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MODELLING OF FGM PIEZOELECTRIC TRANSDUCERS USING GRADED FINITE ELEMENT CONCEPT

Wilfredo Montealegre Rubio, wilfredo.rubio@poli.usp.br
Emilio Carlos Nelli Silva, ecnsilva@usp.br
Flavio Buiochi, fbuiochi@usp.br
Julio C. Adamowski, jcadamow@usp.br
Department of Mechatronics and Mechanical Systems Engineering, Escola Politécnica da Universidade de São Paulo,
Av. Prof. Mello Moraes, 2231 - Cidade Universitária, São Paulo – SP – 05508-900, Brazil

Abstract. *Piezoelectric materials generate displacements when an electric potential is applied and, electric potential when they are subjected to forces or pressure. Functionally Graded Materials (FGM) are composite advanced materials, which are made by changing gradually the properties with position inside material domain. The application of FGM concept to piezoelectric transducer design allows designing composite transducers without interface between materials (e.g. PZT and Aluminum), due to the continuous change of property values. Thus, large improvements can be achieved, as reduction of stress concentration, increasing bonding strength and fatigue-lifetime. Recent works about piezoelectric FGM show lack of computational methods to model these transducers and evaluate their performance considering property gradation function, futhermore, FE comercial softwares have no "tools" to simulate graded continuous materials for while. Thus, this work proposes the development of Finite Element (FE) algorithms to model FGM piezoelectric ceramics and to explore the FGM potential on piezoelectricity field. The continuous change of piezoelectric, dielectric, and elastic properties is achieved by using the graded finite element concept, where these material properties are interpolated inside the finite element using the FE shape functions. A software based on 4-node graded finite element (Q4) is implemented. Dynamic and static analyses are performed. In the examples, the material properties are graded along thickness direction to illustrate the influence of gradation on the output displacements, vibration modes, and resonance frequencies. These examples are compared with results of a homogeneous piezoceramic to show FGM advantages.*

Keywords: *Graded Finite Element, Functionally Graded Material, Piezoelectric ceramic, Dynamic Performance, Static Performance*

1. INTRODUCTION

Piezoelectric materials have the property to convert an electrical energy (electric field and electric potential) into a mechanical energy (stress and strain) and vice-versa. Examples of piezoelectric materials include quartz, ceramics (PZT) and polymers (PVDF). Its main applications are in sensors and electromechanical actuators, as resonators in electronic equipment and acoustic applications, as ultrasound transducers, naval hydrophones, and sonars. Ultrasound transducers are used in medical imaging (Akhnak *et al.*, 2000) and non-destructive tests. Other applications include pressure sensors; piezoelectric actuators for the structural vibration control; performance of nanopositioning and micromanipulation devices as: electronic microscopy instruments; laser interferometry; cell manipulation equipment; microelectromechanical systems "MEMS"; nanotechnology and precision mechanics equipment (Kögl and Silva, 2005). A brief review of piezoelectric applications is offered by Newnham and Ruschau (Newman and Ruschau, 1991).

In order to improve the conventional piezoceramic performance (single and homogeneous materials) are fabricated as piezocomposite materials. However, the interface between materials produces an uneven distribution of stresses which reduces the electric-field-induced displacement characteristics, reliability and lifetime. Other modern approach is to change the piezoelectric properties of ceramic disk through its thickness, specifically, to reduce one echo wave of two produced in each piezoceramic surface and to increase the induced piezoelectric stress gradient. Functionally Graded Material concept has arisen as a solution to reduce the ultrasonic wave generated at one piezoceramic surface (Yamada *et al*., 1998; Ichinose *et al*., 2004; Samadhiya and Mukherjee, 2006).

Functionally Graded Materials (FGM) are materials that possess continuously graded properties with gradual change in microstructure (Hirai, 1996; Suresh and Mortensen, 1998). The materials are made to take advantage of desirable features of its constituent phases. For instance, in a thermal protection system, FGMs take advantage of heat and corrosion resistance, typical of ceramics, and mechanical strength and toughness, typical of metals. A soft property variation supplies advantages such as stress concentration reduction (Suresh and Mortensen, 1998), since they do not present interface among inclusion and matrix materials, therefore, it reduces a common problem in composite materials, the crack arising or damages in these interfaces. Specifically, in Functionally Graded Piezoelectric (FGP) ceramics, the conventional and homogeneous piezoelectric material is replaced by a functionally graded piezoelectric one, see Fig. 1. Therefore, all or some properties vary along a specific Cartesian direction, usually along thickness. Several gradation functions can be used, see Fig. 1. Thus, if the piezoelectric properties change from low to high values, only one ultrasonic wave is radiating, and larger piezoelectric stress gradient together with short-time waveform are obtained (Yamada *et al*., 1998).

Some studies have been reported since 1960s about the advantages of wave generation by FGP transducers (Mitchell and Redwood, 1969); nevertheless, these studies have been highly increased since last 1990s (Wu *et al*., 1996). These works focused in two ways: FGP ceramic fabrication and modelling. FGP ultrasonic transducers can be constructed by forming a number of fine V-grooves on one surface of an active element (Yamada *et al*., 1998), or U-grooves (Guo *et al*., 2005); by applying an appropriate temperature gradient in the thickness direction of a polarized piezoceramic with low Curie temperature (Yamada *et al*., 2000); and by sintering a layer-structured ceramic green body without using any adhesive material (Ichinose *et al*., 2004). On the other hand, FGP ultrasonic transducer can be modelled: (i) by using one-dimensional analytical techniques, such as equivalent network analysis of piezoceramic disk, exploring linear and exponential gradation functions for thickness vibration modes (Yamada *et al*., 2001; Yamada *et al*., 1999); (ii) by using two-dimensional multilayer numerical techniques (Rubio, *et al*., 2007).

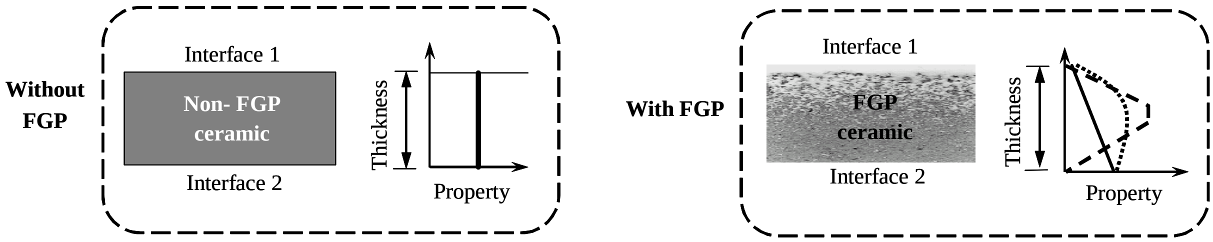


Figure 1. Sketch of a traditional (non-FGP) piezoelectric ceramic and of a FGP piezoceramic disk

Although all works show that the effectiveness of piezoelectric property gradation generates better performances, the modelling is reduced to one-dimensional or multilayer approaches. Former approach neglect lateral vibrations of ceramic disk, and in last one, the gradation functions are not continuous; on the contrary, they are discrete functions. In view of this idea, this work contributes to developing a Graded Finite Element (GFE) implementation for FGP ceramic disk modelling. This computational implementation allows simulating two-dimensional FGP disks with continuous material gradation. The code is implemented by using the MATLAB™ software. It will be considered that the properties varies along the thickness direction, and the FGP results are compared with non-FGP ones.

The paper is built up as follow: first, it is described the GFE formulation of piezoelectric ceramic disks. Then some results and testing of code are presented, and finally, some conclusions are given.

2. GRADED FINITE ELEMENT FOR PIEZOELECTRICITY

The constitutive relationships describing the electrical and mechanical interactions for piezoelectric material are given as (Naillon *et al*., 1983):

$$\begin{aligned} \boldsymbol{T} &= \boldsymbol{C}^{\boldsymbol{E}} \cdot \boldsymbol{S} - \boldsymbol{e}^{\boldsymbol{T}} \cdot \boldsymbol{E} \\ \boldsymbol{D} &= \boldsymbol{e} \cdot \boldsymbol{S} + \boldsymbol{\varepsilon}^{\boldsymbol{S}} \cdot \boldsymbol{E} \end{aligned} \tag{1}$$

where, $\boldsymbol{T}$ is the stress tensor (second order); $\boldsymbol{S}$ is the strain tensor (second order); $\boldsymbol{D}$ is the electric displacement vector; $\boldsymbol{E}$ is the electric field vector; $\boldsymbol{C}^{\boldsymbol{E}}$ is a fourth order elastic tensor and its components are evaluated by constant electric field, $\boldsymbol{\varepsilon}^{\boldsymbol{S}}$ is a second order dielectric tensor under constant strain, and $\boldsymbol{e}$ is a third order piezoelectric tensor where T indicates transpose.

The piezoelectric model is completed by considering the mechanical balance expressed by the Newton's equation for continuous media; the electrical balance corresponds to the charge balance expressed by Gauss's theorem; and the strain and electric field expressions. The mechanical balance is given by:

$$\rho \frac{\partial^2 \boldsymbol{U}}{\partial t^2} = \nabla \cdot \boldsymbol{T} \tag{2}$$

where, ρ is the density of material, t is the time, $\boldsymbol{U}$ the displacement vector, and ∇ the divergence operator. The electrical balance is given by:

$$\nabla \cdot \boldsymbol{D} = 0\,, \tag{3}$$

and respectively the strain and electric field expression by:

$$\boldsymbol{S} = \hat{\nabla} \cdot \boldsymbol{U} \quad \text{and} \quad \boldsymbol{E} = -\nabla \varphi \tag{4}$$

where, φ is the electric potential and $\hat{\nabla}$ is the symmetric gradient operator expressed by (Naillon *et al*., 1983):

$$S_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) \tag{5}$$

The finite element piezoelectric equilibrium equations can be written based on variational principle by using the constitutive piezoelectric equations, Eq. (1) up to Eq. (5). These equations are written in terms of displacement (***U***) and electric potential (***Φ***) vectors at nodal points. Also, the FE equations are written by using the nodal mechanical force (***F***) and electric charge (***Q***) vectors as (without structural damping) (Naillon *et al*., 1983):

$$\begin{bmatrix} \boldsymbol{M}_{uu} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{U}} \\ \ddot{\boldsymbol{\Phi}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{K}_{uu} & \boldsymbol{K}_{u\Phi} \\ \boldsymbol{K}_{u\Phi} & -\boldsymbol{K}_{\Phi\Phi} \end{bmatrix} \begin{bmatrix} \boldsymbol{U} \\ \boldsymbol{\Phi} \end{bmatrix} = \begin{bmatrix} \boldsymbol{F} \\ \boldsymbol{Q} \end{bmatrix} \tag{6}$$

where, $\boldsymbol{M}_{uu}$, $\boldsymbol{K}_{uu}$, $\boldsymbol{K}_{u\Phi}$, and $\boldsymbol{K}_{\Phi\Phi}$ are the mass, elastic, piezoelectric and dielectric matrices. However, in the case of FGP ceramic disks the properties change continuously inside the piezoceramic domain, which means that the matrices of Eq. (6) must be described by some continuous function of Cartesian position (x, y) into the ceramic disk. Thus, the matrices of Eq. (6) are expressed as:

$$\boldsymbol{M}_{\boldsymbol{uu}} = \iiint \boldsymbol{N}_{\boldsymbol{u}}^{\boldsymbol{T}} \rho(x,y) \boldsymbol{N}_{\boldsymbol{u}} dV \tag{7}$$

$$\boldsymbol{K}_{\boldsymbol{uu}} = \iiint \boldsymbol{B}_{\boldsymbol{u}}^{\boldsymbol{T}} \boldsymbol{C}^{\boldsymbol{E}}(x,y) \boldsymbol{B}_{\boldsymbol{u}} dV \tag{8}$$

$$\boldsymbol{K}_{\boldsymbol{u}\Phi} = \iiint \boldsymbol{B}_{\boldsymbol{u}}^{\boldsymbol{T}} \boldsymbol{e}^{\boldsymbol{T}}(x,y) \boldsymbol{B}_{\Phi} dV \tag{9}$$

$$\boldsymbol{K}_{\Phi\Phi} = \iiint \boldsymbol{B}_{\Phi}^{\boldsymbol{T}} \boldsymbol{\varepsilon}^{\boldsymbol{S}}(x,y) \boldsymbol{B}_{\Phi} dV \tag{10}$$

where, $\boldsymbol{N}_{\boldsymbol{u}}$ are the shape functions for the displacements; and $\boldsymbol{B}_{\boldsymbol{u}}$ and $\boldsymbol{B}_{\Phi}$ are the strain-displacement and voltage-gradient matrices, respectively. According to the theory of conventional finite element, the matrices and vectors of piezoelectric constitutive equations result from assembling the vectors and matrices of the single elements.

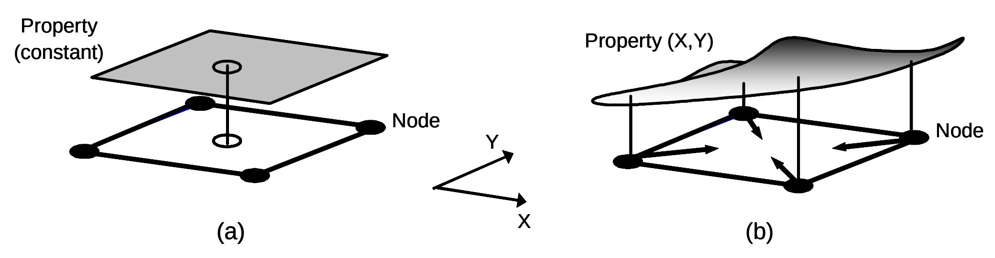


Figure 2. Properties at the element level. (a) Homogeneous Finite Element (HFE); (b) Graded Finite Element (GFE)

To treat the homogeneous material simulations, it is used the traditional Homogeneous Finite Element (HFE) with constant material properties at the element level, which are evaluated at the centroid of each element; see Fig. 2(a). On contrary, for FGP ceramics, the Graded Finite Element (GFE) is implemented, which incorporate the material property gradient at the size scale of the element (Fig. 2(b)). Kim and Paulino (2002) and Santare and Lambros (2000) developed the graded element concept with slightly different formulations. Both studies demonstrated that graded elements result in smooth and accurate change of properties for static problems. In this work, the scheme developed by Kim and Paulino (2002) is extended to piezoelectric materials considering dynamic and static analysis. In FGP ceramics, the GFE concept employs the same shape functions to interpolate the unknown displacements and electric potential, the geometry, and the material parameters. The interpolations for spatial coordinates (x, y), nodal displacements (u), nodal electric potential (φ) and material properties (ρ, $\boldsymbol{C}^{\boldsymbol{E}}$, $\boldsymbol{e}$, $\boldsymbol{\varepsilon}^{\boldsymbol{S}}$) are given by:

- Spatial coordinates:

$$x = \sum_{i=1}^{m} N_i x_i \quad , \quad y = \sum_{i=1}^{m} N_i y_i \tag{11}$$

- Displacements and electric potentials:

$$u = \sum_{i=1}^{m} N_i u_i \quad , \quad \varphi = \sum_{i=1}^{m} N_i \varphi_i \tag{12}$$

- Elastic, Piezoelectric and Dielectric properties:

$$C_{ijkl}^{E} = \sum_{n=1}^{m} N_n \left(C_{ijkl}^{E}\right)_{\boldsymbol{n}} \quad , \quad e_{ikl} = \sum_{n=1}^{m} N_n \left(e_{ikl}\right)_{\boldsymbol{n}} \quad , \quad \varepsilon_{\boldsymbol{ik}}^{\boldsymbol{S}} = \sum_{n=1}^{m} N_n \left(\varepsilon_{\boldsymbol{ik}}^{\boldsymbol{S}}\right)_{\boldsymbol{n}} \quad \text{for} \quad i, j, k, l = 1, 2, 3 \tag{13}$$

respectively, where m is the number of nodes per finite element. Also, when GFE is implemented, the material properties must remain inside the matrices integrals (see Eq. (7) up to Eq. (10)) and must be integrated; in Homogeneous Finite Elements (HFE), these properties are not integrated.

3. IMPLEMENTATION

The GFE for piezoelectricity is implemented by using MATLAB™ code. Two-dimensional four-node quadrilateral finite elements (Q4) for FGP ceramic disks are used in this work, each one with three degrees of freedom: two mechanics (horizontal and vertical displacements), and one electric (electrical potential). Thus, a fully isoparametric formulation is developed in the sense that the same shape functions are applied to interpolate the unknown displacements and electric potentials, the geometry, and the material properties. Therefore, the actual variation of the material properties may be approximated by the element interpolation functions (e.g., a certain degree of polynomial functions).

Three analyses are implemented: Static, Modal, and Harmonic. In static analysis the software solves the following equation system:

$$\begin{bmatrix} \boldsymbol{K}_{uu} & \boldsymbol{K}_{u\Phi} \\ \boldsymbol{K}_{u\Phi} & -\boldsymbol{K}_{\Phi\Phi} \end{bmatrix} \begin{bmatrix} \boldsymbol{U} \\ \boldsymbol{\Phi} \end{bmatrix} = \begin{bmatrix} \boldsymbol{F} \\ \boldsymbol{Q} \end{bmatrix} \tag{14}$$

In modal and harmonic analyses no damping is considered in the dynamic problem. In modal analysis the eigenvalues and eigenvectors are found solving the second-order systems:

$$-\omega^2 \begin{bmatrix} \boldsymbol{M}_{uu} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{U}} \\ \ddot{\boldsymbol{\Phi}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{K}_{uu} & \boldsymbol{K}_{u\Phi} \\ \boldsymbol{K}_{u\Phi} & -\boldsymbol{K}_{\Phi\Phi} \end{bmatrix} \begin{bmatrix} \boldsymbol{U} \\ \boldsymbol{\Phi} \end{bmatrix} = 0 \tag{15}$$

where, ω is the natural frequency. On the other hand, harmonic response analysis seeks the system response when prescribed loads vary sinusoidally with time. Because MATLAB is able to manipulate complex numbers, the harmonic response calculation is based on direct method, in this case the harmonic response equation can be written in the form:

$$-\Omega^2 \begin{bmatrix} \boldsymbol{M}_{uu} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{\ddot{\boldsymbol{U}}} \\ \hat{\ddot{\boldsymbol{\Phi}}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{K}_{uu} & \boldsymbol{K}_{u\Phi} \\ \boldsymbol{K}_{u\Phi} & -\boldsymbol{K}_{\Phi\Phi} \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{U}} \\ \hat{\boldsymbol{\Phi}} \end{bmatrix} = \begin{bmatrix} \hat{\boldsymbol{F}} \\ \hat{\boldsymbol{Q}} \end{bmatrix} \tag{16}$$

where, $\hat{\boldsymbol{U}}$ and $\hat{\boldsymbol{\Phi}}$ are complex magnitudes, respectively, of displacements and electric potentials. $\hat{\boldsymbol{F}}$ and $\hat{\boldsymbol{Q}}$ are complex magnitudes of mechanical and electrical inputs, respectively. All these complex magnitudes depend on frequency Ω.

4. NUMERICAL RESULTS

In this section some results based on GFE implemented with MATLAB are shown. Three analyses are performed: Static, Modal and Harmonic. When homogeneous piezoceramic is simulated only one material is used, in this homogeneous case is used the PZT-5A properties. On contrary, when FGP ceramics are simulated a gradation function is used to represent the continuous change of material properties along thickness. The properties of PZT-5A are shown in Table 1.

Table 1. Material properties for PZT-5A ceramic.

Elastic Constants (*N/m²*)	**Piezoelectric Constants (C/*m²*)**	**Dielectric Constants**
c_{11} = 12.1 x 10^{10} c_{13} = 7.52 x 10^{10} c_{33} = 11.1 x 10^{10} c_{44} = 2.11 x 10^{10}	e_{13} = -5.4 e_{13} = 15.8 e_{15} = 12.3	ε_0 = 8.854188 x 10^{-12} *F/m* ε_{11} = 916 x ε_0 ε_{33} = 830 x ε_0

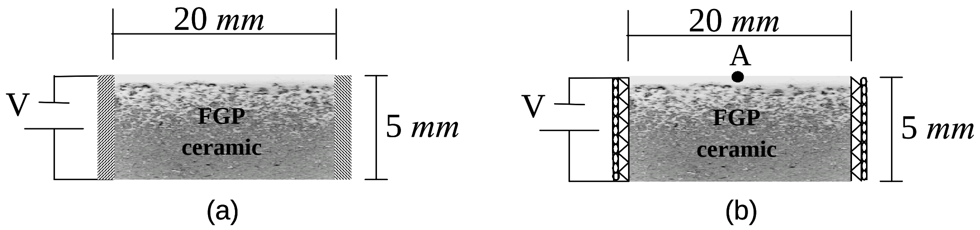


Figure 3. Model used for numerical results. (a) Model for static analysis; (b) Model for modal and harmonic analyses

In Table 1 ε_0 is the permittivity of free space. The density ρ of PZT – 5A material is equal to 7500 kg/m^3. For FGP ceramic disks the following exponential gradation functions for elastic, piezoelectric, and dielectric properties are considered (assuming material properties vary in y Cartesian direction, see Fig. 3) for a 2D problem:

$$\boldsymbol{C}^{\boldsymbol{E}}(y)=\begin{bmatrix} c_{11} & c_{13} & 0 \\ c_{13} & c_{33} & 0 \\ 0 & 0 & c_{44} \end{bmatrix} e^{\beta y} \; ; \quad \boldsymbol{e}(y)=\begin{bmatrix} 0 & e_{13} \\ 0 & e_{33} \\ e_{15} & 0 \end{bmatrix} e^{\gamma y} \; ; \quad \boldsymbol{\varepsilon}^{\boldsymbol{S}}(y)=\begin{bmatrix} \varepsilon_{11} & 0 \\ 0 & \varepsilon_{33} \end{bmatrix} e^{\alpha y} \tag{17}$$

where, β, γ, α are the material gradation parameter elastic, piezoelectric and dielectric properties, respectively. In this work, it is assumed β, γ, and α equal to 85; 322; and 106, respectively. Density is assumed constant along thickness. For all numerical results the simulated models are sketched in Fig. 3, which represents a two-dimensional piezoceramic disk geometry subjected to electric potential between top and bottom disk surfaces. For static analysis the piezoceramic is fixed on both lateral sides (see, Fig 3(a)), and for modal and harmonic analyses the piezoceramic is simulated considering boundary conditions for free response constraining the lateral vibrations, thus the model have only horizontal mechanical constraints in both lateral sides of piezoceramic, see Fig. 3(b).

4.1 Static Results

Initially, a static analysis was developed. An input electric potential equal to 100 V is applied. The σ_{xx}, σ_{zz}, and σ_{xz} stresses, E_z electric field, u_y displacements, and ϕ electric potential values, for FGP and non-FGP implementations, are calculated. Here, only the vertical displacements, stresses σ_{xx}, σ_{zz}, and σ_{xz}, and electric potential are shown in Fig. 4, 5, 6, 7, and 8, respectively. Also, the ANSYS response when is used a homogeneous material is shown. The piezoceramic simulated is sketched in Fig. 3(a), with boundary conditions and dimensions. For all examples the domain is discretized with 30 x 30 four-node bilinear (Q4) isoparametric finite elements, considering both GFE and HFE, and a 3 x 3 Gauss quadrature is employed.

From Fig. 4 up to 8 is observed that FGP models produce similar displacement, stress, and electric potential distribution that the non-FGP models, except for stress σ_{xx}. At the same time, these FGP ceramics have smaller stress magnitude; these magnitudes decrease 37.8 %, 29.4 % and 50 % for the maximum stress σ_{xx}, σ_{zz}, and σ_{xz}, respectively. It is an advantage, because high stress levels accelerate the aging process of piezoelectric materials and they reduce the lifetime of piezoceramic. However, in non-FGP ceramic disk higher displacements are present when an input voltage is applied (although differences with FGP material are small). Also, it is observed in Fig. 4 up to 8 that homogenous responses (non-FGP ceramic simulated with Matlab) are closed to ANSYS results; however, the Matlab and Ansys stress results show some differences, this can be caused due to different stress calculation procedure. Ansys use the four-nodal information to interpolate the stress inside of finite element. The code implemented in this work calculates the stress in middle of finite element.

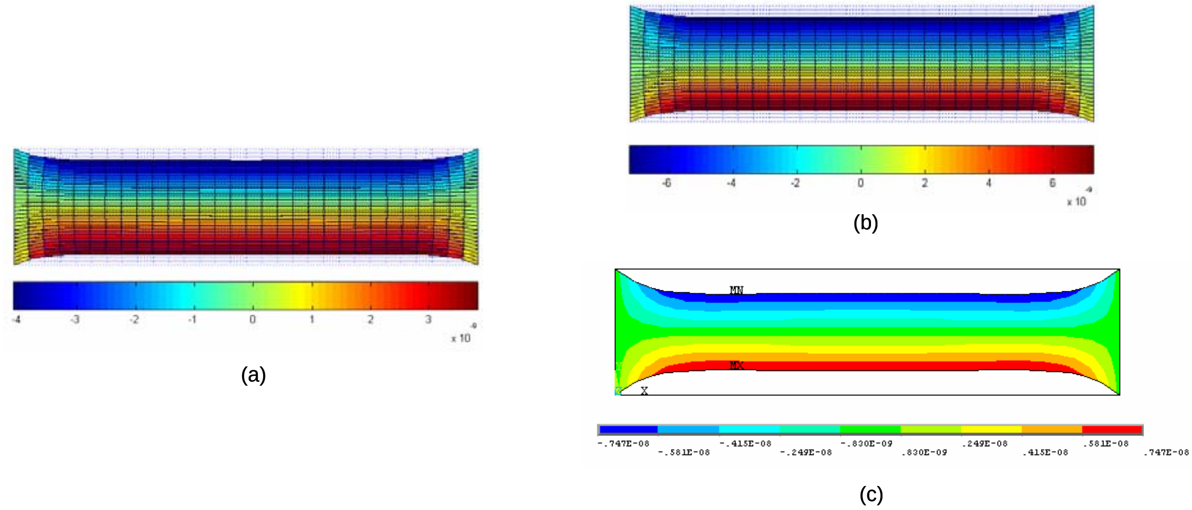


Figure 4. Vertical Displacement (m). (a) FGP model with Matlab; (b) Non-FGP model with Matlab; (c) Non-FGP model with Ansys

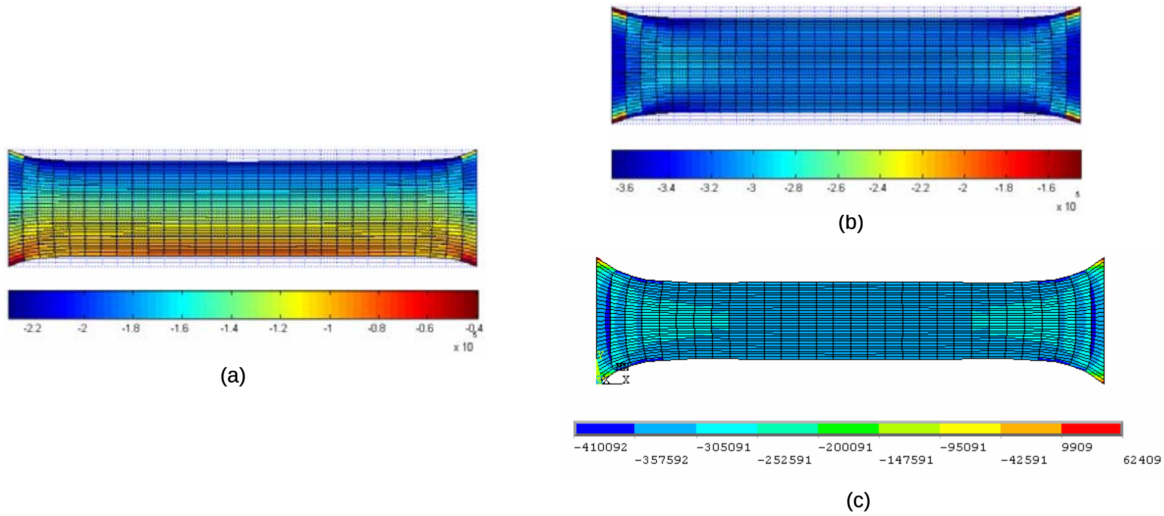


Figure 5. Stress σ_{xx} (Pa). (a) FGP model with Matlab; (b) Non-FGP model with Matlab; (c) Non-FGP model with Ansys

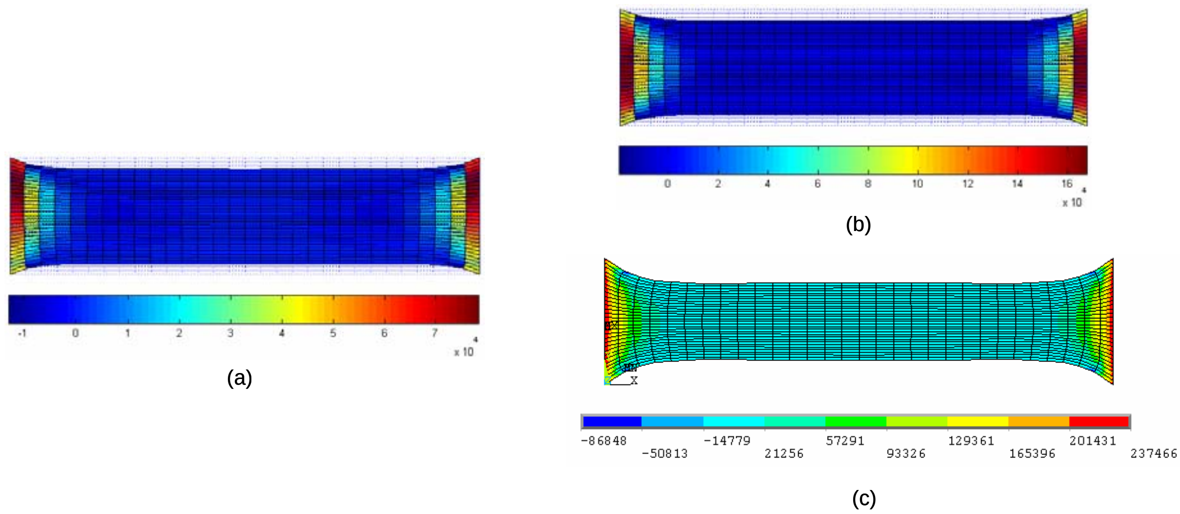


Figure 6. Stress σ_{zz} (Pa). (a) FGP model with Matlab; (b) Non-FGP model with Matlab; (c) Non-FGP model with Ansys

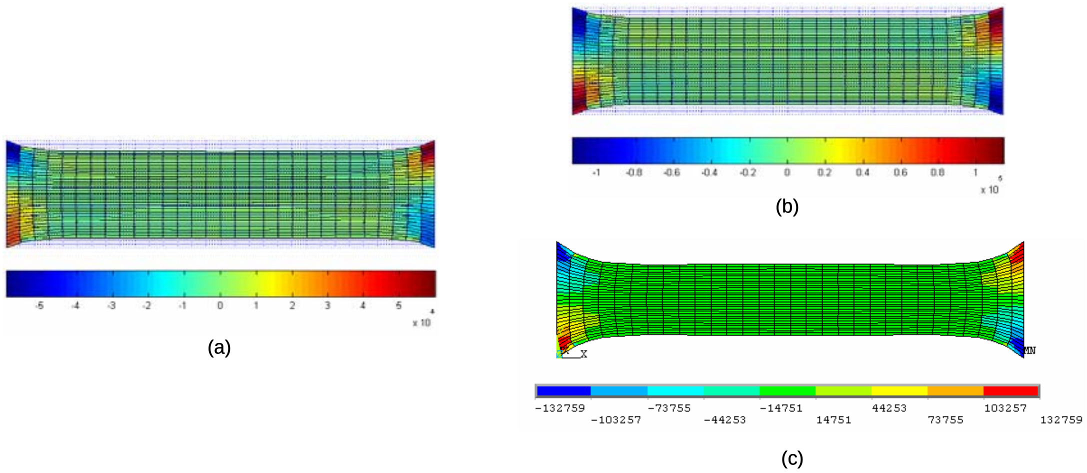


Figure 7. Stress σ_{xz} (Pa). (a) FGP model with Matlab; (b) Non-FGP model with Matlab; (c) Non-FGP model with Ansys

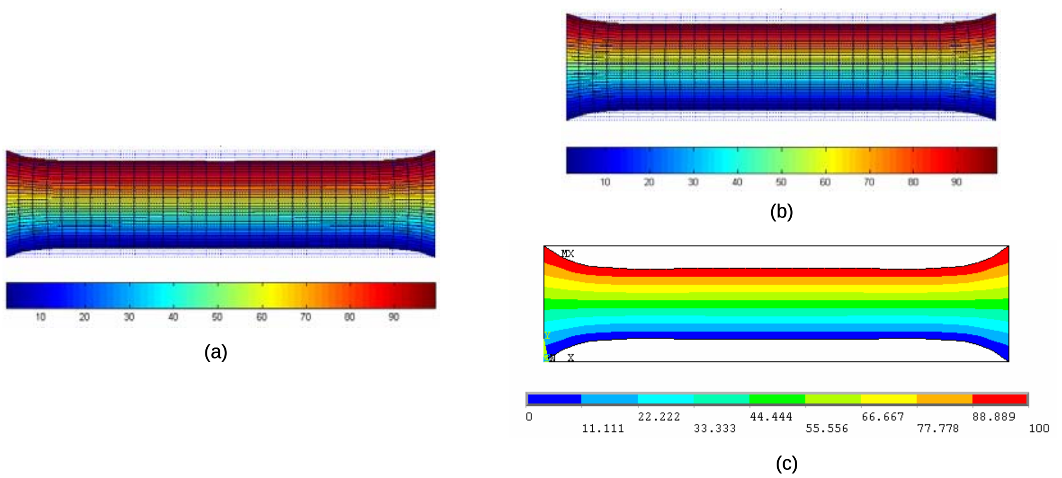


Figure 8. Electric potential (V). (a) FGP model with Matlab; (b) Non-FGP model with Matlab; (c) Non-FGP model with Ansys

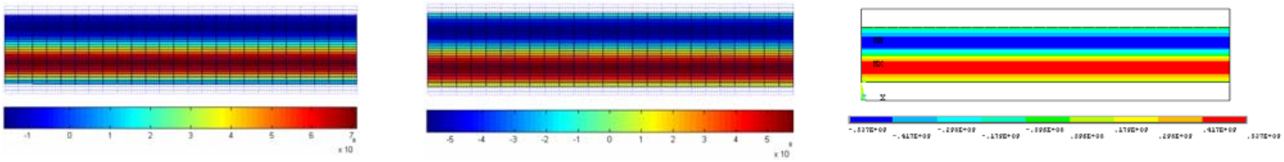
4.2 Modal Results

Modal analysis of this piezoceramic involves two cases which spans the extremes of the piezoelectric coupling effect due to voltage and displacement degrees of freedom. The first case is commonly called the "resonance" condition. A constant voltage equal to zero is applied to the electrical contacts (electrodes) of ceramic disk. This is a "short-circuit" condition, where all voltage potentials are connected to ground. The second case, called "anti-resonance", applies a zero voltage only to one electrode. In this modal analysis, only vibration modes along thickness are calculated, according to model of Fig. 3(b).

Table 2 shows a comparison of the natural frequencies when homogeneous (ANSYS and MATLAB responses) and FGP ceramics are used. The vibration modes are calculated for resonance and anti-resonance conditions. Thus, when resonance and anti-resonance frequencies are equal, it is a mechanical mode and, when resonance and anti-resonance frequencies are different, it is a piezoelectric mode, which is our mode of interest. On contrary, a mechanical mode is identified when resonance and anti-resonance frequencies are equal. The eigenvector plots for first and second piezoelectric modes are shown in Fig. 9 to Fig. 10, respectively.

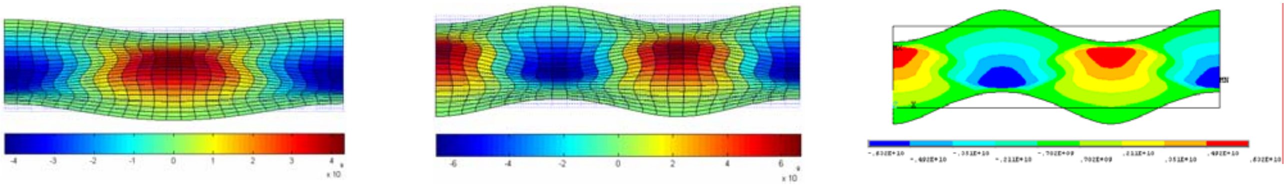
Table 2. Natural frequencies for different material distribution problems

Mode	**FREQUENCIES (MHz)**		
	FGP model with Matlab	**Non-FGP model with Matlab**	**Non-FGP model with Ansys**
First Piezoelectric mode	0.348844	0.393624	0.392914
Second Piezoelectric mode	0.735877	0.928034	0.922429



(a) (b) (c)

Figure 9. First piezoelectric mode. (a) FGP model with Matlab; (b) Non-FGP model with Matlab; (c) Non-FGP model with Ansys

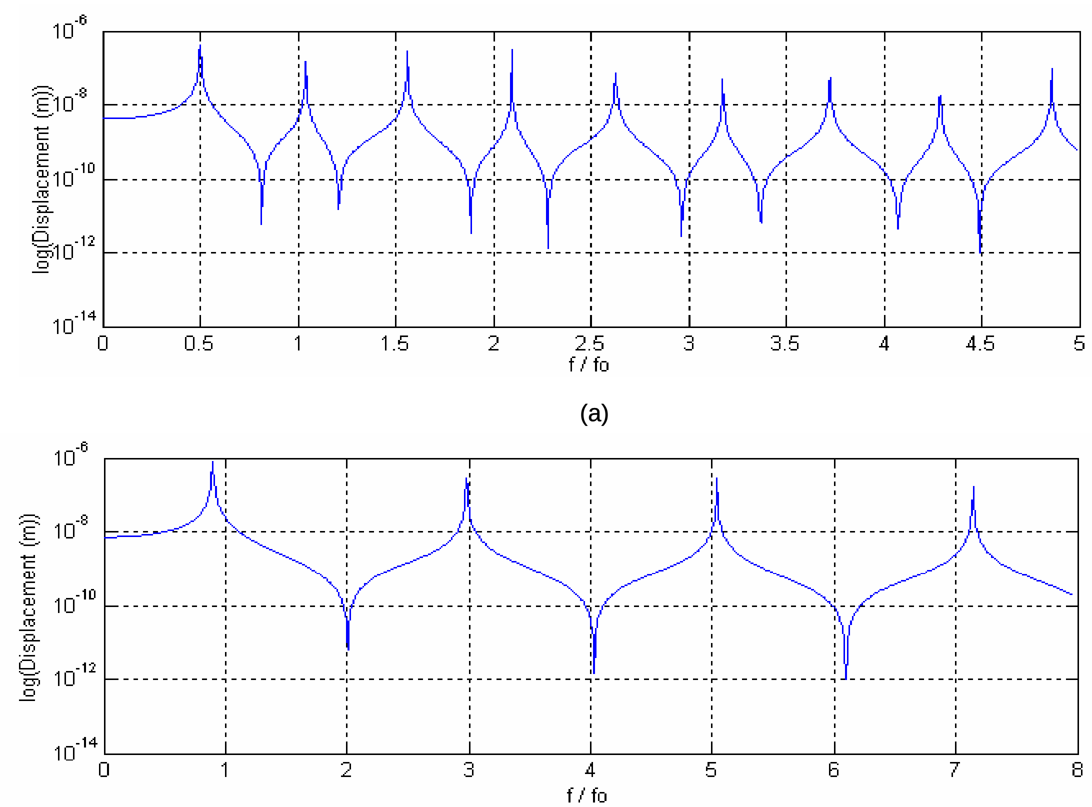


(a) (b) (c)

Figure 10. Second piezoelectric mode. (a) FGP model with Matlab; (b) Non-FGP model with Matlab; (c) Non-FGP model with Ansys

4.3 Harmonic Results

The same 2D model used in modal analysis is simulated by using a harmonic analysis. The input for this simulation was voltage amplitude imposed across the piezoelectric ceramic disk. The input varied sinusoidally between +/-100V.



(a)

(b)

Figure 11. Displacement frequency response. (a) FGP model with Matlab; (b) Non-FGP model with Matlab

The harmonic analysis was performed over a frequency range of 0-3.5 MHz in equal steps of 3.5 kHz. At each frequency, the implemented software computes the steady-state response of the system subjected to a sinusoidally varying input on the FGP or non-FGP ceramic disk. No damping ratio was assumed over all frequencies of the harmonic analysis sweep. The result of particular interest in this solution was the Y (vertical) displacement of the transducer over the frequency sweep, measured at point A in Fig. 3(b) (the top-middle of ceramic disk).

Figure 11 shows the normalized-frequency Y displacement (focused on the thickness vibration modes) for the uniform piezoelectric ceramic (non-FGP) disk and for the FGP model. The resonant responses for the odd order vibration mode appear in the non-FGM. It is noted, however, that both even and odd order vibration modes appear in FGP ceramics, also see Table 2. This result is in accordance with Yamada's results (Yamada *et al*., 2001). They develop an analytical one-dimensional model for FGP ceramics, and found even and odd vibration modes in the admittance response of FGP plates.

5. SUMMARY AND CONCLUSIONS

The modelling of FGP ultrasonic transducers was considerably successful, based on Graded Finite Element modelling. Also, this work provides an investigation of graded piezoceramic disk responses considering static, modal and harmonic analyses. These responses were compared to homogeneous one (non-FGP properties). Generalized isoparametric formulation is employed in FE method to investigate the response of gradation material in piezoceramic disks. This formulation adopts the same interpolation methods of the coordinates and displacements to treat continuous change of material at the element level. This approach results in smooth solution transition across the element boundaries.

The following conclusions can be drawn from these studies:

- The GFE modelling is an accurate technique, consisting of useful tools for designing FGP transducers, thus this work fulfill the lack of computational methods, in commercial softwares, to evaluate the performance of piezoelectric disk when material inhomogeneity at the element level is considered. This approach represents a more accurate technique instead of multi-layer approach to model the gradation behavior.
- With graded piezoceramic disks lower stress levels and more resonance frequencies can be obtained; in other words, even and odd vibration modes are achieved; with non-graded piezoceramics only odd modes are obtained. Thus, large improvements can be achieved in their performance characteristics by using FGP concept.
- In this work, only an exponential gradation function is used to simulate the piezoceramic properties; nevertheless, other gradation functions can be used; however, which gradation function produces better performance? This suggests the use of optimization techniques to design graded piezoceramic disks. Based on these ideas, in a future work, it is proposed the development of optimization algorithms to find this optimized gradation function.

6. ACKNOWLEDGEMENTS

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