

# STIFFNESS IMPROVEMENT OF A 1 – D.O.F CONTROLLED MAGNETIC LINEAR BEARING

## Isaias da Silva

Department of Mechatronics Engineering, Escola Politécnica of São Paulo University, Av. Prof. Mello Moraes, 2231, São Paulo, 05508-900, SP, BRAZIL (email: isaiassv@ig.com.br).

## Oswaldo Horikawa

Department of Mechatronics Engineering, Escola Politécnica of São Paulo University, Av. Prof. Mello Moraes, 2231, São Paulo, 05508-900, SP, BRAZIL (email: ohorikaw@usp.br).

**Abstract.** *In previous works authors have proposed the principle of Magnetic Linear Bearing, M.L.B. in which active control is executed in only one direction of a moving table. Motions in other directions, except for the traverse direction of the table, are restricted only by the action of permanent magnets working in attraction mode. A prototype of the M.L.B. was constructed and experiments showed the effectiveness of the proposed architecture. However, it was observed that the proposed M.L.B. has poor stiffnesses in terms of rotation of the table about directions orthogonal and parallel to the traverse direction. In order to improve these stiffnesses, authors added a pendulum to the table, i.e., using the gravity to improve the stiffnesses. In this work, a new alternative solution is presented. The new solution, called magnetic pendulum, is based on the use of an additional pair of permanent magnets: a short one, attached to the table through a stem and a long one, fixed to the base parallel to the traverse direction of the table. Both permanent magnets are arranged closely to each other and their polarity adjusted in such a way to have an attractive magnetic force. Without corrupting the stiffness of the M.L.B. in other directions, the magnetic pendulum can improve the mentioned stiffnesses. Moreover, since based on magnetic forces, the improvement is valid even without gravity. The principle of the pendulum is presented and by experiments its effectiveness is demonstrated.*

**Keywords:** *magnetic bearing, magnetic suspension, magnetic linear bearing, controlled bearing, mechatronics, magnetic pendulum*

## 1. Introduction

Active magnetic bearings, which support rotors or tables without any mechanical contact, are nowadays widely used in high-speed turbo machinery and precision machinery (see for example, Markert et al, 2002 and Nordmann et al, 2004). Recently the need for a very clean room or a vacuum chamber is increasing particularly in the microelectronics industry, where ICs of latest generation are currently produced. Also, machines used in such environments must be ultra-clean to avoid sample contamination during the handling and the processing. Besides these advantages, magnetic bearings are expected to feature new functional capabilities such as active position control of the rotor or the table, control of the force and stiffness, active vibration control and etc. Therefore, the authors have proposed, implemented and tested a new type of magnetic linear bearing (M.L.B.) (Silva and Horikawa, 2005). The bearing presented was of hybrid type. It consisted of two main parts: passive and active. The passive one used permanent magnets to ensure stable equilibrium in 4-d.o.f. of a moving table. Different of similar type of magnetic bearings, proposed by other authors (Shinshi et al., 2001 and Takeshi; Mitsunori, 2002), the developed M.L.B. was operated by permanent magnets working in attraction mode, reducing problems concerning demagnetization (Campbell, 1994). In the M.L.B., the motion of the table in the direction orthogonal to the traverse direction is unstable. This is stabilized by a control-loop composed of two electromagnetic actuators, a non-contact type gap sensor, a controller and an amplifier. This control-loop constitutes the active part of the system. The constructed M.L.B. showed capability of keeping the table fixed in the orthogonal direction with accuracy better than  $3.5\mu\text{m}$ . However, as mentioned above, the bearing table stiffnesses against rotations about the axes orthogonal and parallel to the traverse direction were poor. In order to solve this drawback, a gravity-based pendulum was added to the center of the table. Experiments in the M.L.B. showed the effectiveness of the pendulum to increasing the stiffnesses. However, this improvement was not enough and disturbances applied onto the table still generated large amplitude oscillations of the table. In order to reduce these oscillations, a magnetic pendulum solution is proposed, implemented and tested.

## 2. The principle of M.L.B.

Figure 1 shows a scheme of the proposed M.L.B.. A table having two electromagnetic actuators (a combination of an electromagnet and a permanent magnet), attached to its lateral ends, is located between two magnets fixed on the base. All permanent magnets have a cuboidal shape and their polarities are set so that an attraction force occurs between

each facing permanent magnets. The attraction force assures the stable flotation of the table in terms of translation  $y$  and also in terms of rotation  $\phi$ ,  $\theta$  and  $\psi$  about  $x$ ,  $y$  and  $z$  axes, respectively. The position of the table in transversal,  $z$  direction, is actively controlled by a system composed of two electromagnetic actuators, a non-contact type gap sensor, a controller and an amplifier. The active control is necessary in this case, since it is impossible to achieve the stability by employing only permanent magnets. This is a consequence of Earnshaw's principle (Earnshaw, 1939). Fig.1 also shows the proposed magnetic pendulum, which is composed of a stem attached to the center of the table and of a small permanent magnet fixed on its end. This magnet operates in attraction mode with a long one, fixed to the base and parallel to the traverse direction of the table.

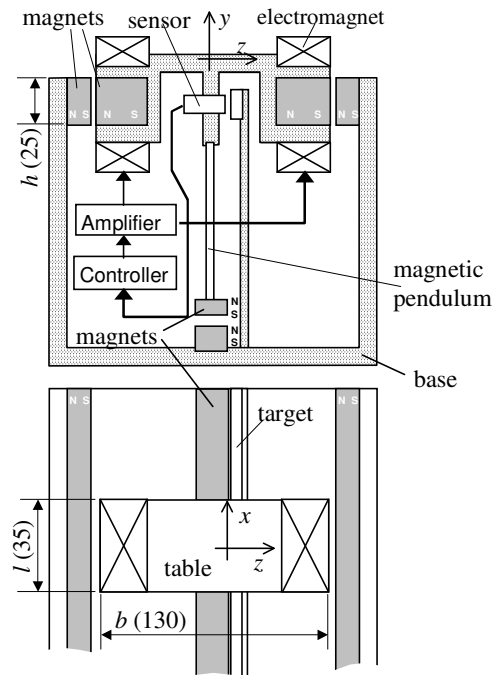


Figure 1. Configuration of the proposed M.L.B..

### 3. Stability requirements

The M.L.B. introduced in this work is based on the principle of a magnetic rotating bearing presented by authors in a previous work (Silva and Horikawa, 2000). Figure 2 illustrates the cross section of the M.L.B., showing the table tilted by  $\phi$ . Here, a few considerations done with respect to the rotary bearing assumed to be valid. Suppose the pairs of permanent magnets at the right side. As the table is tilted, the gap between permanent magnets increases at the upper side but decreases at the lower side. Thus, the magnetic attraction force  $f_z$  at the lower side becomes larger than the corresponding force  $f_z$  at the upper side. The opposite situation occurs with the pair of permanent magnets at the left side of the table. However, the tilting of the table also results a displacement of the permanent magnet fixed to the table, relative to the permanent magnet fixed to the base in  $y$  direction. This displacement generates a vertical force  $f_y$  at both extremities of the table, forcing the table back to its original position. To have a stable suspension in  $\phi$ , the moment generated by  $f_z$  must be smaller than that generated by  $f_y$ . Being  $k_z$  and  $k_y$  respectively the stiffnesses in  $z$  and in  $y$ , and  $\phi$ , a small enough angle, the momentum on the table is approximately:

$$m_\phi(\phi) = 2 \left( k_z \frac{h^2}{4} \right) \phi + 2 \left( k_y \frac{b}{2} \phi \right) \frac{b}{2} = k_z h^2 \phi + k_y b^2 \phi \quad (1)$$

However, the Earnshaw's principle (Earnshaw, 1939) states that  $k_z + k_y = 0$  (stiffness in  $x$ ,  $k_x = 0$ ). Therefore, to have a stable suspension, is required that:

$$b > h \quad (2)$$

Similarly, the stability in  $\theta$ , results in:

$$b > l \quad (3)$$

Here  $l$  is the length of permanent magnets attached to the table, Fig. 3. A stable passive suspension is achieved by satisfying (2) and (3) simultaneously. These conditions, though approximately, are shown to be reasonable since a stable suspension is obtained in the prototype, as will be shown later.

In the rotary bearing, because of the axial symmetry of the bearing, the stiffness  $k_\psi$  about  $z$ -axis was zero. However, in the M.L.B., a finite stiffness has to be achieved. This is assured by a reacting magnetic momentum that occurs when the table rotates by  $\psi$  and a misalignment occurs between facing permanent magnets.

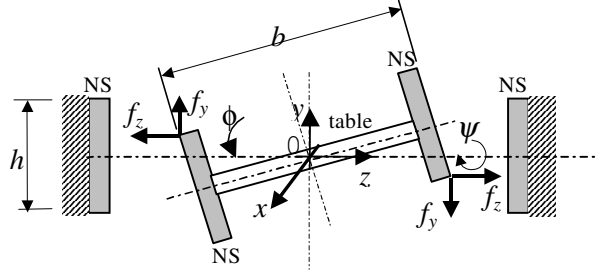


Figure 2. Table inclined by  $\phi$  about  $x$ -axis.

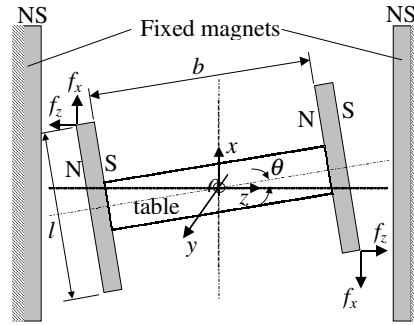


Figure 3. Table inclined by  $\theta$  about  $y$ -axis.

#### 4. M.L.B. stiffness improvement through a magnetic pendulum

Magnetic stiffness  $k_\phi$ ,  $k_\theta$  and  $k_\psi$  against rotations about the  $x$ ,  $y$  and  $z$ -axes, respectively, are provided by attraction forces between permanent magnets attached in the extremities of the table in one side and by those attached to a fixed base in another side, Fig.1.  $k_\phi$ ,  $k_\theta$  and  $k_\psi$  are derived directly from Delamare (Delamare, 1994) who evaluated the stiffness of two facing cuboidal magnets working in attraction mode.

$$k_\phi = b \cdot \left( \frac{b}{2} \cdot k_y + f_z \right) \quad (4)$$

$$k_\theta = b \cdot \left( \frac{b}{2} \cdot k_x + f_z \right) \quad (5)$$

$$k_\psi = \frac{l^2}{2} \cdot k_y \quad (6)$$

Here,  $f_z$  is the magnetic force in  $z$  and  $k_x$  and  $k_y$  are stiffnesses in  $x$  and  $y$  directions. Analytical calculations for obtaining  $f_z$ ,  $k_x$  and  $k_y$  are proposed by Akoun and Yonnet (1984). The extensive terms are not shown here because of the limitation of space.

As discussed above, in experiments made by a M.L.B. first version (Silva and Horikawa, 2005),  $k_\phi$  and  $k_\psi$  were observed to be low. In order to improve these stiffnesses a stem having a weight  $P$  and length  $a$  was attached to the center of the table, as depicted in Figs.1 and 4. This first solution was sufficient to keep the table with no tilting about  $z$ -axis. However, disturbances applied to the table, still caused oscillations of large amplitudes. Therefore, a small permanent magnet is added to the stem and this magnet attracts another one fixed to the base. This interaction produces the forces  $f_y$  and  $f_z$ , as depicted in Fig.4. As shown later, without significant corruption of the M.L.B. vertical stiffness, this new solution improves table's stiffnesses about the  $x$  and  $z$ -axes. This enhancement can be expressed in the following way. Consider the schematics shown in Fig.4. Assuming small values of  $\phi$ , the forces  $f_y$ ,  $f_z$  and  $P$  produce a moment  $m_s$  applied on the table, and improves  $k_\phi$  as follows:

$$m_s(\phi) = P \cdot \frac{a}{2} \phi + a \cdot f_y \cdot \phi + a \cdot f_z \quad (7)$$

but, by definition,  $k_\phi = \frac{dM(\phi)}{d\phi}$ , thus:

$$k_\phi = P \cdot \frac{a}{2} + a \cdot f_y \quad (8)$$

Now, inserting Eq.(8) into Eq.(4), results:

$$k_\phi = b \cdot \left( \frac{b}{2} \cdot k_y + f_z \right) + P \cdot \frac{a}{2} + a \cdot f_y \quad (9)$$

The second and the third terms, at the right side of Eq.(9), show how the stiffness  $k_\phi$  is improved by the length  $a$ , the force  $f_y$  and the weight  $P$  of the magnetic pendulum.

Using Eq.(16) similar enhancement is obtained in terms of  $k_\psi$ , the stiffness about  $z$ -axis, i.e.:

$$k_\psi = \frac{l^2}{2} \cdot k_y + P \cdot \frac{a}{2} + a \cdot f_y \quad (10)$$

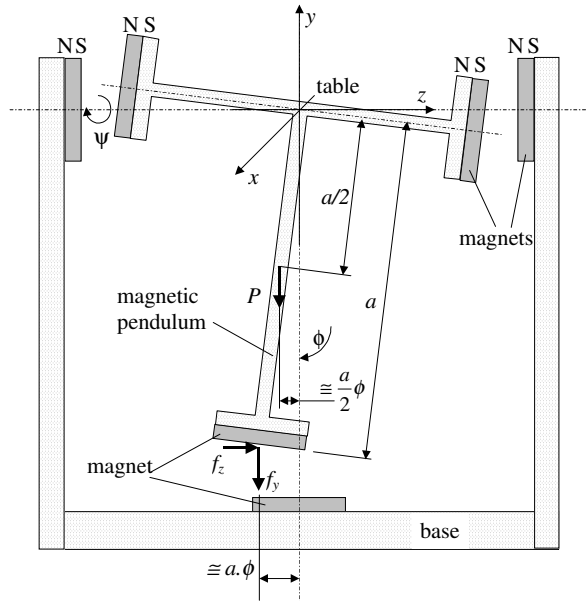


Figure 4. M.L.B. with magnetic pendulum.

## 5. System modeling and control

The control system considering the  $z$  position of the table is modeled assuming some simplifications: (a) the table remains symmetric about the  $x$ ,  $y$  and  $z$ -axes, (b) displacements are small and occur about the equilibrium position and (c) the magnetic attraction force and the electromagnetic force in the  $z$  direction can be linearized with respect to the nominal operating point of equilibrium  $(i_o, z_o)$ . The dynamic model of the M.L.B. is shown in Fig. 5. As depicted in the figure, equal current flow through both electromagnetic coils. Thus, the electromagnetic attraction and repulsion forces are obtained by mounting the coils by reversed polarity in each actuator side. The magnetic force  $f_m$  and the electromagnetic term  $f_{em}$  were linearized with respect to the displacement  $z$  and to the electric current  $i$  as follow:

$$f_m(t) = k_h z(t) \quad (11)$$

$$f_{em}(t) = k_i i(t) \quad (12)$$

Where,  $t$  is the time;  $k_h$  and  $k_t$  are the magnetic and the electromagnetic constants, respectively. The electromagnetic constant  $k_t$ , in Eq.(12), is function of nominal displacement  $z_0$ . On the other hand, considering the use of electromagnets with constant inductance  $L$  and resistance  $R$ , the dynamic behavior of the electromagnetic coil is given by the following equation:

$$L \frac{di}{dt} + Ri(t) = v(t) \quad (13)$$

Using Eqs.(11) ~ (13) the open loop transfer function of the system  $G(s)$  is obtained:

$$G(s) = \frac{Z(s)}{V(s)} = \frac{2k_t}{LM} \cdot \frac{1}{s^3 + \frac{R}{L}s^2 + \frac{k_h}{M}s + \frac{Rk_h}{LM}} \quad (14)$$

Here,  $z$  and  $M$  are the gap deviation from nominal operation point (table  $z$  direction) and the mass of the table, including the magnets, respectively.

In this system, only one gap sensor is used and the measured variable was the  $z$  position. The system described by Eq.(14) is stabilized by a PID (proportional – integral – derivative) – type controller given by:

$$G_c(s) = k \left( 1 + \frac{1}{T_i s} + \frac{T_d s}{\tau s + 1} \right) \quad (15)$$

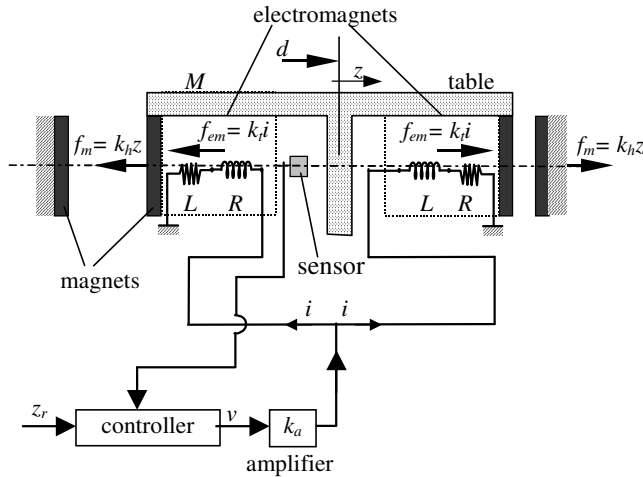


Figure 5. Magnetic linear bearing dynamic model.

Here,  $k$  is the proportional gain of the controller, and  $T_i$  and  $T_d$  are, respectively, the time constant of the integral and derivative elements. Each controller term has a specific effect on the bearing behavior. The proportional, P element makes the current changes proportional to the size of the error signal. This result in a bearing behavior similar of a spring. The force, that push back the table to the center position of the bearing, increases proportionally as the error signal of the  $z$  table position increases. A bearing system with only P element will oscillate and become unstable because there is no damping effect. This damping is introduced by the derivative, D element. The D element produces a force that is proportional to the velocity of the table in  $z$ , working as a shock absorber. Finally, the integrator, I element is introduced in order to eliminate the steady state offset from the set point. The block diagram of the control system for the  $z$ -axis direction of the bearing table is shown in Fig. 6.

## 6. Experimental results

Experiments are performed on a prototype (Fig.7) which was developed using a pair of ferrite (FeBa) permanent magnets fixed to the base and two combinations of rare-earth (NeFeBo) and ferrite permanent magnets in the table. Parameters concerning magnetic forces were calculated by finite element technique and measured experimentally. Parameters of the controller were defined by simulations of Eqs.(14) and (15). All parameters and their values are listed in Tab.1.

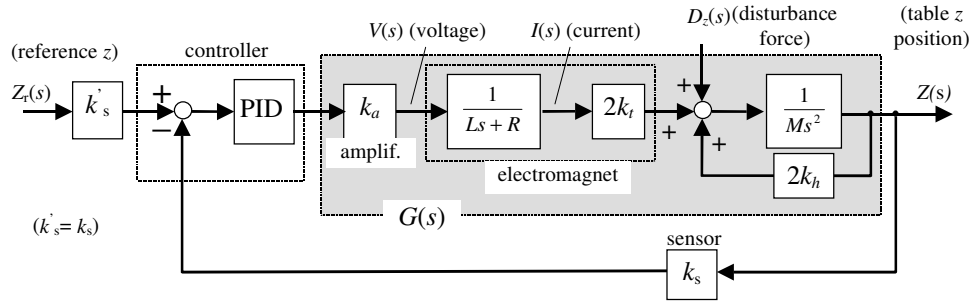


Figure 6. Magnetic bearing control system block diagram.

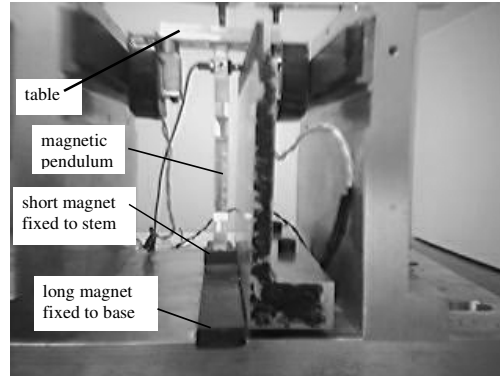


Figure 7. Prototype of the M.L.B.

Table 1. System parameters.

| System                         | Symbol       | Value                |
|--------------------------------|--------------|----------------------|
| Magnetic constant (N/m)        | $K_h$        | $11,75 \times 10^3$  |
| Electromagnetic constant (N/A) | $K_t$        | 5,14                 |
| Mass of table (kg)             | $M$          | 0,9                  |
| Sensor gain (V/m)              | $K_s = K_s'$ | $5 \times 10^3$      |
| Amplifier gain                 | $K_a$        | 10                   |
| Nominal gap (m)                | $Z_0$        | $1,5 \times 10^{-3}$ |
| Inductance (H)                 | $L$          | $65 \times 10^{-3}$  |
| Resistance ( $\Omega$ )        | $R$          | 44                   |
| <b>Controller</b>              |              |                      |
| $K_p$                          |              | 10,8                 |
| $T_i(s)$                       |              | 0,0131               |
| $T_d(s)$                       |              | 0,0061               |

Figures 8 to 10 show the experimental stiffness of the prototype in the  $y$  direction and about  $x$  and  $z$  axes, respectively. In the past work, the M.L.B. with a gap of 1.5mm between the fixed and movable magnets resulted in  $k_y \approx 5500 \text{ N/m}$ ,  $35 \leq k_\phi \leq 52 \text{ Nm/rad}$  and  $2 \leq k_\psi \leq 2.5 \text{ Nm/rad}$ , with and without stem (Silva and Horikawa, 2005). These values can be improved by optimizing the characteristics of the permanent magnets. However, in this work,  $k_\phi$  and  $k_\psi$  are improved by a so-called magnetic pendulum as shown in Eqs.(9) and (10). The pendulum consists of a stem (length  $a=140 \text{ mm}$ , weight  $P=1,5 \text{ N}$ ) and a small ferrite magnet of  $10 \times 25 \times 25 \text{ mm}$ . This small magnet exerts an attraction force against a long ferrite magnet of  $10 \times 25 \times 300 \text{ mm}$ , fixed to the base (see Figs.1, 4 and 7). The intensity of this force is determined using equations described in Akoun and Yonnet (1984). A gap of 2.5mm results in  $f_y \approx 2 \text{ N}$ . Improvements obtained by the pendulum are shown in Figs. 9 and 10.

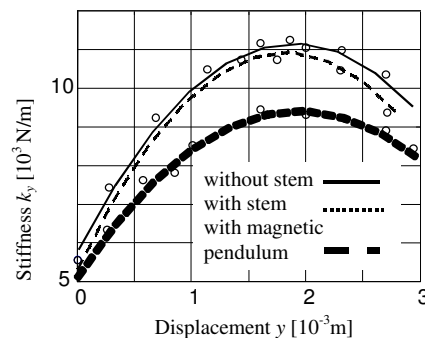


Figure 8. Bearing stiffness in  $y$ -direction.

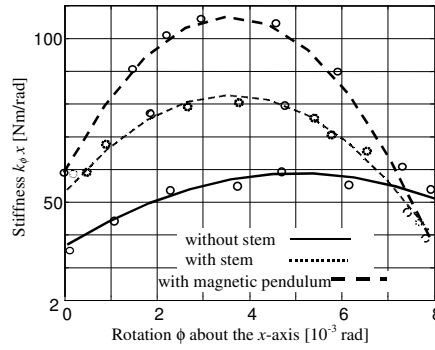


Figure 9. Bearing stiffness about  $x$ -axis.

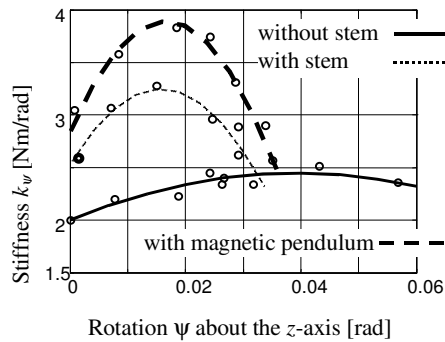


Figure 10. Bearing stiffness about  $z$ -axis.

Figure 11 shows the position of the table in  $z$  direction, with no disturbance being applied to the table and without moving the table along  $x$ . Despite the remained continuous vibration caused by electrical noises, a positioning accuracy better than  $3.5\mu\text{m}$  is obtained.

The controller is also equipped with an input for a reference  $z$  position ( $Z_r$ , Fig.6). Experiments show that the bearing is capable of a  $200\mu\text{m}$  stepwise response, with a response time shorter than  $0.05\text{s}$  (Fig.12). This shows the capability of this bearing to execute fast and precise positioning of the table. By this, the bearing can, for example, compensate systematic motion errors of the table occurring in the  $z$  direction, along its travel. Figs. 13a and 13b show oscillations of the table about the  $x$  and  $z$ -axes when the early mentioned stepwise is applied onto the table along  $z$  direction. These figures indicate how the magnetic pendulum reduces the amplitude of oscillations of the table. The amplitude of oscillations are decreased by almost 70% of that about both  $x$  and  $z$  axes.

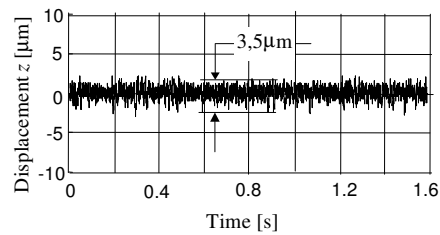


Figure 11. Vibration of the table in  $z$ -direction.

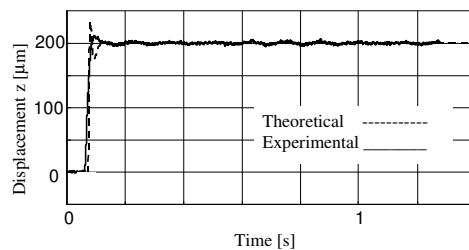


Figure 12. Step response.

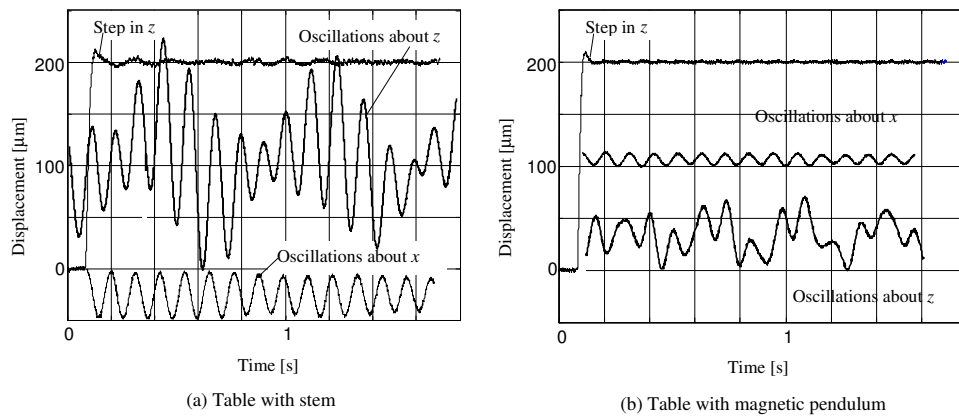


Figure 13. Table oscillations about  $x$  and  $z$ -axes.

## 7. Conclusions

This work presented the stiffness improvement of a novel magnetic linear bearing M.L.B. proposed by the authors in past works. The presented M.L.B. conducts active control in only 1.d.o.f of a magnetically levitated table. Remaining degrees of freedom are restricted only by the action of permanent magnets operating in attraction mode. The principle of this M.L.B. was described, the new proposed magnetic pendulum was developed and the most relevant points to be observed in its design were presented. Finally, by experiments, it was shown that the proposed pendulum is capable of an improvement of more than 70% in the bearing stiffness. Despite the use of the pendulum, the M.L.B. keeps the original features, being capable of: 1) achieving a stable suspension of the table; 2) keeping the table in a fixed  $z$  position with accuracy better than  $3.5\mu\text{m}$ ; 3) executing a fast and precise positioning of the bearing table. The proposed pendulum can be applied for other type of M.L.Bs. It is capable of improving the stiffness of the bearing concerning rotation of the table affecting minimally the bearing stiffness in other directions and it is effective even under absence of gravity.

## 8. Acknowledgment

The authors acknowledge Prof. Richard Markert, Department of Applied Mechanics, Darmstadt University of Technology (TUD), for his valuable advices. This work was partially sponsored by CNPq-Brazil and FAPESP – Brazil and done during the research stay of the first Author at TUD.

## 9. References

- Akoun, G., Yonnet, J.P., 1984, “3D Analytical Calculation of the Forces Exerted Between Two Cuboidal Magnets”, IEEE Trans. on Mag., Vol. 20, pp. 1962-1964.
- Campbell, P., 1994, “Permanent Magnet Materials and Their Applications”, Vol.1, Cambridge University Press, 191p.
- Delamare, J., 1994, “Suspensions Magnétiques Partiellement Passives”, Thèse de doctorat, INPG, Grenoble, France.
- Earnshaw, S., 1939, “On Nature of Molecular Forces”, Trans. Cambridge Philosophical Society, Vol.7, Part 1, pp. 97 - 112.
- Markert, R., Skricka, N. and Zhang, X., 2002, “Unbalance Compensation on Flexible Rotors by Magnetic Bearing Using Transfer Functions. Proc. of the 8th ISMB, pp.417- 422.
- Nordmann, R. and Aenis, M., 2004, “Fault Diagnosis in a Centrifugal Pump Using Active Magnetic Bearings”, Int. J. of Rotating Machinery, Vol.10, No.3, pp.183-191.
- Shinshi, T., Choi, K.B., Cho, Y.G., Li, L., Shimokohbe, A., 2001, “Stabilization of Permanent magnet Repulsive Forces Type Levitation Table with One Degree-of-Freedom Control”, Proc. of the 6th ISMST.
- Silva, I., Horikawa, O., 2000, “An 1-dof Controlled Attraction Type Magnetic Bearing”, IEEE Trans. on Industry Applications, Vol.36, No.04, pp.1138 - 1142.
- Silva, I., Horikawa, O., 2005, “Experimental Development of a 1-D.O.F. Controlled Magnetic Linear Bearing”, Proc. of the Intermag 2005.
- Takeshi, M., Mitsunori, A., 2002, “Repulsive Magnetic Bearing using a Piezoelectric Actuator for Stabilization”, Proc. of the 8th ISMB, pp. 549 – 554.

## 10. Responsibility notice

The authors are the only responsible for the printed material included in this paper.