ABCM Symposium Series in Mechatronics - Vol. 3 - pp.378-385

TRAJECTORIES OF MOVING SLIDERS FOR THE OPTIMAL CONTROL OF A FLEXIBLE MANIPULATOR

Eder Terceiro
Faculdade de Tecnologia de São Paulo & Centro Universitário SENAC
e-mail:ederterceiro@yahoo.com.br
Agenor de Toledo Fleury
Mechanical Engineering Department, Centro Universitário da FEI
e-mail: agfleury@fei.edu.br
Abstract. *In some recent papers (Terceiro and Fleury, 2003;Fleury and Oliveira, 2004), we have shown that the translational motion of one or two masses(sliders) should be used to minimize the vibrations induced by rotational motion on a flexible manipulator. Optimal control methods have been used to generate slider trajectories in the same time period that the manipulator takes to perform a rotational maneuvers between two fixed configurations. Solutions have been achieved for a very light and flexible manipulator even performing quite quick maneuvers.*
In this paper, the composed motion is presented in more details to emphasize the influence of the problem parameters. Initial position of the sliders, for example, may lead to different types of trajectory. A new set of feasible trajectories are also presented using a new Index of Performance applied to the same dynamic problem

Keywords: *Optimal Control, vibration control,flexible robotics, lightweight structures, composed motion.*

1. Introduction

Practical operation of many mechanical machines makes use of several internal motions to accomplish complex tasks. Mechanical design standard procedures assume independent element motions to avoid undesirable induced vibrations and couplings. In most cases, the resulting design leads to heavy structures and slow motions for the machines. From the other side, the dynamic characteristics of modern machines demand increasing larger velocities, accelerations and reduced weights and inertias. For many applications, structural flexibilities must be considered since the early design stages in order to assure good vibration attenuation. Based on the examples of rotating cranes and rotative/prismatic joint robots, this work explores simultaneous rotating/translational motions to minimize vibrations on a light one-link manipulator that performs large rotational maneuvers.

The basic question we intended to investigate is how the motion of independent parts may contribute to reduce the vibration levels of the whole system. Although restricted to simple systems, the results achieved are encouraging to extend the research to other more complex applications.

Here, the objectives are the achievement of suitable system models and the synthesis of optimal controllers using the torque applied to the hub where the flexible arm is fixed and the forces applied to the sliding masses as control variables. The case of a single sliding mass has been presented in the works of Oliveira (2000), Fleury et al. (2002) and Fleury and Oliveira (2004). The investigation has been extended to include any number of sliders and structural modes (Terceiro, 2002). In all cases, the dynamical models of the structural system have been derived through the Extended HamiltonŠs Principle resulting in a set of coupled integro differential non linear equations where system parameters are time and space variant due to changes in the inertia terms. Using substructuring techniques, arm and sliders motions have been separated and systems responses have expanded in products of spatial and time functions. Cases where the sliders move according to pre specified trajectories while the flexible arm performs large angular manoeuvers have been simulated and analyzed (Oliveira, 2000; Terceiro, 2002). The control mode is still time variant, although linear, but if this was helpful to understand the very influence of the composed torque sliders position controls on the elastic vibrations, many questions about design parameters remain inconclusive and results were not considered satisfactory. A second approach, where slider trajectories are control variables, leads to Optimal Control Problems (OCP). The resulting models are non linear and time variant and optimal arm and slider trajectories are investigated through the use of RIOTSŠ95 (Schwartz et al, 1997), a computational package based on the Consistent Approximation Theory (Schwartz and Polak, 1996).

The optimal slider trajectories have shown that movement coupling between different structural elements may be used as an efficient vibration control method (Terceiro and Fleury, 2003). From the other side, choice of system parameters and initial conditions seemed to be critical for the optimal trajectories in terms of the demand of energy to get the objectives. This is the topic we have investigated in this work. In the sequence, a brief summary of system modeling is presented. After this part, old and new results on the optimization problem are discussed.

2. System Model

The conceptual design for the whole system is shown in Figure 1. Hub motion is driven by a DC motor and monitored by an encoder. Two sliders run on each face of the flexible arm, driven by independent motors inside the hub. The description of the complete movement is usually compared to a rigid beam that follows the movemente as showed in figure 2. The virtual structure is knowed "shadow beam".

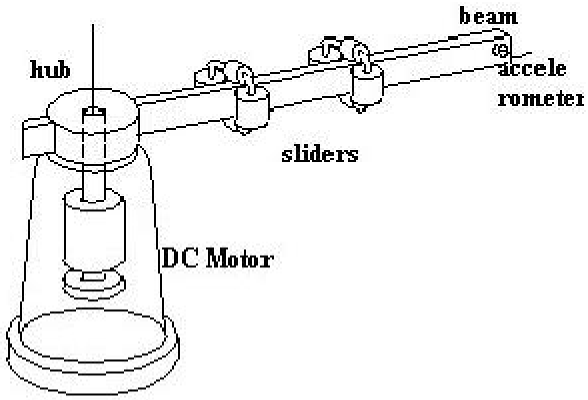

Figure 1. Conceptual design of the flexible mechanism

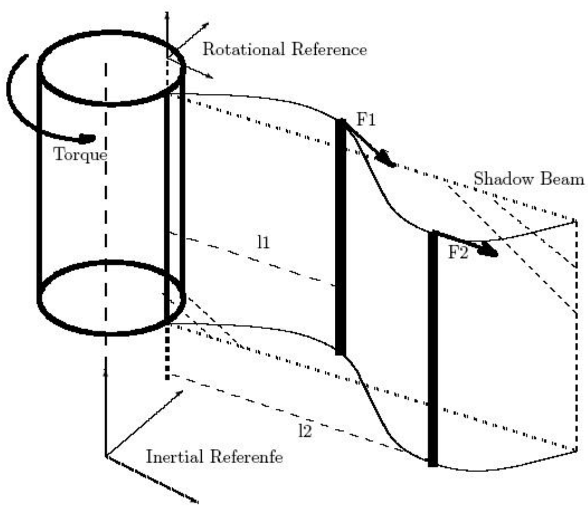

Figure 2. Deformed and Shadow Beam corresponding to the flexible arm

The motion of the arm is supposed to occur in the horizontal plane (Oxy) in such a way that gravitational forces can be disregarded. In order to apply the Extended Hamilton's Principle and get the corresponding model one has to consider the kinetic energies of the arm, of the two sliders and the hub, the elastic potential energy of the arm and virtual work of the non conservative forces on the system, namely the torque of the hub (τ) and the force on the each sliders (F1 and F2). The resulting model is a set of 6 partial integro-differential equations whose solution is very difficult. One uses substructuring techniques to get simpler models. Here, the substructures are the flexible arm and each of the sliders.

In the sequence, substructure motions are synchronized and normalized to get a set of matrix equations in the form (Terceiro, 2002):

$$\ddot{\eta}_{rpx1} = -[T]^{-1}_{pxp}\left([W]_{pxp}\eta_{rpx1} - 2M_i\dot{l}_i\dot{\theta}\left\{\int_0^L \phi_r dx\right\}_{px1} + \tau\dot{\phi}_r|_{x=0_{px1}}\right) \quad (1)$$

With $[S]_{pxp} = [T]^{-1}_{pxp}$ was given:

$$S_{rs} = \frac{M_i + M_i^2 \displaystyle\sum_{k=1,k\neq r}^{p} \phi_k\big|_{x=l_i} \int_0^L \phi_k dx}{1 + \displaystyle\sum_{k=1}^{p} M_i\phi_k\big|_{x=l_i} \int_0^L \phi_k dx} \quad \text{for} \quad r = s$$

$$= \frac{-M_i^2 \phi_r\big|_{x=l_i} \int_0^L \phi_s dx}{1 + \sum_{k=1}^{p} M_i \phi_k\big|_{x=l_i} \int_0^L \phi_k dx} \qquad \text{for} \quad r \neq s \tag{2}$$

In Equations (1) and (2), η is the system response considering a number p of vibration modes ϕ_k, $k = 1, \cdots, p$ included in the model. M_i, i=1,2, are the slider masses and l_i correspond to the slider positions. And θ is the angular position of the beam and τ is the applied torque. Using Newton's Law on the rotational motion , one can get:

$$\left(J_B + J_C + M_L l_I^2 \ddot{\theta} \right) = \tau + F_{Ej}(l_i) l_i \tag{3}$$

Substituting the values above, the angular motion is given by:

$$\ddot{\theta} = \frac{\sum_{r=1}^{p} \sum_{s=1}^{p} S_{rs} \left(\omega_s^2 \eta_s^2 \right) \phi_r\big|_{l_i}}{J_b + J_C + M_i l_i^2} + \frac{2 M_i \dot{l}_i \dot{\theta} \left(\sum_{r=1}^{p} \sum_{s=1}^{p} S_{rs} \left(\int_0^L \phi_s \right) \phi_r\big|_{l_i} - 1 \right)}{J_b + J_C + M_i l_i^2} + \frac{\left(1 - \sum_{r=1}^{p} \sum_{s=1}^{p} S_{rs} \left(\dot{\phi}\big|_{x=0} \right) \phi_r\big|_{l_i} \right) \tau}{J_b + J_C + M_i l_i^2} \tag{4}$$

where J_B and J_C are the mass moments of inertia of the arm and of the hub, respectively. Equations (1) and (4) describe the motions of the arn and of the two sliders. For more details, see, for example, Terceiro (2002).

3. Optimal Control Problem

Considering the hub/flexible arm structure, the control variables are the external forces, due to reaction to the slider movements, besides the torque applied to the hub. The system is highly non-linear. The state model is given:

$$\dot{x}_{2r-1} = x_{2r} \tag{5}$$

$$\dot{x}_{2r} = -\frac{\sum_{s=1}^{p} w_s^2 S_{rs} x_{2s-1}}{M_1 + M_2} - \frac{2 \left(M_1 x_{2p+4} + M_2 x_{2p+6} \right) x_{2p+2} \sum_{s=1}^{p} S_{rs} \int_0^L \phi_s dx}{M_1 + M_2} + \frac{\sum_{s=1}^{p} S_{rs} \dot{\phi}_s\big|_{x=0}}{M_1 + M_2} u_i \tag{6}$$

$$\dot{x}_{2p+1} = x_{2p+2} \tag{7}$$

$$\dot{x}_{2p+2} = \ddot{\theta} = \frac{\sum_{r=1}^{p} \sum_{s=1}^{p} \omega_s^2 \phi_r(l_i) S_{rs} x_{2s-1}}{J_{BC} + M_1 x_{2p+3}^2 + M_2 x_{2p+5}^2} + \frac{1 - \sum_{r=1}^{p} \sum_{s=1}^{p} S_{rs} \dot{\phi}\big|_{x=0} \phi_r(l_i)}{J_{BC+M_1 x_{2p+3}^2 + M_2 x_{2p+5}^2}} \tau + \\ +2 \frac{\left(M_1 x_{2p+4} + M_2 x_{2p+6} \right) x_{2p+2} \left(\sum_{r=1}^{p} \sum_{s=1}^{p} \phi_r(l_i) S_{rs} \left(\int_0^L \phi_s dx \right) - 1 \right)}{J_{BC} + M_2 x_{2p+5}^2 + M_1 x_{2p+3}^2} \tag{8}$$

$$\dot{x}_{2p+3} = x_{2p+4} \tag{9}$$

$$\dot{x}_{2p+4} = \frac{u_2}{M_1} + x_{2p+3} x_{2p+2}^2 \tag{10}$$

$$\dot{x}_{2p+5} = x_{2p+6} \tag{11}$$

$$\dot{x}_{2p+6} = \frac{u_3}{M_1} + x_{2p+5} x_{2p+2}^2 \tag{12}$$

The elements of matrix S are written acoording to in Eq. 2 with the necessary adaptations.

4. Simulation Results

Although the dynamical model has been derived for an arbitrary number of vibration modes, from now on just the first mode is considered. This is justified by the complexity of the numerical problem that has to be solved, a fact that does not allow as many modes as desired. For the solutions of the Optimal Control Problems (OCP), numerical simulations have been performed using the $RIOTS_{95}$ package (Schwartz et al ., 1997). One has considered as a valid case the simulated problems that satisfy the rigid convergence criterium of the RIOTS algorithm. Table 1 shows the main parameters of the flexible arm. Running optimal control problems means inclusion of several new parameters. As the number of different cases increased a lot (Terceiro, 2002), a more detailed analysis brought about aspects on the system behavior that were not perceived before. One of the key aspects is the control energy that has to be sent to each of the two sliding masses.

Table 1. Physical parameters of the Optimal Control Problem

Module of Young of the aluminium	$E = 7.110^{10}$ Pa	Length of the arm	L = 0.7 m
Thickness of the arm	h = 0,001 m	Width of the arm	b=0.0254 m
Mass of the arm	$m = \rho_B * L * b * h = 0,0482$ kg	Linear density mass of the arm	$\rho_0 = m/L$
Density of aluminium	$\rho_B = 2710 kg/m^3$ (Al)	Moment of inertia of the hub	$J_c = 1.3510^{-4}$
Moment of inertia of the transversal section of the arm	$I_v = b * \frac{h^3}{12}$	Slider mass 1	$M_1 = 0.05 * m$
Moment of inertia of the arm	$J_v = \rho_0 * \frac{L^3}{3}$	Slider mass 2	$M_2 = 0.05 * m$

Table 2. Main conditions for the optimal control problem

	Case 1	Case 2
Index of the performance	$\int_0^T \left((4x_1^2(t)\right) dt$	$\int_0^T \left((4x_1^2(t) + x_2^2(t)\right) dt$
Final Angular Velocity	0	0
Final Tip Vibration of the arm	free	free
Final Position of sliders	free	free
Final Velocity of sliders	0	0

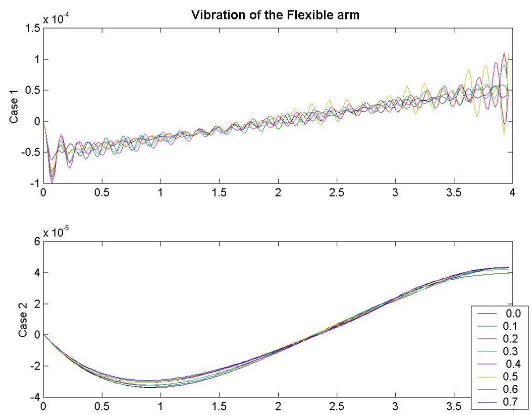

Figure 3. Arm Tip Vibration

The first new result compares 2 cases (1 and 2) where the first one uses an Index of Performance that deals with displacements and velocities of the rotating arm and the second one includes just the tip displacement of the flexible beam. In both simulations, to the contrary of the results already presented for two sliders, there are no constraints on the trajectories. Conditions for simulation are resumed in Table 2.

Figures 3 to 6 below show the system response when using two sliders and conditions given in Table 2.

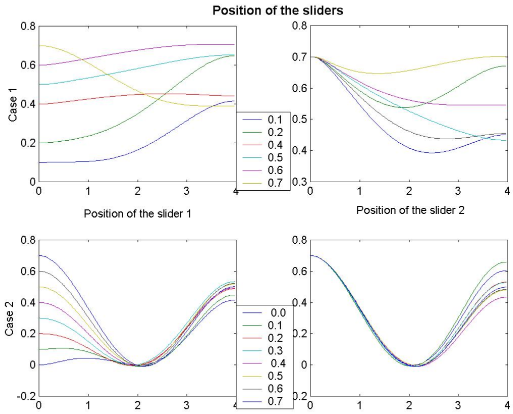

Figure 4. Sliders Trajectories

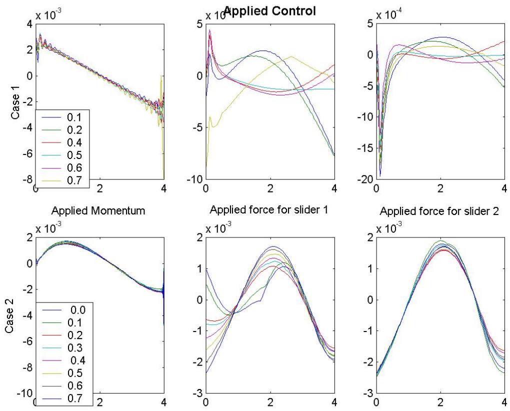

Figure 5. Torque on the hub and forces on sliders 1 and 2

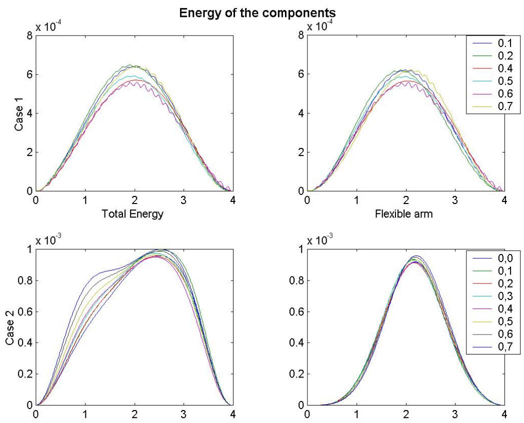

Figure 6. Energy delivered to slider movements

From these results, one can observe that the exclusion of mode velocities from the Objective Function leads to very smooth trajectories. From the other side, the total energy is one order of magnitude above. Case 2 shows very interesting curves for mass movements. From Figure 4, it is possible to verify that one of the sliders almost stops in the middle of the trajectory. Besides that, it seems that the vibration attenuation is reached using just one of the sliders. The optimal solution leads to a large movement of a ŞchosenŤ slider while the other one performs quite small movements. Another important remark is that, although there is no constraint, the two sliders remains over the beam during all the manoeuver.

5. One Slider Results

These results motivated the investigation on the influence of other parameters on the system response. First, it was decided to run the system with just one slider and no trajectory constraints, leading to a situation partially explored by Oliveira(2000) and Fleury and Oliveira (2004). Only arm and sliding mass speeds at the time corresponding to the end of the manoeuver were imposed null. The main differences are related to the Indexes of Performance and to the initial control used to start the simulation.

Four new cases have been selected and simulated. For Case 3, the IP is $\int_0^4 4x_1^2(t) + x_2^2(t)dt$ and, for Case 4, it is $\int_0^4 4x_1^2(t) + x_2^2(t) + F^2(t) + \tau^2(t)dt$ both running on the same initial control guess. For Case 5, the IP is the same as for Case 3 and Case 6 repeats the same IP as for Case 4, but running on a different initial controls. The results for the tip arm vibrations are shown in Figure 7. The box in the right of the figures indicate the slider initial position. The x axis is the time to complete the rotation, that is four seconds.

The influence of the new Index of Performance on the response shape is significant (compare the two graphics at left to the ones at right). From the other side, the initial control does not seem having a great influence on the response, as expected. Small differences appear in the slider trajectories, as depicted in Figure 8, below.

Even in these cases, it is worth observing the smoothness of the trajectories when torque and forces are minimized among the other variables. Comparing the new results to those obtained with just one slider in Fleury and Oliveira (2004), one must point out that part of the trajectories achieved in Cases 3 and 5 correspond to the slider moving back of the hub, a situation already detected and technically feasible for the flexible mechanism.

In terms of energy, the inclusion of torque and forces on the Objective Function also leads to better performances. Figures 9 and 10 show the torque applied to the hub and the force applied to the slider to get the optimal trajectories. One can see that torque and force are almost one order of magnitude bigger for Cases 3 and 5.

Torque and force should be resumed to the total energy delivered to the system to perform the rotational manoeuver while minimizing beam vibrations. This is done in Figure 11.

From Figure 11, one can see that except for the situations corresponding to the initial position of the slider near the arm tip, the energy supplied to the mechanism is one order of magnitude smaller when torque and force are minimized.

6. Concluding Remarks

Some interesting remarks can be extracted from the 6 cases presented and analyzed. The most important is the improvement on the system behavior when the control energy is included in the Objective Function for joint minimization. Trajectories become smoother, less control energy should be expended. Second, one must point out that this situation occurs when the flexible mechanism carries one or two sliders, thus signaling that both cases can lead to good responses. Movements are easier to implement when two sliding masses are employed but almost the same performance can be reached with one slider.

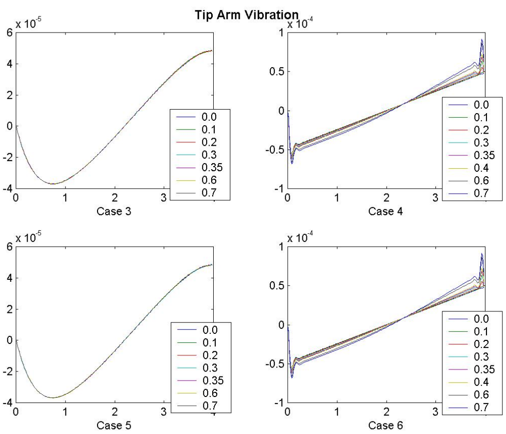

Figure 7. Arm Tip Vibration

The other aspect we intended to emphasize in this paper is the independence of the optimal solutions relative to the initial control guesses. Although the influence of the initial control proved to be very small, the algorithm seems to have a "memory" associated to the initial guess. A definitive answer to this question is still lacking.

The authors are still investigating other aspects on the optimal slider trajectories. The most urgent is the extension of

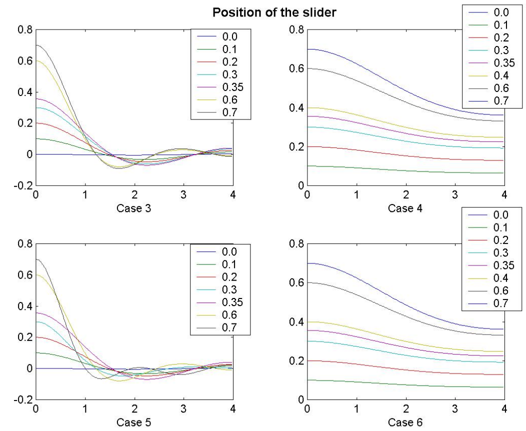

Figure 8. Position of the sliders during the manoeovers

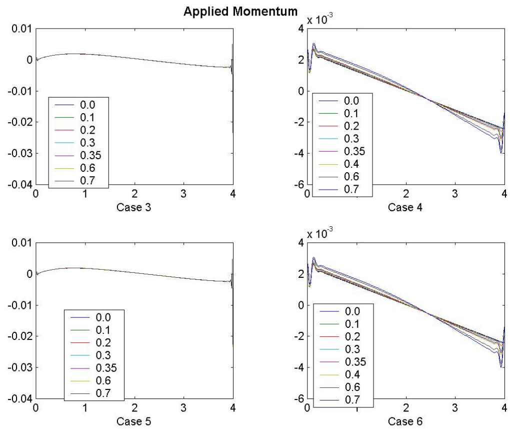

Figure 9. Torque applied to the hub

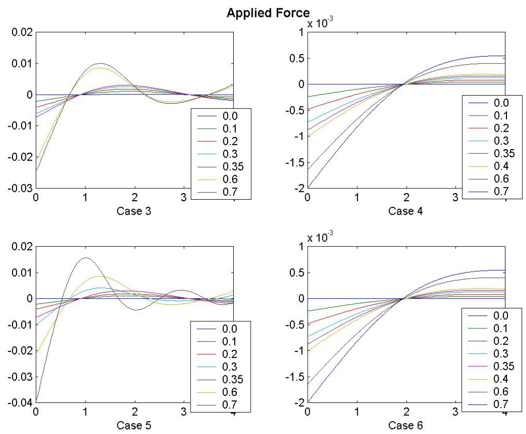

Figure 10. Force applied to the slider

the simulation programs to include second and third modes. The problem here is the model dimension, that leads to many hours of computer simulation to achieve reasonable results. At the same time, an experimental set up in order to validate models and solutions is of fundamental importance to proceed in the investigations.

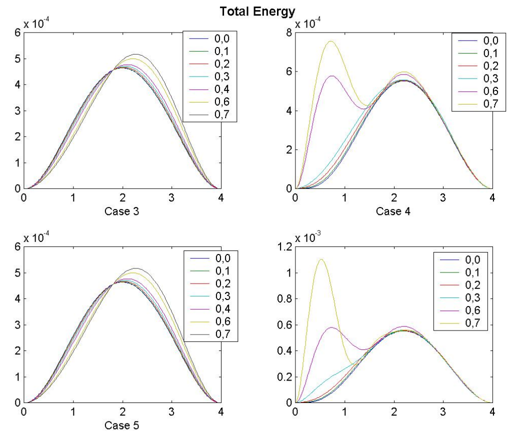

Figure 11. Total energy demanded for the flexible mechanism

7. Acknowledgements

The second author is gratefully acknowledged to CNPq, Brazilian National Research Council, for financial support of part of this project.

8. REFERENCES

Fleury, A. T., Oliveira, F. R. F, 2004, "Dynamics and Control of a Flexible Rotating Arm through the Movement of a Sliding Mass", In: Firdaus Udwadia, Hans Ingo Weber, George Leitmann. (Org.). Dynamical Systems and Control. 01 ed. Boca Raton, Florida: Chapman & Hall/CRC, v. 22, p. 299-318.

Fleury, A. T. ; Oliveira, F. R. F. DE ; Lima, Raul Gonzalez ; Terceiro, Eder, "On the Use of Moving Sliders to the Optimal Control of a Flexible Rotating Arm" In: CBA 2002/14^o Congresso Brasileiro de Automática, 2002, Natal, RN. Anais do CBA 2002/14^o Congresso Brasileiro de Automática. Campina Grande, PB : SBA/Sociedade Brasileira de Automática, 2002. v. Pa 590.

Oliveira, F.R.F., 2000, "Vibration Control of a Flexible Rotating Arm through the Movement of a Sliding mass over the arm" (In Portuguese), MSc. Dissertation, University of São Paulo, Brazil, 140p.

Schwartz, A. L., Polak, E., Chen Y. , 1997, "RIOTS-Recursive Integration Optimal Control Trajectory Solver. A MATLAB toolbox for solving optimal control problems. Version 1.0", Dept. of E1ectrica1 Engineering and Computer Science, University of California at Berkeley.

Schwartz, A. L., 1996, "Theory and Implementation of Numerical Methods Based on Runge-Kutta Integration for Solving Optimal Control Problems", Ph.D. dissertation, University of California at Berkley

Terceiro, Eder ; Fleury, A. T., 2004, "Optimal Control Approach to the Vibrations of a Flexible One-Link Manipulator Carrying Moving Sliders" In: Julio Adamowski; Edilson Hiroshi Tamai; Emilia Villani; Paulo Eigi Miyagi. (Org.). Mechatronics. 01 ed. Rio de Janeiro: ABCM/Associação Brasileira de Engenharia e Ciências Mecânicas, v. 01, p. 233-242.

Terceiro E., 2002, "Use of the sliders masses to optimal control of the flexible rotate arm vibrations: Generalization, Simulation and Results." (In Portuguese), MSc. Dissertation, University of São Paulo, Brazil, 139p.

9. Responsibility notice

The author(s) is (are) the only responsible for the printed material included in this paper