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TWO DOF ROBUST CONTROLLER FOR A SIX DOF UNDERWATER AUTONOMOUS VEHICLE

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Abstract. *In this paper, a two degree of freedom (TDOF) robust controller is developed to control the motions of an Autonomous Underwater Vehicle (AUV) in six degree of freedom. Mathematical models used to develop controllers for AUVs usually involves a number of uncertainties, mainly due to the complex nature of this problem. A robust controller is then needed to achieve desired levels of robustness of stability and performance. To tackle the problem of the controller synthesis, a mixed* $\mathcal{H}_\infty$ *approach with a TDOF structure is used, which leaded to a controller that improves the AUV performance whereas guarantees stability specifications. The centralized controller was also evaluated with an AUV non-linear model through a large number of numerical simulations. Responses both in frequency and time domains, including trajectory tracking, are produced and analyzed, showing that planar and spatial motion control was fully achieved.*

Keywords: $\mathcal{H}_\infty$ *, Two DOF Robust Controller, AUV.*

1. INTRODUCTION

Autonomous Underwater Vehicles (AUVs) are designed to operate in adverse environments with good performance concerning trajectory tracking and disturbance rejection. Control design is always challenging due to the non-linear behavior of the vehicle, the multivariable nature of the problem and the uncertainties of parameters and environmental conditions. $\mathcal{H}_\infty$ approaches and non-linear control are popular methods to deal with robustness and performances issues, and were successfully applied in the control of AUVs (Kaminer et al., 1981; Innocenti and Campa, 1999; Healey and Lienard, 1993; Song et al., 2002; Ryoo et al., 2005). In former works, authors (Donha and Luque, 2006a, 2206b) tried to use a $\mathcal{H}_\infty$ controller synthesis to solve the control problem of an under-actuaded AUV, with limited success regarding tracking performance. The objective of this work is to develop a robust controller for the aforementioned vehicle with improved performance concerning tracking and disturbance rejection, which in a near future will be implemented and used in the navigation of a torpedo like AUV, under development in the University of São Paulo.

This work tackle the tracking problem of reference signals, $roll$, $pitch$, $yaw\ rate$ and $depth\ rate$ using the $\mathcal{H}_\infty$ mixed sensitivity approach. For this purpose a Two Degree of Freedom (TDOF) Controller is developed, which structure presents more advantages concerning stability and performance, against traditional structure of One Degree of Freedom (ODOF) controller (Donha and Luque, 2006a).

This paper is organized as follows: a brief description of the AUV model is presented in Section 2. The $\mathcal{H}_\infty$ mixed sensitivity approach using a TDOF controller is exposed in section 3. In section 4, numerical results are presented and analyzed. Finally, results are discussed and summarized in the conclusion section.

2. AUV LINEAR MODEL: brief description

This section gives a brief description of the AUV model. The AUV, shown in Fig. 1, is a torpedo-like vehicle, 5.99 m long, with 0.62 m maximum cross-section diameter and 1460 kg mass. It is equipped with a thruster for cruising and fully moving control surfaces (rudder and stern), to steer the vehicle in marine environment. The AUV nonlinear model is composed of six differential nonlinear ordinary equations that represent the dynamics of the underwater vehicle and six equations for coordinate transformations between inertial frame and body frame.

Table 1 defines the nomenclature used for this type of vehicles (Fossen, 1994). As shown in Fig. 1, in this case two reference frames are needed: an inertial frame for position and orientation coordinates (x, y, z, ϕ, θ and ψ), and a body frame for linear and angular velocities of the AUV (u, v, w, p, q and r). Since two frames of coordinate systems are necessary to determine the position and orientation of vehicle at sea, a transformation should be realized using Euler angles (Fossen, 1994) (see Eq. (1)):

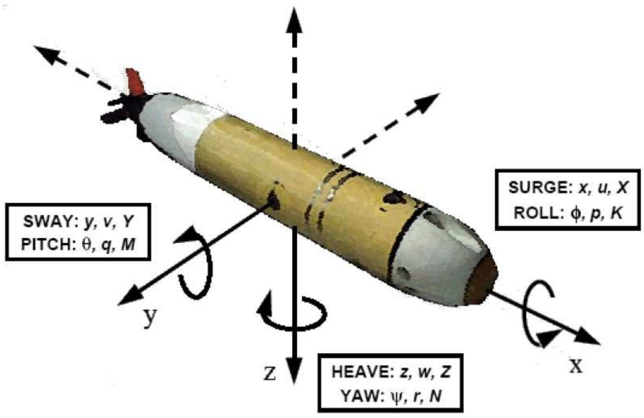


Figure 1. Underactuated Torpedo Like AUV

Table 1. Notation of coordinate systems

motion	description	forces and moments	linear and angular velocities	position and attitude
surge	motion in the x direction	X	u	x
sway	motion in the y direction	Y	v	y
heave	motion in the z direction	Z	w	z
roll	rotation around x axis	K	p	ϕ
pitch	rotation around y axis	M	q	θ
yaw	rotation around z axis	N	r	ψ

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} u\cos\psi\cos\theta + v\left(\cos\psi\sin\theta\sin\psi - \sin\psi\cos\phi\right) + w\left(\sin\psi\sin\phi - \cos\psi\cos\phi\sin\theta\right) \\ u\sin\psi\cos\theta + v\left(\cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi\right) + w\left(\cos\phi\sin\theta\sin\psi - \cos\psi\sin\phi\right) \\ -u\sin\theta + v\cos\theta\sin\phi + w\cos\phi\cos\theta \\ p + q\sin\phi\frac{\sin\theta}{\cos\theta} + r\cos\phi\frac{\sin\theta}{\cos\theta} \\ q\cos\phi - r\sin\phi \\ q\frac{\sin\phi}{\cos\theta} + r\frac{cos\phi}{\cos\theta} \end{bmatrix} \tag{1}$$

Dynamics of the vehicle is determined relatively to the body frame using second law of Newton in matrix form (Luque, 2007).

$$\{F\} = [M]\{\dot{v}\} \quad \text{or} \quad \{\dot{v}\} = [M]^{-1}\{F\} \tag{2}$$

Where $v = \begin{bmatrix} u & v & w & p & q & r \end{bmatrix}^T$ is a velocity vector, F is a force vector that contain forces and moments actuating on the vehicle, their effects are due body lift, body drag, fin lift, fin drag, propeller force, and hydrostatic principles. M is a 6×6 matrix that contain body mass, added mass and inertial coefficients. For a complete explanation and numerical data for these coefficients see the work of Luque (2007).
Equations (1) and (2) describe the AUV model in body frame and inertial frame. After linearization, it is observed that four states are totally uncoupled and the controller model is reduced to eight states. The uncoupled variables are controller by SISO controllers, whereas the coupled part is controlled by a centralized controller, synthetized by a mixed sensitivity procedure, shown in section 3. The model is linearized using a cruise speed of 2 m/s, obtaining a Linear Time Invariant (LTI) model expressed in classical form:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), x(t_o) = x_o \\ y(t) &= Cx(t) + Du(t) \end{aligned} \tag{3}$$

In the above model, $x = \begin{bmatrix} u & v & w & p & q & r & \phi & \theta \end{bmatrix}^T$, is the state vector; $u = \begin{bmatrix} \delta_r & \delta_s \end{bmatrix}^T$, is input vector where δ_r is displacement of rudder and δ_s is displacement of stern flaps; $y = \begin{bmatrix} \phi & \theta & \dot{\psi} & \dot{z} \end{bmatrix}^T$, output vector. $A_{8\times8}$, $B_{8\times2}$, $C_{4\times8}$

and $D_{4\times 2}$ are adequately dimensioned. As said before, this is an under-actuated vehicle, where movements are controlled only by rudder and flaps deflection. Output vector y was choose considering available sensors and variables considered important for tracking.
To facilitate the design and analysis of the control system, the linear model is then scaled using the physical saturation limits on the control surfaces: 30 degrees for the rudder and 25 degrees for the stern. According to Logan (1994), the scale values for the control states can be assumed as the maximum expected tracking errors, as follows: 1 m for depth, 1 m/s for depth rate, 10 ° for heading, and 10 $^\circ/s$ for heading rate. The scale value for the $roll$ and $pitch$ was 10 °, and the scale value (maximum expected) for both $roll\ rate$ and $pitch\ rate$ was 10°. The scale values for both axial an vertical velocity is 1 m/s. For major details and numerical values of this procedure see Donha and Luque (2006b).
After scaling, the system used for control design was finally written in the usual way as follows:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1 w(t) + B_2 u(t), x(t_o) = x_o \\ z(t) &= C_1 x(t) + D_{11} w(t) + D_{12} u(t) \\ y(t) &= C_2 x(t) + D_{21} w(t) + D_{22} u(t) \end{aligned} \quad (4)$$

where $x(.)$ is the state vector, $x(t_o)$ is the known initial state, t is the time, $u(.)$ is the input vector, $w(.)$ is the dynamic disturbance, which may have random and deterministic components, $z(.)$ is the controlled state vector and $y(.)$ is the measured state vector.

3. $\mathcal{H}_\infty$ MIXED SENSITIVITY

An usual approach to characterize the closed-loop performance objectives in the modern control theory is the measurement of certain closed-loop transfer matrices using different matrix norms (Skogestad and Postlethwaite, 1996). These norms provide a measure of how large output signals can get for certain classes of input signals, which is a measure of the gain of the system. A mathematically convenient measure of a closed-loop matrix $T_{zw}(s)$ in the frequency domain is the $\mathcal{H}_\infty$ norm defined as:

$$\|T_{zw}\|_\infty := \max_{\omega\in\mathbb{R}}(\overline{\sigma} T_{zw}(j\omega)) \quad (5)$$

There are several ways of setting up the control problem and consequently the selection of the weighting functions related to the system performance (Donha and Katebi, 2007). One of the most popular procedures is the mixed sensitivity loop-shaping approach where direct bounds on important system transfer functions such as the system sensitivity: $S(s) = (I + GK(s))^{-1}$, the control sensitivity $S(s) = KS(s)$ or the complementary sensitivity $T(s) = I - S(s)$ are considered. S determines the tracking performance and the disturbance attenuation, C limits the actuator action in reducing the unnecessary cost, normally in high frequencies. T is associated with closes-loop system response. Therefore, before synthesis of an optimal controller, specifications of well shaping of these sensitivity functions are given bellow:

E1 Closed-loop stability;

E2 $\overline{\sigma}(S) < 1$ for $\omega < 0,7$rad/s;

E3 $\overline{\sigma}(T) < 1$ for $\omega > 2$rad/s;

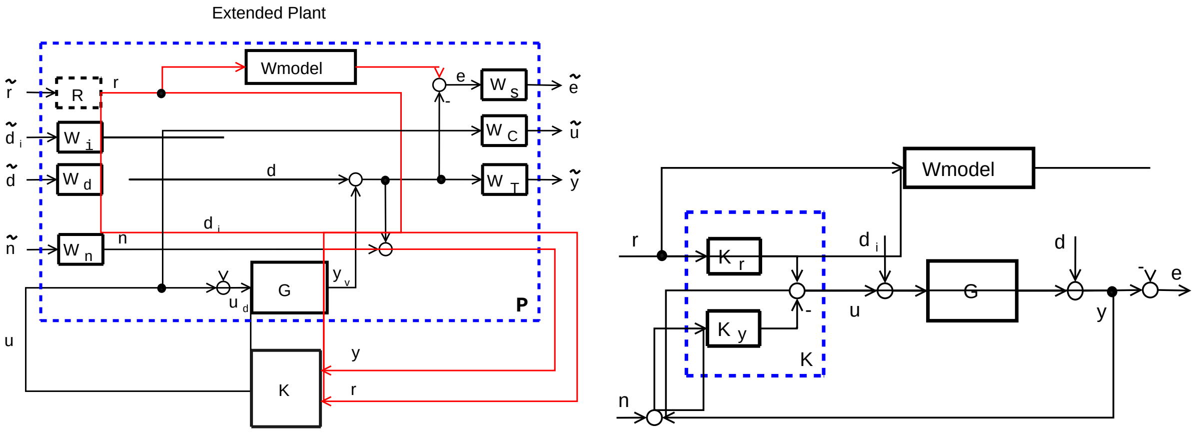
E4 $\overline{\sigma}(C) < 1$ for $\omega > 7$rad/s e

E5 Time responses at step signal

Table 2. System response, assuming a first order system response with settling time $t_s = 4\tau$

Controlled signal	Maximum overshoot	Settling time t_s	$1/\tau$
roll	20%	20s	0,2
pitch	10%	20s	0,2
yaw rate	10%	20s	0,2
depth rate	10%	80s	0,05

Input weighting functions are the used to reflect the available knowledge about the input and output disturbances (d_i and d), and measurement noises (n). On the output side, other output function should be used to reflect the requirements on



(a) TDOF Controller (b) TDOF controller without weighting

Figure 2. Two Degrees of Freedom Controller (TDOF) configuration

the shape of the $\mathcal{H}_\infty$ controller, to reflect restriction on the control signals and to shape the complementary function to modify, for example, tracking features of the system.
Thus, the first step of the $\mathcal{H}_\infty$ controller procedure in this case involve the minimization of a performance index, formulated as follows:

$$\|T_{zw}\| = \|W_S S \quad W_T T \quad W_c C\|_\infty^T \tag{6}$$

Where T_{zw} is the closed loop transfer function from exogenous input w to controlled signal z in a two port configuration (see Fig. 2(a)); the weight W_S on S will determine the tracking performance and the disturbance attenuation (Donha and Luque, 2006a); the weight W_C on C will limit the actuator action in high frequencies, ensuring a desired roll-off frequency; weighting W_T on T is used to set the closed-loop roll-off frequency.
Assuming that the matrix involved satisfy necessary detectability and observability, and based on well-known results, there exist an optimal controller $K(s)$ such that a closed-loop function between z and w satisfies:

$$\|T_{zw}\|_\infty \leq \gamma \tag{7}$$

Finally, the system is put in the two port form (Fig. 2(a)), for which there are many commercial and non commercial softwares for computing the $\mathcal{H}_\infty$ suboptimal controller of n states equivalent to the extended plant P. Mathematical synthesis for this structure are well exposed in (Skogestad and Postlethwaite, 1996). The controller synthesis is get using μ-analysis toolbox (Balas et al., 1991), which solves this problem finding a suboptimal γ knowing systems and weighting functions. If an adequate γ value is not get, then another weighing should be calculated. Whereby the synthesis should be realized again until finding a suboptimal value for γ nearly to 1 if find.

TDOF Controller

It is common to observe in multivariable systems with a large number of degrees of freedom, that specifications like good rejection of disturbances and good tracking are not reached simultaneously with total success (Donha and Luque, 2006a). Therefore, other structures must be investigated to minimize or to eliminate these problems, mainly in underactuated systems, as it is the case in this study. Figure 2(a) shows an alternative structure, where K is a controller of two degrees of freedom (TDOF) with the following structure $K = \begin{bmatrix} K_r & K_y \end{bmatrix}^T$. The advantage in using a TDOF controller, also used by Lundström, Skogestad and Doyle (1999), is to improve tracking and the specifications of temporal responses.
The prefilter K_r is destined to reaching these specifications in time domain, while K_y guarantees the stability of the system. Eventually, frequency specifications were satisfied adjusting a second order function W_{model}. After these modifications, the procedure of the synthesis of the controller TDOF follows the same steps of the ODOF synthesis controller (Donha and Luque, 2006a).
Figure 2(b) shows a TDOF structure of controller, without weighting functions W_* neither the filter R. Sensitivity functions of the system controlled is defined by S, T and C relative to output e, y and u, respectively. Therefore the matrix transfer from exogenous input to exogenous output can be expressed as Eq. (8):

$$\begin{bmatrix} e \\ y \\ u \end{bmatrix} = \begin{bmatrix} SGK_r - W_{model} & SG & S & -T \\ SGK_r & SG & S & -T \\ (I+K_yG)^{-1}K_r & -T & -K_yS & -K_yS \end{bmatrix} \begin{bmatrix} r \\ d_i \\ d \\ n \end{bmatrix} \tag{8}$$

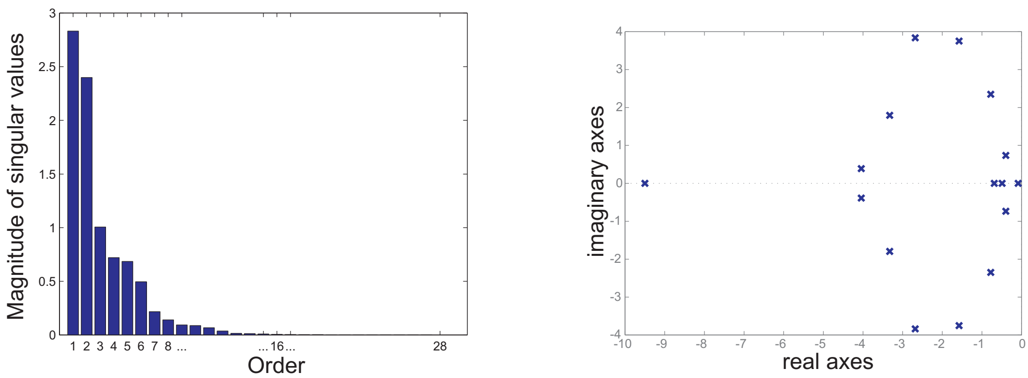
The input filter R was designed to improve performance and it was connected to right references input (see Fig. 2). In this case, R was designed like a first order filter with proportional gain k and thus it was possible to achieve E5 specification (see Tab. 2).

$$R_i = \frac{k_i}{\tau_i s + 1} \tag{9}$$

Where τ_i and t_s are obtained of Tab. 2, t_s is a settling time which the output remains within $\pm 2\%$ of its final value.
From this point of view, for multivariable systems, R was defined as a diagonal matrix given in the appendix section.

4. RESULTS

Using a common structure of One Degree of Freedom (ODOF) controller, specifications E1 to E4 was achieved with excellent robustness specification (Donha and Luque, 2006a), however time responses still showed a tracking error because the performance attenuation was poor in low frequencies. Controller synthesized with ODOF presented 27 states 4 input and 2 output. Additionally, a R filter could be implemented to solve tracking problem, but a better approach for this purpose was using a two degree of freedom (TDOF) structure (see Fig. 2(a))



(a) *Hankel singular values plot (full order)* (b) *Poles of K (reduced order)*

Figure 3. Optimal Hankel norm approximation, from 28th to 16th order

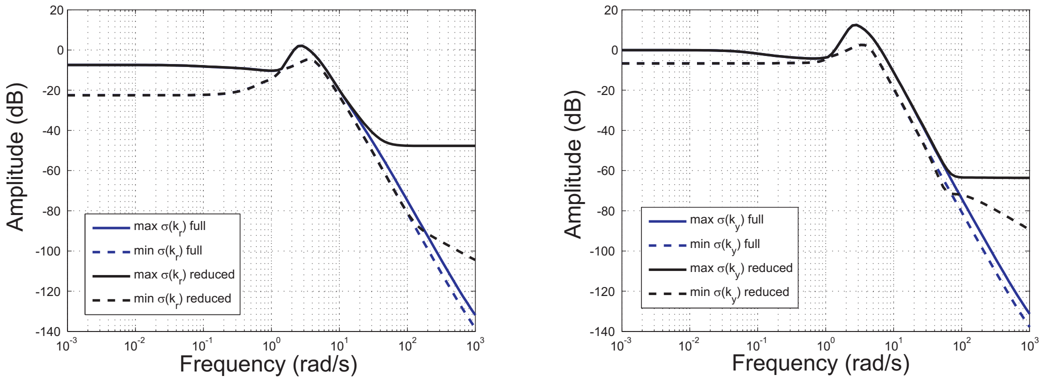
Controller synthesized with TDOF presented 28 states, 8 input and 2 output. Using a well know Hankel norm, the controller order was reduced to 16 states. This approximation was elected because the most representative singular values are in these states, see Fig. 3(a). The practical implementation was defined achievable (Skogestad and Postlethwaite, 1996). Hankel norm approximation is shown in Fig. 3(a). For simples inspection, it is observed that the magnitude of singular values are larger, and that the 16 first states contain the main information about the system. The controller was thus reduced to 16 order and their poles are shown in Fig. 3(b), all poles are located in the open left s-plane.

Figure 4 shows the reduced order controller, where K_r is the tracking part, whereas K_y is the feedback part. The roll-off frequencies of both are less than 7 rad/s, satisfying E3. In the full order controller declines continuously, however in the reduced order K_r and Ky lead to lost similarity with the full order. This is due to the Hankel norm approximation. But this is not a problem when the attenuation is less than -40 dB, as shows Fig. 4.

Figure 5(a) shows robust performance relative to sensitivity function. Figure 5(b) shows robust stability relative to complementary sensitivity function. Both plots reflect robustness of the controlled system.

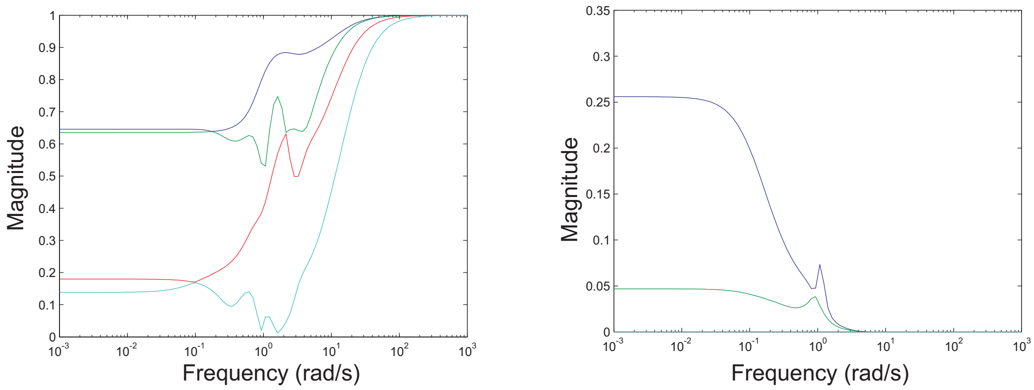
For multivariable systems, specification E1 to E4 are the same for all signals to be controlled, which reduced effort in seeking parameters for weighting functions. After determining a suboptimal value of gamma γ, it is verified if the function shapes S, T and C satisfied specifications E1 to E4. E5 was only possible to achieve using the TDOF controller.

Figure 6 shows the sensitivity functions for the controlled system with a TDOF controller. These plots have relation with specification E2 to E4. The underactuated system is very sensible to disturbance output, thus in Figure 6(a) curves



(a) Singular values of K_r (b) Singular values of K_y

Figure 4. Controller K, full order 28 and reduced order 16



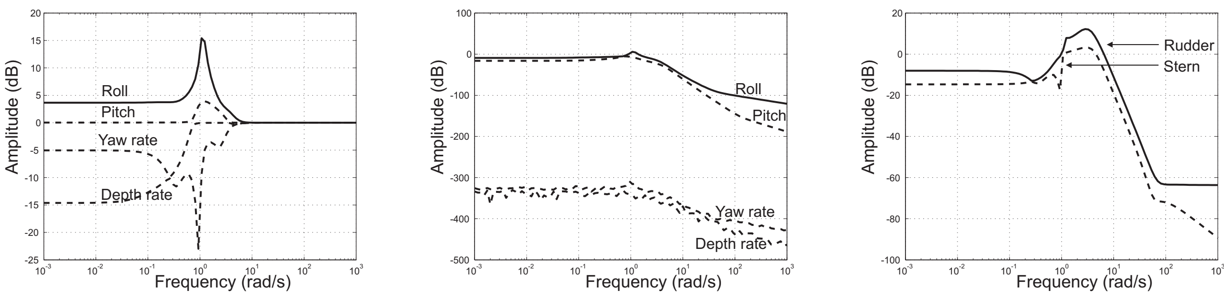
(a) *Performance Stability* (b) *Robust Stability*

Figure 5. Robustness of controlled system using a TDOF controller

related to $roll$ and $pitch$ present poor characteristics of sensibility, and the values in low frequencies are above 0 dB. Therefore, the specification E2 was not guaranteed for $roll$ and $picth$ movements, however E2 is guaranteed for $yaw\ rate$ and $depth\ rate$.

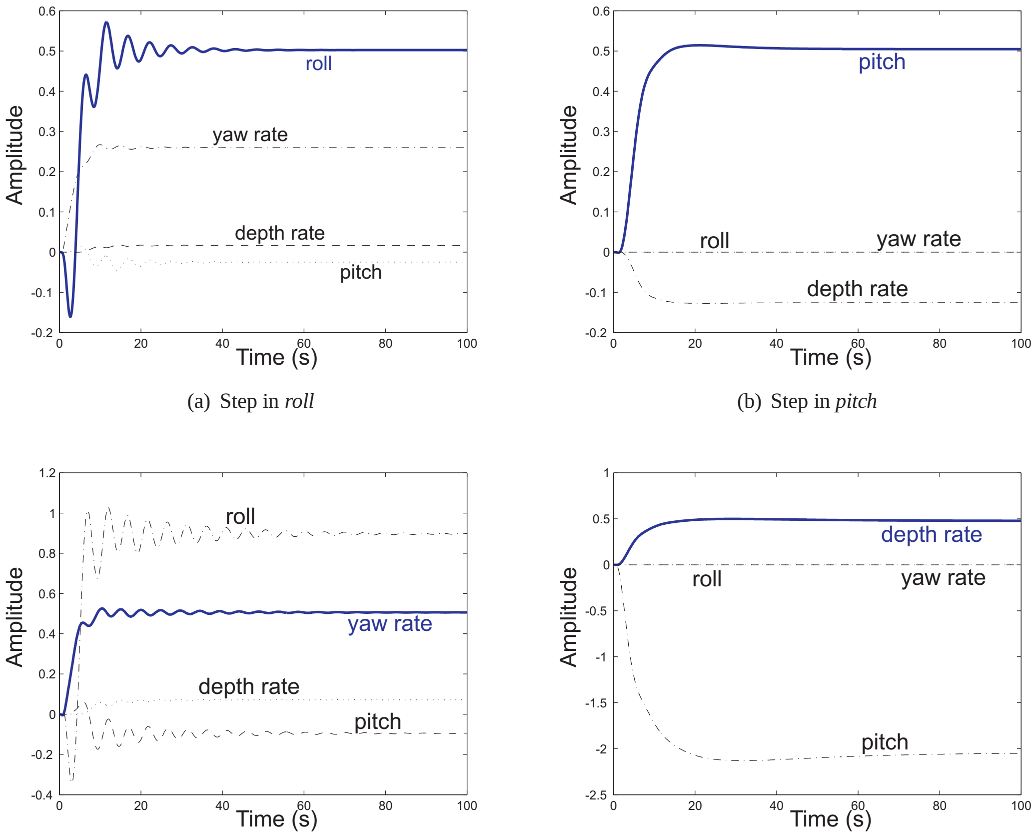
Although E2 was not satisfied, the system presents robust performance (see Fig. 5(a)). E3 was completely achieved for all controlled signals (see Fig. 6(b)) and robustness stability was also achieved (see Fig. 5(b)), although it was observed high separations between singular values, explained by the high coupling between $roll/yaw\ rate$ and $pitch/depth\ rate$, because the system is underactuated. E4 was also achieved for all controlled signals (see Fig. 6(c)).

In the linear model, step responses with full order and reduced order were similar. In nonlinear cases both full order and reduced order showed saturation with steps inputs in $roll$ and $yaw\ rate$, probably because the system underactuated. Figure 7 shows step responses of the system, with 0.5 of amplitude in each channel. All variables controlled ($roll$, $pitch$, $yaw\ rate$ and $depth\ rate$) reflected good tracking capability and E5 was finally achieved using a TDOF controller.



(a) Sensitivity S (b) Complementary Sensitivity T (c) Control Sensitivity C

Figure 6. Sensitivity functions



(a) Step in *roll* (b) Step in *pitch*

(c) Step in *yaw rate* (d) Step in *depth rate*

Figure 7. Non-linear step response with 0.5 of amplitude

5. CONCLUSIONS

The synthesis of a robust $\mathcal{H}_\infty$ controller based on the mixed sensitivity design using a TDOF approach was successfully used to improve tracking performance on an underactuated AUV. To mitigate the problem of high controller order (28 states), which is a handicap of this synthesis procedure, a Hankel norm reduction was employed leading to a suitable structure with a small performance impact. The centralized robust controller was developed using only the coupled part of the model, obtained after linearization. The main problem during the synthesis procedure using the mixed sensitivity approach remains in the choice of the weighting functions used to tune the controller to achieve the desired performance. This choice was a matter of time and skill and perhaps an alternative approach is advisable. Nevertheless, robust stability and performance were achieved and verified not only by the usual measures, but also by a number of simulations with different operational conditions and disturbances. The control technique employed here assumes non-structured uncertainties, which may lead to very conservative designs. This problem can also be mitigated by an alternative approach such as the μ-synthesis, and which will be the next step in this research. A guidance system for this AUV based on the control procedure used here is under consideration, and some new and interesting results are already being produced.

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7. APPENDIX

The weighting function used for synthesis of TDOF controller are given below:

$$W_i = I_{2\times 2} \quad W_d = I_{4\times 4} \quad W_n = I_{4\times 4} \quad W_{model} = I_{4\times 4} \tag{10}$$

$$\begin{aligned} W_{S_\phi} &= \frac{s^3 + 6.91s^2 + 2.76s + 8.3}{s^3 + 14.04s^2 + 19.95s + 19.43} \\ W_{S_\theta} &= \frac{s^3 + 4.57s^2 + 3.72s + 3.61}{s^3 + 8.38s^2 + 10.63s + 4.81} \\ W_{S_{\dot\psi}} &= \frac{s^3 + 1.32s^2 + 2.37s + 4.12}{s^3 + 19.98s^2 + 11.75s + 12.91} \\ W_{S_{\dot z}} &= \frac{s^3 + 11.2s^2 + 14.46s + 7.51}{s^3 + 12.9s^2 + 18.81s + 11.93} \end{aligned} \tag{11}$$

$$\begin{aligned} W_{T_\phi} &= \frac{0.29}{7.12s + 1} \\ W_{T_\theta} &= \frac{0.74}{10.14s + 1} \\ W_{T_{\dot\psi}} &= \frac{0.35}{8.83s + 1} \\ W_{T_{\dot z}} &= \frac{0.67}{13.88s + 1} \end{aligned} \tag{12}$$

$$\begin{aligned} W_{C_{\delta_r}} &= \frac{s^2 + 3.44s + 2.96}{1\text{e-}4s^2 + 0.32s + 256.19} \\ W_{C_{\delta_s}} &= \frac{s^2 + 3.44s + 2.96}{1\text{e-}4s^2 + 0.32s + 256.19} \end{aligned} \tag{13}$$

$$W_S = \begin{bmatrix} W_{S_\phi} & 0 & 0 & 0 \\ 0 & W_{S_\theta} & 0 & 0 \\ 0 & 0 & W_{S_{\dot\psi}} & 0 \\ 0 & 0 & 0 & W_{S_{\dot z}} \end{bmatrix} \quad W_T = \begin{bmatrix} W_{T_\phi} & 0 & 0 & 0 \\ 0 & W_{T_\theta} & 0 & 0 \\ 0 & 0 & W_{T_{\dot\psi}} & 0 \\ 0 & 0 & 0 & W_{T_{\dot z}} \end{bmatrix} \tag{14}$$

$$W_C = \begin{bmatrix} W_{C_{\delta_r}} & 0 \\ 0 & W_{C_{\delta_s}} \end{bmatrix} \tag{15}$$

$$R = \begin{bmatrix} \frac{1.76}{s+0.2} & 0 & 0 & 0 \\ 0 & \frac{0.6}{s+0.2} & 0 & 0 \\ 0 & 0 & \frac{9.8}{s+0.2} & 0 \\ 0 & 0 & 0 & \frac{-10}{s+0.05} \end{bmatrix} \tag{16}$$