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A NEW INTEGRATION METHOD FOR DIFFERENTIAL INVERSE KINEMATICS OF CLOSED-CHAIN ROBOTS

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Abstract. *Inverse kinematics algorithms based on numerical integration involves drift phenomena of the solution; as a consequence, errors are generated when the end-effector location differs from the desired one. This problem is worse for robots with closed kinematic chains, where minimal errors open the chain or generate excessive torques at the joints. To solve this problem, a novel integration method is presented in this paper, and its exponential stability is showed. An simulation example outlines its main properties.*

***Keywords**: Numerical integration method, Closed kinematic chains, Trajectory control*

1. INTRODUCTION

A kinematic chain is a system of bodies composed by an assemblage of links connected by joints. When every link in a kinematic chain is connected at least to two other links, the kinematic chain forms one or more closed loops and it is called a closed-loop chain or, shortly, closed chain. On the other hand, if every link is connected to every other link by one and only one path, the kinematic chain is called an open-loop chain or, shortly, an open chain. It is also possible for a kinematic chain to be made up of both closed and open chains. We call such a chain a hybrid kinematic chain.

A robotic system typically consists of a mechanical manipulator, an end-effector, a microprocessor-based controller and a computer. A mechanical manipulator is made up of several links connected by joints constituting a kinematic chain. Some of the joints in the manipulator are actuated; the others are passive. Typically, the number of actuated joints is equal to the degrees of freedom.

A robot is said to be a serial robot or open-loop manipulator if its kinematic structure takes the form of an open-loop chain; a parallel manipulator if it is made up of a closed-loop chain, and a hybrid manipulator if it consists of both open- and closed-loop chains.

In this paper we focus on the kinematics of closed chains present in parallel and hybrid manipulators. More specifically we intend to obtain the positions of the passive joints given the positions of the active joints by integrating the differential kinematics of the closed chain using numerical techniques.

It is well known that in open-loop kinematic chains the numerical integration of the differential kinematics involves a drift phenomena of the solution and, as a consequence, the end-effector location corresponding to the computed joint variables differs from the desired one (see Sciavicco and Siciliano, 2004, for example).

In closed-loop kinematic chains, beside the drift phenomena the integration errors in the joint positions reflect in an opening of the closed chain, which can be measured by a closure error (Pavlin, 1995).

The main contribution of this paper is to introduce a new method to integrate numerically the differential kinematics of closed chains guaranteeing the exponential convergence of the closure error.

To present the method we first state the problem conceptually. Next, the differential kinematics based on the screw representation the Davies Method are shortly reproduced for completeness. In the sequence, we present the proposed algorithm and state its convergence properties. An example outlines the main algorithm features.

2. PROBLEM STATEMENT

In closed kinematic chains not all the joints can be controlled independently. Thus, some of the joints are driven by actuators whereas others are passive. In this paper, the joints driven by actuators are called primary joints, and the passive ones are named secondary joints.

Let the primary joint variables be denoted by a vector q_p and the secondary joint variables be described by a vector q_s. Then the kinematic constrains imposed by the limbs can be written in the general form (Tsai (1999))

$$f(q_p, q_s) = 0 \tag{1}$$

where f is an n-dimensional implicit function of q_p and q_s and 0 is an n-dimensional zero vector. Equation (1) is sometimes referred as the closure equation.

Differentiating Eq. (1) with respect to time, we obtain a relationship between the input joint rates (primary joint velocities $\dot{q}_p$) and the output joint rates (secondary joint velocities $\dot{q}_s$) as follows:

$$N_p(q_p)\dot{q}_p + N_s(q_s)\dot{q}_s = 0 \tag{2}$$

where $N_p(q_p) = \dfrac{\partial f}{\partial q_p}$ and $N_s(q_s) = \dfrac{\partial f}{\partial q_s}$

From Eq. (2) we have

$$\dot{q}_s = -N_s^{-1}(q_s)N_p(q_p)\dot{q}_p \tag{3}$$

The secondary joint position can be calculated by integrating Eq. (3) as follows:

$$q_s(t) - q_s(0) = \int_0^t \dot{q}_s dt = -\int_0^t N_s^{-1}(q_s)N_p(q_p)\dot{q}_p dt \tag{4}$$

It should be remarked that this technique for calculating the secondary joint positions is independent of the solvability of the kinematic structure. Nonetheless, it is necessary that the secondary matrix $N_s(q_s)$ be square and of full rank. In case this matrix has more columns that rows infinite solutions exist to Eq. (3) and a viable solution method is to formulate the problem as a constrained linear optimization problem, as is usual in redundant manipulators analysis. This approach and a method to deal with singularities can be both found in Sciavicco and Siciliano (2004).

The integration can be performed in discrete time by resorting to numerical techniques. The simplest technique is based on the Euler integration method; given an integration interval Δt, if the joint positions and velocities at time t_k are known, the joint positions at time $t_{k+1} = t_k + \Delta t$ can be computed as

$$q_s(t_{k+1}) = q_s(t_k) - N_s^{-1}(q_s(t_k))N_p(q_p(t_k))\Delta q_p(t_k) \tag{5}$$

where

$$\Delta q_p(t_k) = q_p(t_{k+1}) - q_p(t_k) \tag{6}$$

By calculating the secondary joint positions using Eq. (5), a cumulative error in q_s is introduced. Therefore, Eq. (1) is not satisfied and an opening in the closed-loop chain is introduced. To illustrate this, consider the four-bar mechanism in Fig. (1).

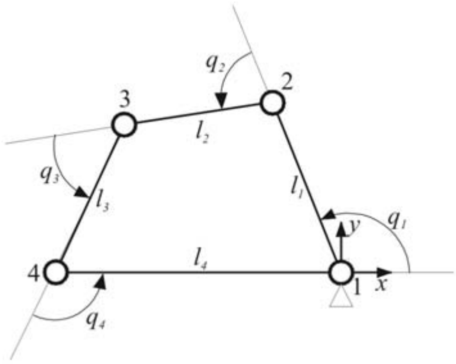

Figure 1. Four bar plane closed chain.

The cumulative error in q_s generate an opening in the closed-loop chain, as depicted in Fig. (2). To solve this problem, we present a new method to integrate Eq. (3) using numerical techniques in which the closure error converges exponentially. To describe it first we present the fundamental kinematic tools used in this work.

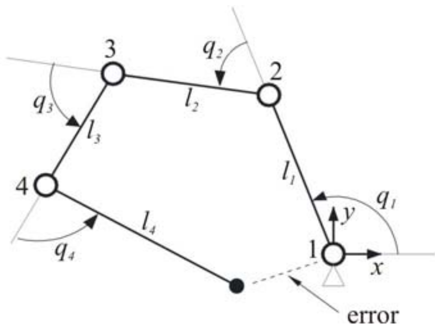

Figure 2. Closure error in a four bar close-loop chain.

3. FUNDAMENTAL KINEMATIC TOOLS

Our approach is based on the method of successive screw displacement, on the screw representation of differential kinematics, on the Davies method, and on the Assur virtual chain concept, which are shortly presented in this section.

3.1. Method of Successive Screw Displacements

In this subsection we describe a method of representing a location of a rigid body in a kinematic chain with respect to a coordinate frame, based on the successive screw displacement concept. First, we present the transformation matrix associated with a screw displacement, and then the concept of the resultant screw of two successive screw displacements is described.

Homogeneous transformation screw displacement representation

Chasles's theorem states that the general spatial displacements of a rigid body is a rotation about and a translation along some axis. Such a combination of translation and rotation is called a screw displacement (Bottema and Roth, 1979). In what follows, we derive a homogeneous transformation that represents a screw displacement (Tsai, 1999).

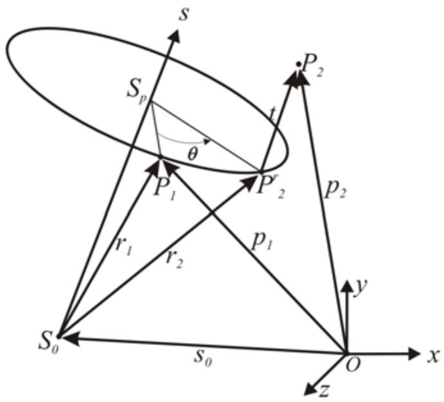

Figure 3. Vector diagram of a spatial displacement.

Figure 3 shows a point *P* of a rigid body that is displaced from a first position P_1 to a second position P_2 by a rotation θ about a screw axis followed by a translation of *t* along the same axis. The rotation brings *P* from P_1 to P_2^r, and the translation brings *P* from P_2^r to P_2. In the figure, $s = \begin{bmatrix} s_x & s_y & s_z \end{bmatrix}^T$ denotes a unit vector along the direction of the screw axis, and $s_0 = \begin{bmatrix} s_{0x} & s_{0y} & s_{0z} \end{bmatrix}^T$ denotes the position vector of a point lying on the screw axis. The rotation angle θ and the translation *t* are called the screw parameters. These screw parameters together with the screw axis completely define the general displacement of a point attached to a rigid body and, consequently, of a rigid body.

Representing the first position P_1 by the vector $p_1 = \begin{bmatrix} p_{1x} & p_{1y} & p_{1z} \end{bmatrix}^T$ and the second position P_2 by the vector $p_2 = \begin{bmatrix} p_{2x} & p_{2y} & p_{2z} \end{bmatrix}^T$, the general screw displacement for a rigid body can be given by the Rodrigues's formula as:

$$p_2 = R(\theta)p_1 + d(t) \tag{7}$$

where $R(\theta)$ is the rotation matrix corresponding to the rotation θ about the screw axis and $d(t)$ is displacement vector corresponding to the translation *t* along the screw axis.

Considering the augmented vectors $\hat{p}_1 = \begin{bmatrix} p_1^T & 1 \end{bmatrix}^T$ and $\hat{p}_2 = \begin{bmatrix} p_2^T & 1 \end{bmatrix}^T$ the general displacement of a rigid body (Eq. (7)) can be represented by a homogeneous transformation given by:

$$\hat{p}_2 = A(\theta, t)\hat{p}_1 \tag{8}$$

where

$$A(\theta, t) = \begin{bmatrix} R(\theta) & d(t) \\ 0 & 1 \end{bmatrix} \tag{9}$$

and the elements of $R(\theta)$ and of $d(t)$, according (Tsai (1999)), are given by:

$$R(\theta) = \begin{bmatrix} \cos\theta + s^2{}_x(1-\cos\theta) & s_y s_x(1-\cos\theta) - s_z\sin\theta & s_z s_x(1-\cos\theta) + s_y\sin\theta \\ s_x s_y(1-\cos\theta) + s_z\sin\theta & \cos\theta + s^2{}_y(1-\cos\theta) & s_z s_y(1-\cos\theta) - s_x\sin\theta \\ s_x s_z(1-\cos\theta) - s_y\sin\theta & s_y s_z(1-\cos\theta) + s_x\sin\theta & \cos\theta + s^2{}_z(1-\cos\theta) \end{bmatrix}$$

$$d(t) = ts + [I - R(\theta)]s_0$$

Successive screw displacements

We now use the homogeneous transformation screw representation to express the composition of two or more screw displacements applied successively to a rigid body.

Figure 4 shows a rigid body σ which corresponds to a second moving link and is moved by two successive screw displacements: a first one, called the fixed joint axis $\$_1(q_1)$, applied to the joint axis situated between the ground (fixed base) and the first link (first link screw axis), and a second one, called the moving joint axis $\$_2(q_2)$, applied to the joint axis between the first and the second link (second link axis).

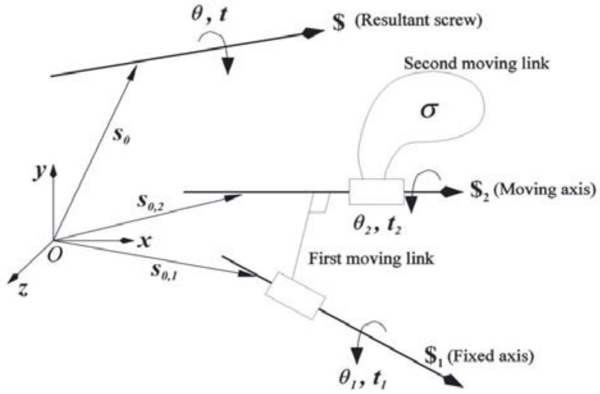

Figure 4. Two-link chain and its associated screw displacements.

As the rigid body is rotated about and/or translated along these two joint axes, the best way to obtain its resultant displacement is to displace the rigid body σ about/along the fixed axis and, in what follows, displace the body about/along the moving axis. In this way, the initial location of the moving joint axis can be used to derive the transformation matrix $A_2(q_2)$, which represents the $\$_2(q_2)$ screw displacement while the fixed joint axis is used for derivation of matrix $A_1(q_1)$, which represent the $\$_1(q_1)$ screw displacement (see details in Tsai (1999)).

Consequently the resulting transformation matrix is given by a premultiplication of the two successive screw displacements; that is,

$$A_r(q_1, q_2) = A_1(q_1)A_2(q_2) \tag{10}$$

3.2. Screw representation of differential kinematics

The Mozzi theorem states that the general spatial differential movement of a rigid body consists of a differential rotation about, and a differential rotation along a axis named instantaneous screw axis. In this way the velocities of the points of a rigid body with respect to an inertial reference frame *O-xyz* may be represented by a differential rotation ω about the instantaneous screw axis and a simultaneously differential translation τ about this axis. The complete movement of the rigid body, combining rotation and translation, is called screw movement or twist and is here denoted by $\$$. Figure 5 shows a body "twisting" around the instantaneous screw axis. The ratio of the linear velocity and the angular velocity is called pitch of the screw $h = \|\tau\| / \|\omega\|$.

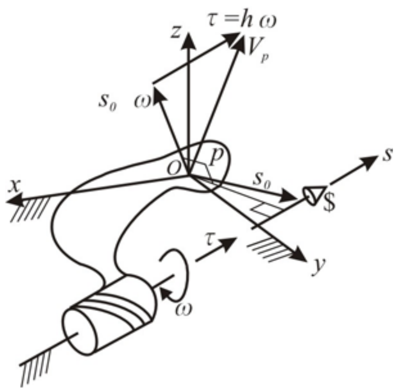

Figure 5. Screw movement or twist.

The twist may be expressed by a pair of vectors, i.e. $\$ = \left[\omega^T; V_p^T\right]^T$, where ω represents the angular velocity of the body with respect to the inertial frame, and V_p represents the linear velocity of a point *P* attached to the body which is instantaneously coincident with the origin *O* of the reference frame. A twist may be decomposed into its amplitude and its corresponding normalized screw. The twist amplitude, denoted as $\dot{q}$ in this work, is either the magnitude of the angular velocity of the body, $\|\omega\|$, if the kinematic pair is rotative or helical, or the magnitude of the linear velocity, $\|V_p\|$, if the kinematic pair is prismatic. The normalized screw, $\hat{\$}$, is a twist in which the magnitude is factored out, i.e.

$$\$ = \hat{\$}\dot{q} \tag{11}$$

The normalized screw coordinates (Davidson and Hunt, 2004) may be given by,

$$\hat{\$} = \begin{bmatrix} s \\ s_0 \times s + hs \end{bmatrix} \tag{12}$$

where, as above, the vector $s = \begin{bmatrix} s_x & s_y & s_z \end{bmatrix}^T$ denotes a unit vector along the direction of the screw axis, and the vector $s_0 = \begin{bmatrix} s_{0x} & s_{0y} & s_{0z} \end{bmatrix}^T$ denotes the position vector of a point lying on the screw axis.

So, the twist given in Eq. (11) expresses the general spatial differential movement (velocity) of a rigid body with respect to an inertial reference frame *O-xyz*. The twist can also represent the movement between two adjacent links of a kinematic chain. In this case, the twist $\$_i$ represents the movement of link *i* with respect to link $(i-1)$.

3.3. Davies' method

Davies' method is a systematic way to relate the joint velocities in closed kinematic chains. Davies derives a solution to the differential kinematics of closed kinematic chains from the Kirchhoff circulation law for electrical circuits. The resulting Kirchhoff-Davies circulation law states that "The algebraic sum of relative velocities of kinematic pairs along any closed kinematic chain is zero" (Davies, 1981).

We use this law to obtain the relationship among the velocities of a closed kinematic chain as in Campos et al. (2005). So, considering that the velocity of a link with respect to itself is null, the circulation law can be expressed as

$$\sum_{i=1}^{n} \$_i = 0 \tag{13}$$

where 0 is a vector which dimension corresponds to the dimension of the twist $\$_i$.

According to the above introduced normalized screw definition this equation may be rewritten as

$$\sum_{i=1}^{n} \hat{\$}_i \dot{q}_i = 0 \tag{14}$$

where $\hat{\$}_i$ and $\dot{q}_i$ represent the normalized screw and the magnitude of twist $\$_i$, respectively.

Equation (14) is the constraint equation which, in general can be written as

$$N\dot{q} = 0 \tag{15}$$

where $N = \begin{bmatrix} \hat{\$}_1 & \hat{\$}_2 & \ldots & \hat{\$}_n \end{bmatrix}$ is the network matrix containing the normalized screws which signs depend on the screw definition in the circuit orientation, and $\dot{q} = \begin{bmatrix} \dot{q}_1 & \dot{q}_2 & \ldots & \dot{q}_n \end{bmatrix}$ is the magnitude vector.

A closed kinematic chain has actuated joints, here named primary joints, and passive joints, named secondary joints. The constraint equation, Eq. (15), allows calculating the secondary joint velocities as functions of the primary joint velocities. To this end the constraint equation is rearranged highlighting the primary and secondary joint velocities and Eq. (15) is rewritten as follows:

$$\begin{bmatrix} N_p & \vdots & N_s \end{bmatrix} \begin{bmatrix} \dot{q}_p \\ \ldots \\ \dot{q}_s \end{bmatrix} = 0 \tag{16}$$

where N_p and N_s are the primary and secondary network matrices, respectively, and $\dot{q}_p$ and $\dot{q}_s$ are the corresponding primary and secondary magnitude vectors, respectively.

Equation (16) can be rewritten as

$$N_p\dot{q}_p + N_s\dot{q}_s = 0 \tag{17}$$

which is the Eq. (2) derived in another way.

3.4. The Assur virtual kinematics chain concept

The Assur virtual kinematic chain concept, virtual chain for short, is essentially a tool to obtain information about the movement of a kinematic chain or to impose movements on a kinematic chain (Campos et al. (2005)).

This concept was first introduced by Campos (2004) which defines the virtual chain as a kinematic chain composed of links (virtual links) and joints (virtual joints) satisfying the following three properties: a) the virtual chain is open; b) it has joints whose normalized screws are linearly independent; and c) it does not change the mobility of the real kinematic chain, in other words, it is an Assur group (Baranov, 1985).

The orthogonal PPR Assur virtual chain

The PPR virtual chain is composed by two virtual links (*C1*, *C2*) connected by two prismatic joints, whose movements are in the *x* and *y* orthogonal directions, and a rotational joint, whose the movement is in the *z* directions, see Fig.6. The prismatic joints are called *px* and *py*, and the rotative joint is called *rz*.

The first prismatic joint (*px*) and the rotative joint (*rz*) are attached to the chain to be analyzed (real chain). Joint *px* connects the link *R1* with virtual link *C1*, joint *py* connects virtual link *C1* with virtual link *C2*, and joint *rz* connect the virtual link *C3* with real link *R2* (see Fig.6).

Let the twist $\$_{px}$ represent the movement of link *C1* in relation to link *R1*, twist $\$_{py}$ represent the movement of link *C2* in relation to link *C1*, twist $\$_{rz}$ represent the movement of link *R2* in relation to link *C2*. Therefore, the movement of link *R2* in relation to real link *R1* may be expressed by $\$_{px} + \$_{py} + \$_{rz}$.

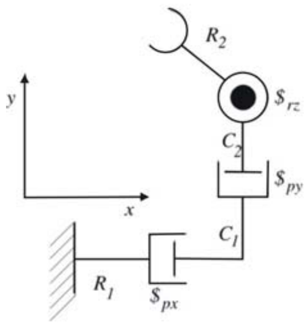

Figure 6. PPR Assur virtual chain.

Consider the C-reference system (*C-system*) attached to the virtual link *C2* at the *rz* joint. Therefore, there is no rotation between the *C-system* and the *B-system* (attached to the inertial base), and the *rz* joints are aligned with *z* axes. So, the normalized screws corresponding to the virtual joints represented in the *C-system* are

$$ {}^{C}\hat{\$}_{rz} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}; {}^{C}\hat{\$}_{px} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}; {}^{C}\hat{\$}_{py} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{18} $$

It may be observed that the orthogonal PPR Assur virtual chain represents the movements in a planar Cartesian system. Other Assur virtual chains can be found in Campos (2004) and Campos et al. (2005).

4. INTEGRATION ALGORITHM USING ASSUR VIRTUAL CHAINS

An integration algorithm is necessary to integrate the kinematic differential equation to obtain the joint positions. The algorithm proposed in this paper has two steps. The first one is to introduce a virtual chain to represent the closeness error resulting from the integration error as depicted in Fig. 1. Introducing the virtual chain to the four-bar planar mechanism in Fig. 1, the resulting closed chain is shown in Fig. 7.

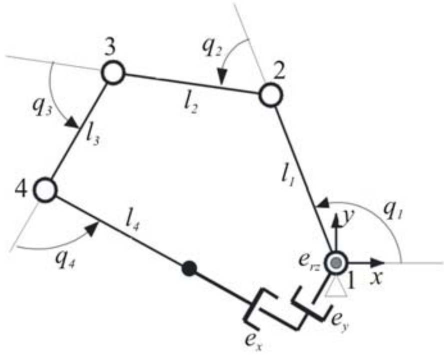

Figure 7. Closure error represented by a virtual chain in a four bar mechanism.

The constraint equation of this closed-loop chain results:

$$N_p(q_p)\dot{q}_p + N_s(\overline{q}_s)\dot{q}_s + N_e\dot{q}_e = 0 \tag{19}$$

where $N_p(q_p)$ is the primary network matrix as before, $N_s(\overline{q}_s)$ is the secondary network matrix corresponding to the secondary positions obtained by integration ($\overline{q}_s$), $\dot{q}_p$ and $\dot{q}_s$ are the primary and secondary magnitude vectors, respectively, N_e is the error network matrix, and $\dot{q}_e$ is the error magnitude vector.

The second step consists in replacing Eq. (3) by

$$\dot{q}_s = -N_s^{-1}(\overline{q}_s)N_p(q_p)\dot{q}_p - N_s^{-1}(\overline{q}_s)N_eK_eq_e \tag{20}$$

where the gain matrix K_e is chosen to be positive definite and q_e is the position error vector.

4.1. Stability

The algorithm stability can be verified substituting Eq. (20) in Eq. (19), obtaining:

$$N_e\dot{q}_e + N_eK_eq_e = 0$$

Multiplying all terms by N_e^{-1} (which ever exists because the virtual chain joints normalizes screws are ever linearly independent), results:

$$\dot{q}_e + K_eq_e = 0 \tag{21}$$

As the gain matrix is positive definite, Eq. (21) states that the position error vector $q_e \to 0$ as $t \to \infty$ exponentially.

Theoretically, the gains can be as great as desired, in practice they are limited by the possibility of introducing numerical problems, such as numerical oscillations and, in some limit cases, instabilities. This gain limit can result in undesirable errors. This difficulty is overcome by using gains that do not introduce numerical problem combined with various iterations which allow control the error, as is out depicted in the next subsection.

4.2. Error control

The exponential convergence is guaranteed in the general case when the primary positions and velocities are time varying. It has the same property in case the primary positions and velocities are constant, as in a second iteration. If the error is greater than a desired value, new iterations can be performed until the error is as short as an admissible tolerance. Consequently, the proposed algorithm allows control the error.

4.3. Position error vector

The screw displacement of a link in a kinematic chain can be expressed by a homogeneous matrix, and that the resulting screw displacement in a link *j* can be calculated using the successive screw displacement method (see section 3.1) by premultiplying the homogeneous matrices corresponding to the preceding joint movements, i.e.:

$$A_j = \prod_{i=1}^{j-1} A_i \tag{22}$$

Considering that, in a closed chain, the last and the first links are the same, and that the orientation and the position of a link with respect of itself is given by a homogeneous matrix equal to the fourth order identity matrix. In a closed-loop chain with *np* primary joints and *ns* secondary joints Eq. (22) the closed-loop equation results:

$$\prod_{i=1}^{np}\left[A_p\right]_i \prod_{i=1}^{ns}\left[A_s\right]_i = I \tag{23}$$

where $\left[A_p\right]_i$, i = 1…*np*, are the homogeneous matrices corresponding to the primary joints, and $\left[A_s\right]_i$, i = 1…*ns*, are the homogeneous matrices corresponding to the secondary joints.

Consider a closed-loop that has an error chain like is depicted in Fig. 7. As in Pavlin (1995), we represent the closure error by a homogeneous matrix *E*, and the closed-loop equation becomes:

$$\left\{\prod_{i=1}^{np}\left[A_p\right]_i \prod_{i=1}^{ns}\left[A_s\right]_i\right\} E = I \tag{24}$$

the closure error is calculated by

$$E = \left\{\prod_{i=1}^{np}\left[A_p\right]_i \prod_{i=1}^{ns}\left[A_s\right]_i\right\}^{-1} = \begin{bmatrix} R_e & p_e \\ 0 & 1 \end{bmatrix} \tag{25}$$

where $p_e = \left[p_{ex} \quad p_{ey} \quad p_{ez}\right]^T$ is the position error vector and R_e is the rotation matrix error. The matrix R_e corresponds to three Euler angles r_{ex}, r_{ey} and r_{ez}, see Sciavicco and Siciliano (2004).

The "position" error (which is a posture error involving position and orientation) is given by the position error vector $q_e = \left[r_{ex} \quad r_{ey} \quad r_{ez} \quad p_{ex} \quad p_{ey} \quad p_{ez}\right]^T$.

4.4. Numerical implementation

Applying the Euler integration method in Eq. (20) we obtain:

$$q_s(t_{k+1}) = q_s(t_k) - N_s^{-1}(q_s(t_k))N_p(q_p(t_k))\Delta q_p + N_s^{-1}(q_s(t_k))N_e K_e q_e(t_k)\Delta t \tag{26}$$

5. EXAMPLE

The presented method is illustrated by solving the position kinematic of a four-bar planar mechanism (Fig. 1) by integrating its differential kinematics.

In this example, joint 1 (Fig. 1) is considered primary while the others are secondary. Joint 1 moves from initial position $\pi/4$ to the final position $\pi/2$ according to $q_p(t) = \frac{\pi}{4} + \frac{\pi}{4}\sin\left(\frac{\pi t}{8}\right)$ from t = 0 to 4s.

The kinematic parameters are $l_1 =$ 0.5 m; $l_2 =$ 1.0 m; $l_3 =$ 0.5 m; $l_4 =$ 1.0 m. The initial position vector is $q_s(0) = \left[\frac{-\pi}{4} \quad \frac{-3\pi}{4} \quad \frac{-\pi}{4}\right]^T$ rad. The integration interval is $\Delta t =$ 0.001 s.

Considering the reference frame attached to the last link of the error chain, the network matrices result:

$$N_e = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} ; \; N_p = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} ; \; N_s = \begin{bmatrix} 1 & 1 & 1 \\ y_2 & y_3 & y_4 \\ -x_2 & -x_3 & -x_4 \end{bmatrix}$$

The secondary position vector is calculated using Eq. (26) and the resulting vector components are compared with an analytical solution presented in (Waldron and Kinzel, 1999).

The solution of the planar four bar mechanism is obtained choosing $K_e = K \quad I_3$, where *K* is a positive scalar and I_3

is an identity matrix. Varying *K* from 100 to 1000 and the iterations number from 1 to 4, we have the results presented on Fig. 9.

Fig. 9 presents the maximum error between the numerical solution and the analytical solution in function of *K*. It is observed that the error decreases with increasing gains and decreases strongly in the first iterations. So, to obtain the admissible errors the gains can be chosen in order to avoid numerical problems if they are combined with iterations.

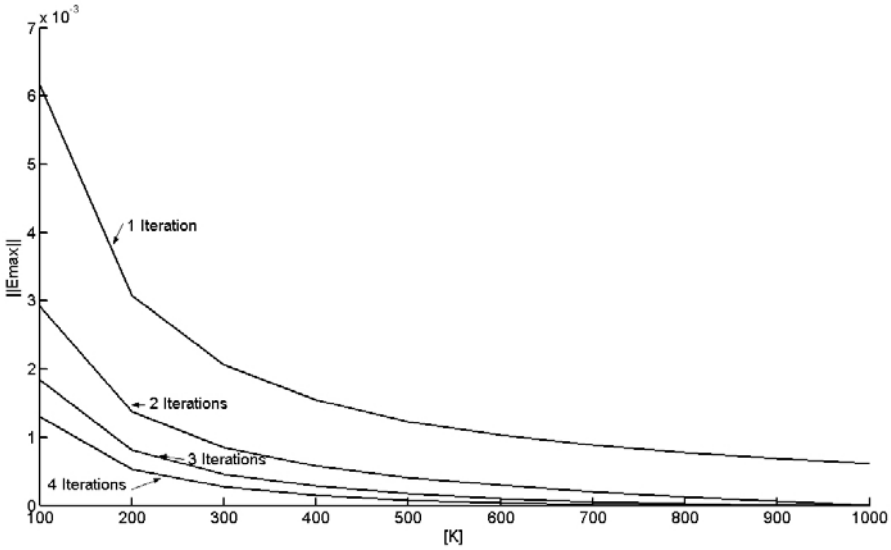

Figure 9. Numerical solution results.

6. CONCLUSIONS

A new method to integrate numerically the differential kinematics of closed chains guaranteeing the closure error exponential convergence is presented and its stability properties are shown theoretically.

The error control is outlined and the influence of various iterations is analyzed considering an example. These results show that there is a compromise between error, number of iterations (or calculating time) and algorithm gains.

7. ACKNOWLEDGEMENTS

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