planetary gearbox gear ratio, K_t is the cart motor torque constant, K_m is the cart back electromotive force constant, V_m is the cart motor armature voltage, R_m is the cart motor armature resistance and r_{mp} is the cart motor pinion radius.

According to the system's state space representation defined by equations (1), (2), (3) and (4), the equations of motion are obtained as Santos (2007):

$$\begin{aligned}
&M_c M_{f2}\ddot{x}_c(t) - M_c K_{f2}x_{f2}(t) = (M_c + M_{f2})F_c(t)\\
&M_{f1}\ddot{x}_{f1}(t) + K_{f1}x_{f1}(t) - K_{f2}x_{f2}(t) = -M_{f1}\ddot{X}_g(t)\\
&(M_{f1}M_c M_{f2}r_{mp}^2)\ddot{x}_{f2}(t) - (M_c M_{f2}r_{mp}^2 K_{f1})x_{f1}(t) + ((M_{f1}M_c + M_{f2}M_c)r_{mp}^2 K_{f2})x_{f2}(t) =\\
&= -M_{f1}M_c r_{mp}^2 F_c(t)
\end{aligned} \tag{7}$$

where M_{f1} is the first floor mass, M_{f2} is the second floor mass and M_c is the cart mass.

Taking into account equation (6) in order to convert force ($\mathbf{F_c}(t)$) to voltage input ($V_m(t)$) and using the model parameter values provided in *User Manual* the state space matrices **A**, $\mathbf{B_1}$, $\mathbf{B_2}$, **C** and **D** result to be as follows:

$$\mathbf{A}=\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 278.43 & -18.69 & 0 & 0 \\ 0 & -431.03 & 431.03 & 0 & 0 & 0 \\ 0 & 431.03 & -766.49 & 5.98 & 0 & 0 \end{bmatrix} \qquad \mathbf{B_1}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} \qquad \mathbf{B_2}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 3.00 \\ 0 \\ -0.96 \end{bmatrix}$$

$$\mathbf{C}=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -431.03 & 431.03 & 0 & 0 & 0 \\ 0 & 431.03 & -766.49 & 5.98 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{D}=\begin{bmatrix} 0 \\ 0 \\ -0.96 \end{bmatrix}$$

4. H_∞ OUTPUT FEEDBACK CONTROL BY LMI APPROACH

A standard form of a general feedback system is given by Fig. 4, where **w** is the exogenous vector, **z** is the regulated output vector, **y** is the measurement output signal used to feedback the system, **K** is the output feedback controller and **u** is the signal of control.

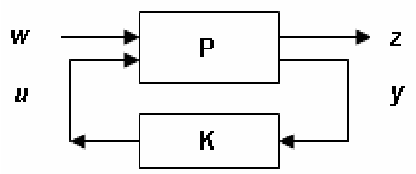


Figure 4: Standard Form Design in the Focuses Convex Optimization.

Mathematically, the general model is given by:

$$\begin{Bmatrix} \mathbf{z} \\ \mathbf{y} \end{Bmatrix} = \begin{bmatrix} \mathbf{P}_{zw} & \mathbf{P}_{zu} \\ \mathbf{P}_{yw} & \mathbf{P}_{yu} \end{bmatrix} \begin{Bmatrix} \mathbf{w} \\ \mathbf{u} \end{Bmatrix} = \mathbf{P} \begin{Bmatrix} \mathbf{w} \\ \mathbf{u} \end{Bmatrix} \tag{8}$$

Where $\mathbf{P}_{zw}$, $\mathbf{P}_{zu}$, $\mathbf{P}_{yw}$ and $\mathbf{P}_{yu}$ are the transfer functions matrices from the inputs to the outputs, as indicated by their subscripts.

The main goal is remain the vector **z** small face to disturbances **w**. The classical H_∞ theory can be used to solve this problem, (Burl, 1999) or (Moreira *et al*., 1999).

In the convex domain, the closed-loop system has the following transfer function (here, **I** is the identity matrix),

$$\mathbf{H}_{zw}=\mathbf{P}_{zw}+\mathbf{P}_{zu}\mathbf{K}(\mathbf{I}-\mathbf{P}_{uy}\mathbf{K})^{-1}\mathbf{P}_{yw} \tag{9}$$

Problem: Find an output controller **K** such that:

$$\left\|\mathbf{H}_{zw}\right\|_\infty < \lambda \tag{10}$$

where λ is the cost of the design. The infinity norms above are related with the maximum gain for any transfer function, (Burl, 1999).

In the present paper only the performance problem is considered. The augmented plant model that includes the performance is show in Fig. 5, (Moreira, 1998).

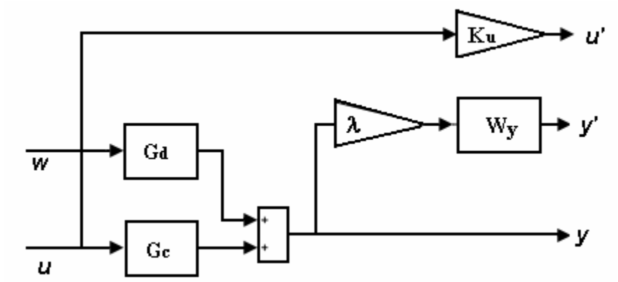


Figure 5: Augmented Plant for Performance Problem Used to Design the Controller.

where $\mathbf{G_c}$ is the transfer function of system and $\mathbf{G_d}$ is relative to disturbance input. There are two regulated outputs, **u'** and **y'**, the weighting control and measured signal respectively. The signal **u'** refers to the control energy constraint and the signal **y'** is related to the increase in damping.

To achieve the desired specification, weighting function $\mathbf{W_y}$ and a gain $\mathbf{K_u}$ are used to shape the regulated outputs. The weighting function $\mathbf{W_y}$ is typically a pass-low function in the frequency domain to enforce an increase in the closed-loop damping of the system. The gain $\mathbf{K_u}$ is selected to restrict the control signal level, and hence limit power consumption. This is detailed, for example, in Moreira, 1998.

The augmented plant of Eq. (8) has the following matrix functions,

$$\mathbf{P}_{zw} = \begin{bmatrix} \lambda \mathbf{W_y G_d} \\ 0 \end{bmatrix}, \qquad \mathbf{P}_{zu} = \begin{bmatrix} \lambda \mathbf{W_y G_c} \\ \mathbf{K_u} \end{bmatrix}, \qquad \mathbf{P}_{yw} = \mathbf{G_d}, \qquad \mathbf{P}_{yu} = \mathbf{G_c} \tag{11}$$

with **w** = **w** and **z** = [**u' y'**]^T. Substituting Eq. (11) into Eq. (9), the closed-loop matrix of the system becomes,

$$\mathbf{H}_{zw} = \begin{bmatrix} \lambda \mathbf{W_y G_d} & 0 \\ 0 & \mathbf{K_u} \end{bmatrix} \begin{bmatrix} \mathbf{S} \\ \mathbf{U} \end{bmatrix} \mathbf{G_d} \tag{12}$$

where, from the definition of the sensitivity and energy restriction functions, **S** and **U**, respectively (Moreira *et al.*, 1999), we have the definitions,

$$\begin{aligned} \mathbf{S} &= \mathbf{I} + \mathbf{G_c K}(\mathbf{I} - \mathbf{G_c K})^{-1} = (\mathbf{I} - \mathbf{G_c K})^{-1} \\ \mathbf{U} &= \mathbf{K}\,(\mathbf{I} - \mathbf{G_c K})^{-1} \end{aligned} \tag{13}$$

This augmented plant was solved using toolbox LMI from Matlab with aid from *sconnect* command, (Gahinet *et al.*, 1995). The H_∞ problem, Eq. (10), was solved using the command *hinflmi*, which applies convex optimization technique, (Gahinet *et al.*, 1995) and (Gahinet and Apkarian, 1994).

5. EXPERIMENTAL APPLICATION

Reponses in the Frequency Domain

In this section, it was used the following configurations to verify the proposed methodology. The first step is to choose the W_y filter as a second order function with cut-off frequency between the first and the second natural modes. The W_y filter parameters were obtained by adjustment, in order to have a good performance. The value of the gain used was specified as $\mathbf{K_u}$= 0.00599 to limit the energy restriction function. The cost of the design was λ= 0.02. The transfer function is given by:

$$\mathbf{W}_y = \frac{7.994}{s^2 + 16.96\,s + 799.4}$$

Figure 6 shows the singular value plot of this function.

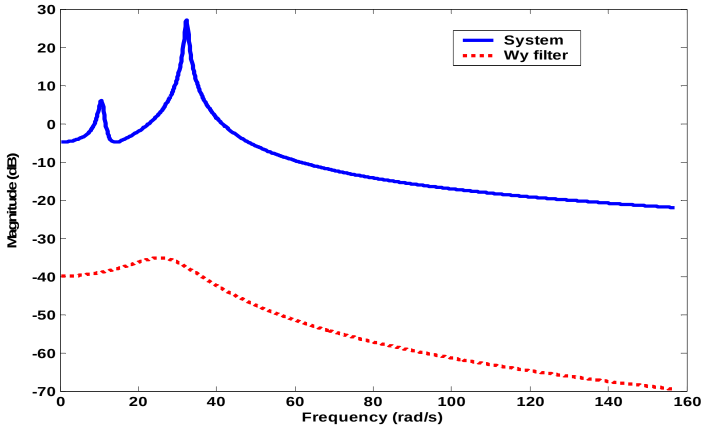


Figure 6: Weighting Function for the Sensibility Function.

The closed-loop response (FRF between disturbance input **w** and measured output signal **y**) of this system in frequency domain is shown in Fig. 7. There is suitable amplitude attenuation in the two modes. The performance characteristics of the resulting controller are shown in Fig. 8. The specification given by Eq. (10) is reached, because the sensitivity function is limited by the inverse of filter $\mathbf{W}_y$ (Moreira, 1998), as shown in Fig. 8a.

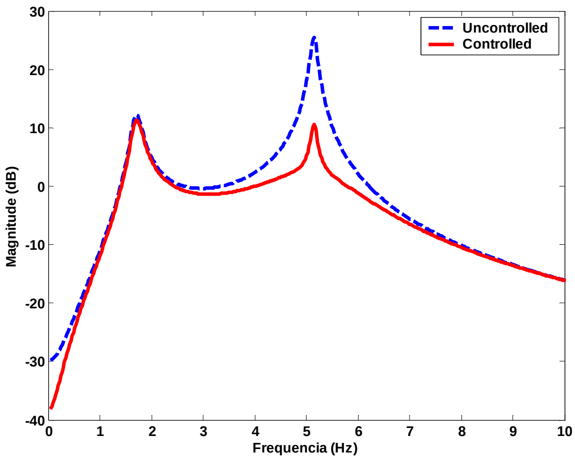
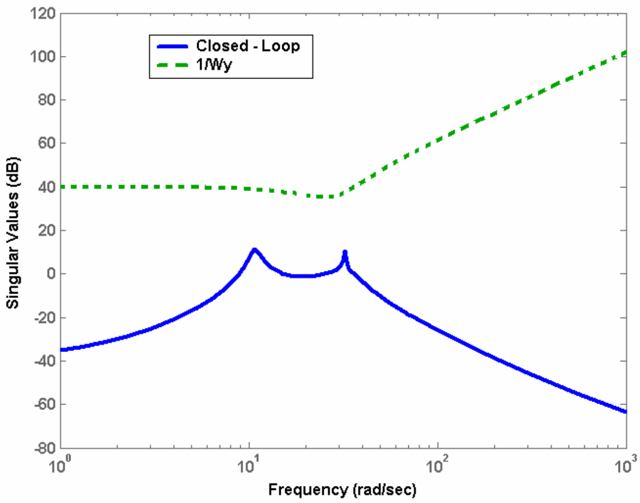
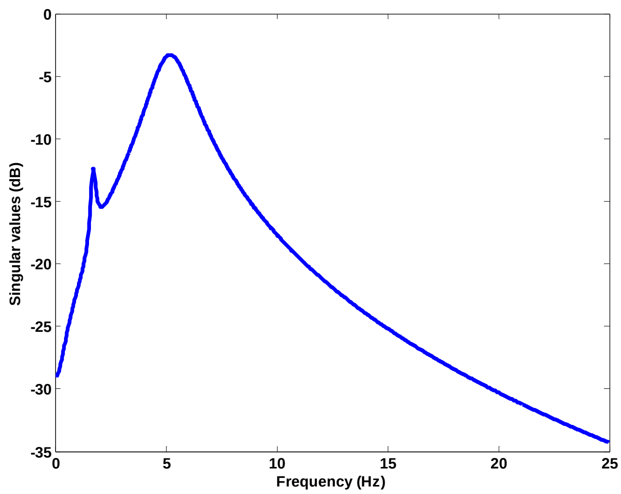


Figure 7: FRF of the Controlled and Uncontrolled Response.

When the system is controlled, the open loop value of the first mode attenuation reaches 0.9 dB. While, for the second mode an attenuation of 14.9 dB is achieved.



(a) **S** Sensitivity Function



(b) **U** Energy Restriction Function

Figure 8: Design Performance.

Another important point can be understood by Fig. 8b; the large peaks correspond to low frequency modes, it means that input control signal is distributed in the two first modes.

Reponses in the Time Domain

Figs. 9 and 10 shows the acceleration of the first floor and of the second floor of the bench–scale structure shown in Fig.1, respectively, when excited by ground acceleration shown in Fig. 12. Figure 11 shows the control force applied by the actuator.

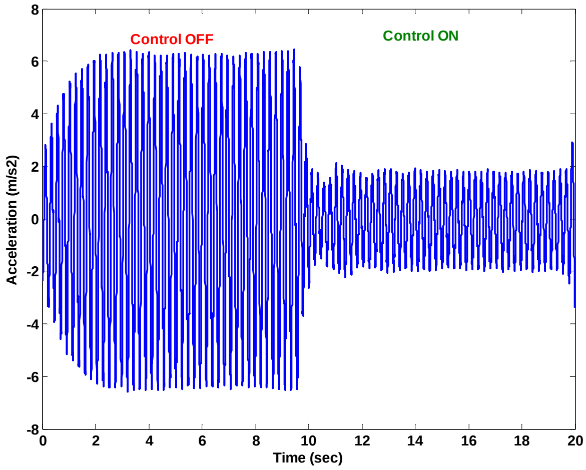


Figure 9: Acceleration of First Floor

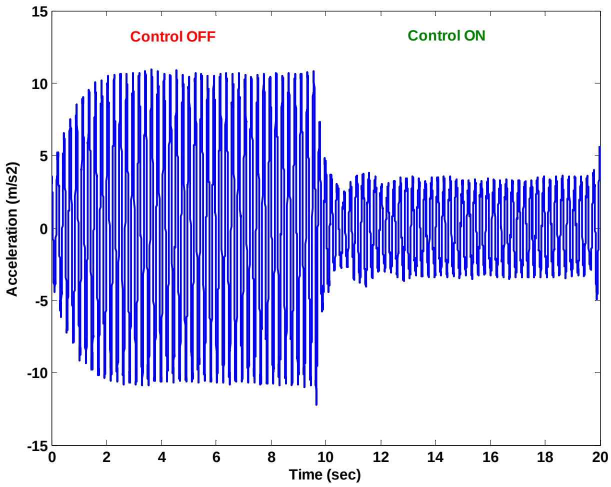


Figure 10: Acceleration of Second Floor

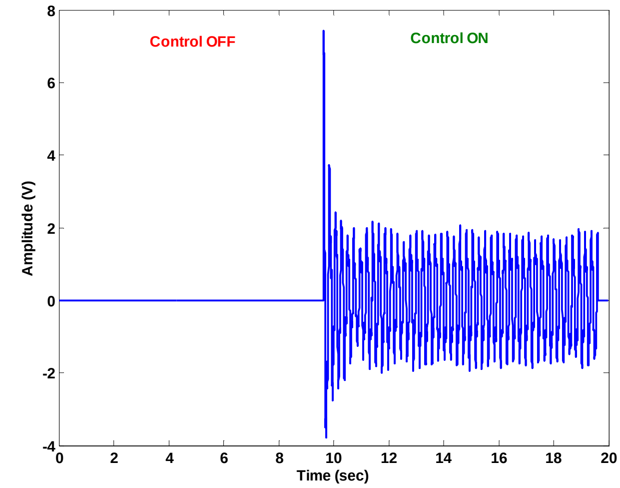


Figure 11: Control Force

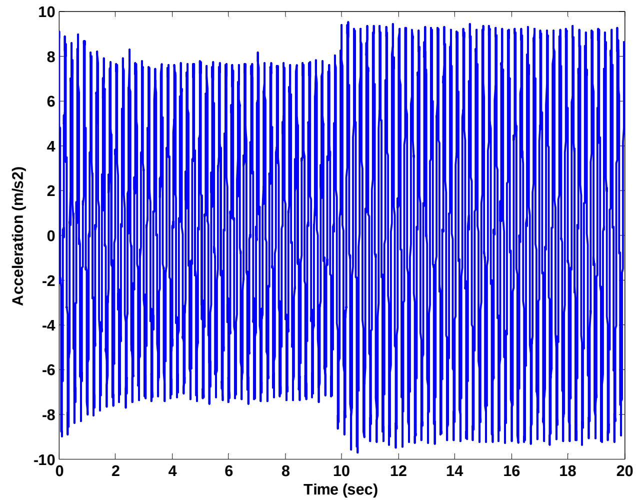


Figure 12: Ground Acceleration

6. CONCLUSION

The study of algorithms for active vibrations control in flexible structures became an area of enormous interest, mainly due to the countless demands of an optimal performance of mechanical systems as aircraft, aerospace and automotive structures. Smart structures, formed by a structure base, coupled with actuators and sensor are capable to guarantee the conditions demanded through the application of several types of controllers.

This paper is addressed to explore the abilities of LMI approach in the design of controllers. An H_∞ output feedback control strategy was used actively to control the two modes of a bench-scale structure that represent a flexible building like structure. Once formulated in terms of LMI a problem can be solved efficiently by convex optimization algorithms. The experimental application showed that the vibration attenuation reaches 0.9 dB and 14.9 dB for the first and second modes, respectively. This paper showed that H_∞controller, solved through LMI and considering only the performance problem, can reduce structural vibration appropriately.

7. ACKNOWLEDGEMENTS

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