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DYNAMICS AND JACOBIAN ANALYSIS OF A PARALLEL ARCHITECTURE ROBOT: THE HEXA

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Abstract. *Many applications in the field of production automation, such as assembly and material handling, require machines capable of very high speeds and accelerations. Since 1965 with Stewart's work the parallel robots proved to be a strong supplement to the serial robots. The parallel robots are able to work on some tasks with a much better performance. The study of a robot dynamics is necessary for its mechanical design and synthesis, providing the forces and moments that must be resisted by the joints, links and actuators. The model of the inverse dynamics developed in this work is based on Newton-Euler's formulation and is capable of calculating forces and moments along the Hexa architecture robot during its operation. This work also presents a general method to calculate the Jacobian matrix for parallel robots and to search for the singular positions for both direct and inverse kinematics. Tools are developed to analyze the efforts transmitted from the actuators to the mobile platform. The analysis of the efforts transmitted leads to the calculation of m-dimensional ellipsoids of forces around the end-effector's point of interest for any point of the Hexa reachable workspace. This analysis provides the information for the mapping of the workspace to obtain performance indexes for each one of Hexa's six degrees of freedom which can be used for optimization purposes.*

Keywords: *parallel architecture, robot, dynamics, Jacobian, Hexa.*

1. Introduction

The focus of this work is the six degree of freedom (d.o.f) Hexa robot, proposed by Uchiyama (1994), which is actually an extended concept of the 3 d.o.f. Delta robot, proposed by Pierrot (1991). In the Hexa robot the actuated revolute joints are responsible for the movement of six rods (actuated rods), which are linked to other six rods (passive rods) by spherical joints. Lastly, these passive rods are linked to the moving platform also by spherical joints. By controlling the actuated rods movement it is possible to control the moving platform that carries the end-effector. Figure 1 presents a Hexa schematic and a real Hexa machine built by IWF.

A well know problem in robotics is the singularity detection and avoidance, especially in parallel robots. In some cases for those types of robots, singularities can change their operational modes or cause critical performance problems like the complete loss of rigidity. Also, by studying the Jacobian matrix eigenvalues and vectors it is possible to reach the force ellipsoid concept, or what a single unit of torque in the motors can accomplish in the moving platform, which is one of the most important performance metrics. In this work, we present a method to easily obtain the Jacobian matrix and some studies in force analysis and singularity detection.

Also a vital step in the project of a robot is to know its dynamical behavior, which allows knowing the forces and torques demanding in each mechanical element during the operation. The mechanical project of the equipment must consider dynamical responses. Another purpose is to provide a fast solving equation system for the control algorithm, adequate for real-time processing. Some of the state-of-the-art models are even capable to measure and correct trajectory errors caused by external forces, although that is not the focus of this work. The dynamics model that this work focuses is the Hexa's inverse dynamics, i.e., knowing the data related to the moving platform (speed, position, acceleration), the other needed information (for instance, motor's torques) are calculated.

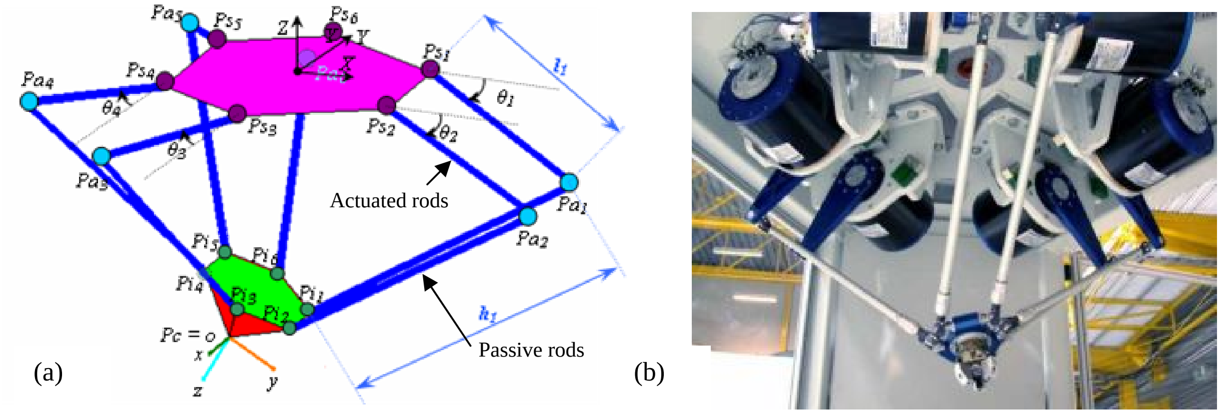

Figure 1. (a) Hexa's schematic; (b) An example of a real Hexa machine (from IWF).

As shown in Fig. 1, the set of points $Pi_j = \{Pi_1, Pi_2, ..., Pi_6\}$ represents the centers of the spherical joints that link the passive rods to the moving platform; the set $Pa_j = \{Pa_1, Pa_2, ..., Pa_6\}$ represents the centers of the universal joints that links the actuated rods to the passive rods; finally the set $Ps_j = \{Ps_1, Ps_2, ..., Ps_6\}$ represents the centers of the revolute actuated joints. The point Pc is the point in which all movements are based from (for instance, a tool tip). The coordinate system *o-xyz* has its origin in Pc and is fixed to the moving platform. *O-XYZ* is global, being fixed in the base. The set of actuated and passive links are, respectively, $l_j = \{l_1, l_2, ..., l_6\}$ and $h_j = \{h_1, h_2, ..., h_6\}$ and the set of angles $\theta_j = \{\theta_1, \theta_2, ..., \theta_6\}$ represents the actuation angles.

2. Jacobian Matrix and Singularities

First, the Jacobian Matrix for the Hexa robot will be calculated. Considering the schematic presented in Fig. 1, follows that:

$$\overline{OP_C} + \overline{P_C Pi_j} = \overline{OPs_j} + \overline{Ps_j Pa_j} + \overline{Pa_j Pi_j} \tag{1}$$

Deriving Eq. (1), follows:

$$\mathbf{v_{Pc}} + \boldsymbol{\omega}_\mathbf{p} \times \mathbf{b}_j = \underbrace{\dot{\boldsymbol{\theta}}_j \times \mathbf{l}_j}_{\text{Derivate of } \overline{\text{PsjPaj}}} + \underbrace{\dot{\boldsymbol{\theta}}_j \times \mathbf{l}_j + \boldsymbol{\omega}_{\mathbf{h}j} \times \mathbf{h}_j}_{\text{Derivate of } \overline{\text{PajPij}}}, \text{ for } j = 1,\ldots,6 \tag{2}$$

The vectors presented in Eq. (2) are: $\mathbf{v_{Pc}}$, $\boldsymbol{\omega}_\mathbf{P}$, $\boldsymbol{\omega}_{\mathbf{h}j}$, $\dot{\boldsymbol{\theta}}_j$, $\mathbf{l_j}$, $\mathbf{h}_j$ and $\mathbf{b}_j$ representing, respectively, the linear velocity of point Pc, the angular velocity of the moving platform, the angular velocity of the passive rods, the angular velocity of the actuated joints, the actuated rods, the passive rods, and the distance $\overline{P_C Pi_j}$.

Since the angular velocities of the passive rods are not relevant we multiply Eq. (2) by $\mathbf{h_j}$, leading to:

$$\mathbf{h}_j \mathbf{v_{Pc}} + \boldsymbol{\omega}_\mathbf{p}\left(\mathbf{b}_j \times \mathbf{h}_j\right) = 2\dot{\boldsymbol{\theta}}_j\left(\mathbf{l}_j \times \mathbf{h}_j\right), \text{ for } j = 1,\ldots,6 \tag{3}$$

$$\mathbf{h}_j \mathbf{v_{Pc}} + \boldsymbol{\omega}_\mathbf{p}\left(\mathbf{b}_j \times \mathbf{h}_j\right) = 2 \cdot \frac{\dot{\boldsymbol{\theta}}_j}{\left\|\dot{\boldsymbol{\theta}}_j\right\|}\left(\mathbf{l}_j \times \mathbf{h}_j\right)\dot{\boldsymbol{\theta}}_j, \text{ for } j = 1,\ldots,6 \tag{4}$$

The left side of Eq. (4) is dependent on linear and angular velocities of the moving platform and the right side is dependent on the actuator's velocity. Equation (4) can be written in matricidal form, separating the Jacobian matrix into two parts: one associated with the direct kinematics and another associated with the inverse kinematics, like proposed by Gosselin and Angeles (1990). Therefore, follows Eq. (5) where **Jx** is the direct kinematics associated matrix and **Jq** is the inverse kinematics associated matrix. Both matrixes have dimensions 6x6.

$$\underbrace{\left[\mathbf{h}_j \quad \mathbf{b}_j \times \mathbf{h}_j\right]}_{\mathbf{Jx}} \cdot \underbrace{\begin{Bmatrix} \mathbf{v_{Pc}} \\ \boldsymbol{\omega}_\mathbf{p} \end{Bmatrix}}_{\dot{\mathbf{x}}} = \underbrace{\left[diagonal\left(2 \cdot \frac{\dot{\boldsymbol{\theta}}_j}{\left\|\dot{\boldsymbol{\theta}}_j\right\|}\left(\mathbf{l}_j \times \mathbf{h}_j\right)\right)\right]}_{Jq} \underbrace{\dot{\boldsymbol{\theta}}_j}_{\dot{\mathbf{q}}}, \text{ for } j = 1,\ldots,6 \tag{5}$$

If the determinant of the matrixes **Jx**, **Jq**, or both equals zero, then the position in study is a singularity with concern to the associated kinematic of each matrix (direct, inverse or both). The Jacobian matrix, by convention, is the inverse of the serial ones, being written as $\mathbf{J} = \mathbf{Jq}^{-1}\,\mathbf{Jx}$. Matrix **Jq** is diagonal.

3. Force and Velocity Transmission Analysis

Consider that the stiffness of each actuated joint is several times inferior to the machine structural stiffness. Consider also that $\boldsymbol{\tau}$ represents their torque vector and $\boldsymbol{\chi}$ their stiffness matrix (diagonal), composed by each joint stiffness k_j. If $\Delta\mathbf{q}$ represents the actuated joints deflection, then:

$$\boldsymbol{\tau} = \chi\Delta\mathbf{q} \tag{6}$$

If it is defined that **x** represents Hexa’s pose (platform’s position + orientation), it is possible to establish that:

$$\Delta\mathbf{q} = \mathbf{J}\Delta\mathbf{x} \tag{7}$$

Admitting that any energy loss due to friction forces at the joints are neglected, as well as the gravitational forces, the virtual work δW of all active forces can be written as:

$$\delta W = \boldsymbol{\tau}^{\mathrm{T}}\delta\mathbf{q} - \mathbf{F}^{\mathrm{T}}\delta\mathbf{x} = 0 \tag{8}$$

Where **F** represents the forces at the moving platform and can be obtained as shown in Eq. (9):

$$\mathbf{F} = \mathbf{J}^{T}\boldsymbol{\tau} \tag{9}$$

One of the most important performance metrics is the analysis of the forces magnitude produced at the moving platform by a unit of force on the actuators. In other words, the question to be answered is: What magnitude of forces can be resisted at the interest point Pc considering that $\|\boldsymbol{\tau}\| = 1$. Thus,

$$\boldsymbol{\tau}^{T}\boldsymbol{\tau} = 1 \Rightarrow \mathbf{F}^{T}\mathbf{J}^{-1}\mathbf{J}^{-T}\mathbf{F} = 1 \Rightarrow \mathbf{F}^{T}[\mathbf{J}^{T}\mathbf{J}]^{-1}\mathbf{F} = 1 \tag{10}$$

This equation represents an m-dimensional ellipsoid, where m is the number of degrees of freedom of the moving platform. The matrix $[\mathbf{J}^{T}\mathbf{J}]^{-1}$ is positive definite, so its eigenvectors are orthogonal to each other. The main ellipsoid axes coincide with the matrix eigenvectors and their lengths are inversely proportional to the eigenvalues of this matrix.

The ellipsoid format depends on the point of the workspace considered as well as the moving platform’s orientation. However, the closer this ellipsoid is to a sphere, the better the forces transmission relationship are. In the case that a sphere is reached, it is said that an isotropic transformation occurred (best force transmission regarding directions, although for magnitude there is need for further analysis).

Being n the number of actuators, in an isotropic transformation an n-dimensional sphere in the joint’s space is mapped into an m-dimensional sphere in the end-effector’s space. In the same way, in a singularity position, the n-dimensional ellipsoid is mapped into a cylinder (one of the eigenvalues is zero), or in an ellipsoid that has one of its axis reduced to a point (one of the eigenvalues is infinite).

If the eigenvalues of the matrix $[\mathbf{J}^{T}\mathbf{J}]^{-1}$ are described as the set $\lambda_1, \lambda_2 \ldots, \lambda_6$; the eigenvectors are $\boldsymbol{\xi i}_1, \boldsymbol{\xi i}_2 \ldots, \boldsymbol{\xi i}_6$, for $i = 1, 2 \ldots, 6$ and the forces and torques are $F_j = \{f_x, f_y, f_z, T_x, T_y, T_z\}$, the ellipsoid equation will be:

$$\sum_{i=1}^{6}\left\{\lambda_i \cdot \left[\sum_{j=1}^{6}\left(\xi_{ij} \cdot F_j\right)\right]^2\right\} = 1 \tag{11}$$

In Eq. (11), ξ_{ij} represents one of the components of each eigenvector. The same is valid for F_j. For example: $F_2 = f_y$ and ξ_{34} represents the forth component of the third eigenvector found.

Another analysis possible to be made is that when a parallel architecture robot is near a singularity, a small movement of the actuators can generate a high speed at the end-effector. This implies in a reduction in the positioning accuracy near those positions because the measurement sensors resolution is amplified several times. Besides that there is also a speed amplification which is equivalent to a loss of rigidity in that particular direction.

Some researches such as Chablat and Wenger (2003) studied the effects of the speed amplification and proposed the mapping of the end-effector speed ellipsoid based on the actuators speed to establish limits among the eigenvalues of the ellipsoid matrix. Those limits also imply constraints in the useful machine workspace in order to enhance the performance during the operation. In the work of Chablat and Wenger (2003) it was suggested that the eigenvalues of $\mathbf{J}^T\mathbf{J}$ should be kept between 1/9 and 9 as a method of smoothness and optimization of the velocities transmission and eliminating singularities from the machine's workspace. For these studies, it was used that:

$$\dot{\mathbf{q}}^T\dot{\mathbf{q}} \le 1 \Rightarrow \dot{\mathbf{x}}^T[\mathbf{J}^T\mathbf{J}]\dot{\mathbf{x}} \le 1 \tag{12}$$

For the purpose of this work and the continuation of the studies done so far, the approach of the Force Ellipsoid and the Chablat and Wenger (2003) approach seem much more feasible to be applied in synthesis and optimization.

4. Inverse Dynamics

The dynamics analysis of a parallel architecture is always difficult if compared to an equivalent serial one, due the existence of several kinematic chains all connected by the moving platform.

Several approaches were used to describe the problem and prepare a model, including the Newton-Euler formulation, the Lagrange's formulation and the Virtual Work Principle; each method carries with it some advantages and disadvantages.

In the Newton-Euler formulation, the reaction forces and torques between all moving parts, including actuators demands can be easily calculated, providing an excellent model for mechanical projects. Although, due the high number of equations and their high complexity, the computational efficiency of the model is poor, making it difficult to use in real-time control.

In the Lagrange's formulation it is possible to obtain a much more computational efficient model, eliminating the reaction forces that are of no interest to a control algorithm. Although, due the imposed constraints of a closed loop kinematic chain, it becomes very difficult to derive the equations using independent generalized coordinates. To solve these problems, some researches recommend the use of extra coordinates in addition to the Lagrange's multipliers (Tsai, 1999).

The Virtual Work Principle has a very good cost / benefit relationship, what allows knowing the model main information without a high computational cost.

There is still no consensus proving which formulation is best to describe parallel robot dynamics. Some researches like Tsai (1999) recommend the use of the Virtual Work Principle. Others, like Knapp e Cobet (2000) used Lagrange for their Hexaglide machine and obtained excellent results. Dasgupta e Mruthyunjaya (1998a and 1998b) preferred the Newton-Euler approach and also obtained a successful model.

For this work, due the main objective and focus is on the synthesis problem (mechanical project) the Newton-Euler approach was chosen.

4.1. Newton-Euler Formulation

Next is presented a roadmap for the Newton-Euler formulation. The main steps to obtain the Newton-Euler formulation model are:

1. Derive the inverse kinematics equations, obtaining the velocities and accelerations in all interest points.
2. Draw the free body diagram, isolating all main components and designating action / reaction forces and torques in the contact points.
3. Apply the Angular Moment Theorem and Newton's 2nd Law to the robot's passive rods for all kinematic chains (cut them open)
4. Unite the closed loop chain equations in one single system of equations to obtain the moving platform reaction forces with the rods.
5. Derive the motor action forces and ground forces equations.

4.1.1. Step 1: Inverse Kinematic Model

The inverse kinematic model used in this work use a geometric formulation as described by Tsai (1999). Due to the necessity of knowing the velocity and acceleration in the interest points, the analysis needs to go further. There are two basic equation needed to perform step 1. Consider A and B as generic points that belong to the same rigid body C. If $\mathbf{v}$ represents the linear velocity vector and $\boldsymbol{\omega}$ the angular velocity vector, it is possible to write:

$$\mathbf{v}_{\mathbf{B}} = \mathbf{v}_{\mathbf{A}} + \boldsymbol{\omega}_{\mathbf{C}} \times \overline{BA} \tag{13}$$

$$\dot{\mathbf{v}}_{\mathbf{B}} = \dot{\mathbf{v}}_{\mathbf{A}} + \dot{\boldsymbol{\omega}}_{\mathbf{C}} \times \overline{BA} + \boldsymbol{\omega}_{\mathbf{C}} \times \left(\boldsymbol{\omega}_{\mathbf{C}} \times \overline{BA}\right) \tag{14}$$

In the pool of known variables, there are: angular velocity and acceleration of the moving platform; linear velocity and acceleration of the interest point Pc (which belongs to the moving platform); velocity and acceleration of the fixed base (equal to zero). Using Eq. (13) and Eq. (14) it is possible to derive both velocities and accelerations to the set Pi_j

(j varies from 1 to 6). From this point beyond there are just 4 unknown vector variables (velocity and acceleration of the set Pa_j and angular velocity and acceleration of the passive rods h_j) and 2 real variables (velocity and acceleration magnitudes of the actuated joint) in each kinematic chain.

In summary, there are 14 real variables to be found (4 x 3 + 2), but there are only 12 equations from Eq. (13) and Eq. (14) applied to the passive and actuated rods. The piece of data missing is that there are two arbitrary variables: the angular velocity and acceleration of the passive rods around their own axes. Applying transformation matrixes to convert the global coordinates into local ones (passive rods h_j), 2 variables are eliminated (considered zero). With this insight, the velocities and accelerations of all interest points are calculated.

4.1.2. Step 2: Free Body Diagram

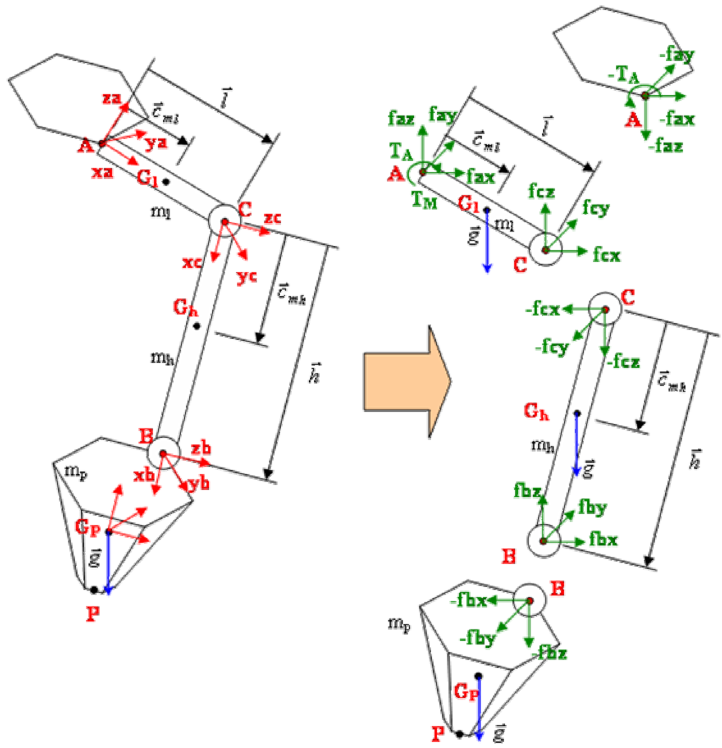

Figure 2. Hexa's free body diagram. For simplicity, the notation was changed: $Ps_j = A$, $Pi_j = B$, $Pa_j = C$, $Pc = P$. G indicates the gravity centers points (which have their relative position measured by c_{mh} and c_{ml} for both types of rods); $\vec{g}$ the gravity, m the mass, T the torques. External forces are not outlined.

The free body diagram is also important to remind how many unknown variables do exist: forces at the rods h_j (2 per rod = 12 vectors); ground forces at the fixed base (6 vectors); torques at the fixed base (2 x 6 = 12 vectors). A total of 30 vectors need to be calculated.

4.1.3. Step 3: Apply the Theorems to the Passive Rods (Angular Moment and Newton's 2nd Law)

Both theorems are important to obtain a fully determined system. Their equations are, respectively, described by Eq. (15) and Eq. (16):

$$\sum \mathbf{T}_O = \dot{\mathbf{H}}_O \tag{15}$$

$$\sum \mathbf{F} = m\mathbf{a} \tag{16}$$

Equation (15) indicates that the sum of Torques applied in an object with respect to a point O, $\mathbf{T_O}$, equals the object's angular moment at the same point ($\mathbf{H_O}$). Equation (16) indicates that the sum of Forces applied at an object, $\mathbf{F}$, equals the product between its own mass, m, and acceleration $\mathbf{a}$ (2nd Newton's Law).

Using the free body diagram presented in Fig.2 and Eq. (15) for point C, we calculate the angular moment for the rods h_j. Using also transformation matrixes to allow the calculations to be done in the rod's own coordinate system (one of the system's axis follows the rod's axis) the equations obtained possess only two unknown variables: the forces perpendicular to the rods at point B. These variables are found, and only the forces along the rod's axis remain unknown. Another purpose of the transformation matrixes is to have an equation were the inertia matrixes are fixed.

Using Eq. (16) it is also possible to obtain six more equations relating both force vectors applied to rods h_j.

4.1.4. Step 4: Uniting the chains

To unite the chains Eq. (15) and Eq (16) must be applied to the moving platform. The easiest point to apply Eq. (15) is the platform's center of mass, *Gp* (see Fig. 2). The forces at the joints that link the moving platform to the passive rods are almost known (only one component is missing – the forces component along the passive rod's axis). Both equations together produce a linear system made by six equations and as explained before, there are only six unknown variables, leading to the calculation of the last component of forces $\mathbf{f_b}$ (see Fig. 2).

As for the forces at the joints between passive and actuated rods, they are calculated using Eq. (16) applied in step 3.

4.1.5. Step 5: Motors and Ground Forces

In the last step, Eq. (15) and (16) are applied to the active rods. From Eq. (15) applied at the point C the torques at point A are obtained (including actuation torque) and from Eq. (16) the ground reaction forces are calculated.

5. Results

Two tool sets were built using Matlab: one for the force ellipsoid and another for dynamics. For the force ellipsoids, the tool was build using the machine's simulator developed previously to this work, enabling a real time view of the results depending on the machine's current pose. Fig. 3 illustrates the force analysis (due the purpose of illustration the scales were altered to allow a more comprehensive view). Fig. 4 presents the dynamics tool.

5.1. Simulator: Force Ellipsoid Tool

As an example, consider the case presented in Fig. 3: Hexa's platform is moving through the global coordinate X, from 0 mm to 200 mm (almost in the workspace border). To accomplish this calculation, Eq. (11) was used. The torques at the mobile platform were considered zero, so the forces produced at the motors are focused in generating translational movement and resisting external forces only. Observe that the initial ellipsoid diminishes and deforms when Hexa moves to a point near the workspace border, reaching a portion of space were the transmitted forces are very small. Hexa's geometry parameters are also presented in Fig. 3. Hexa's geometry parameters used here are the following:

- Side of the moving platform and tool length: 100 mm each;
- Length of the rods: h = 500 mm, l = 300 mm;
- Distance between parallel actuated rods l: 200 mm.

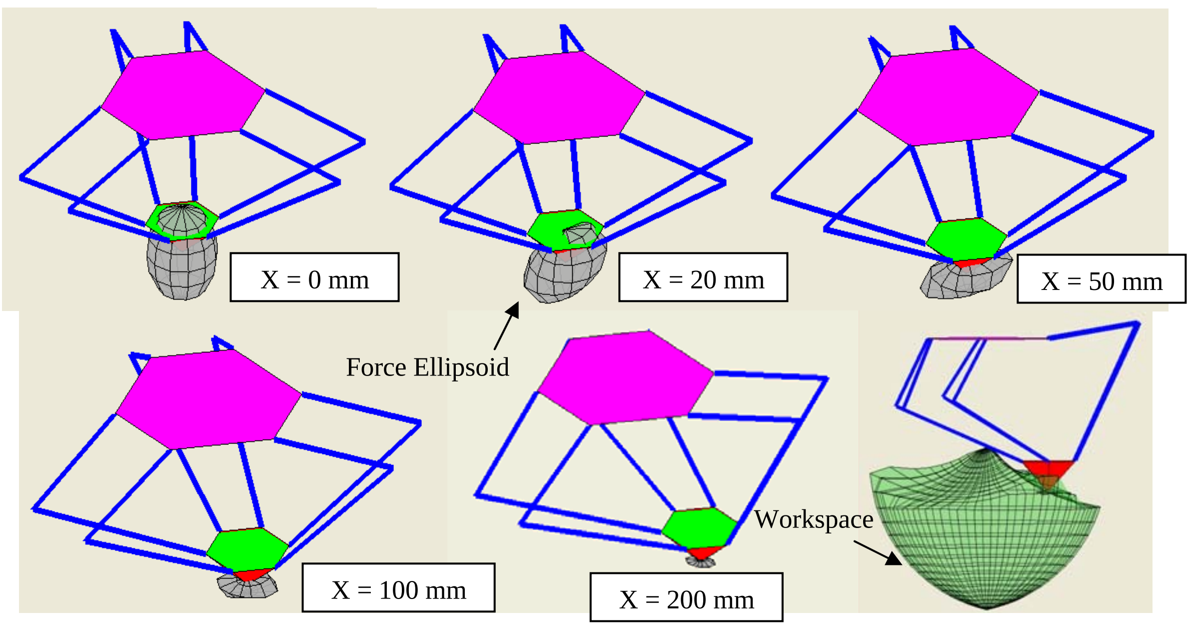

Figure 3. Hexa Force Ellipsoids in a few positions along X global axis

5.2. Dynamics Tool

For the inverse dynamics calculation, a tool set was made to allow knowing the position, velocity and acceleration of the main interest points and also forces and torques. As inputs, the tool needs the platform's pose, geometrical characteristics (rod's lengths, platform size, and others), external forces being applied, masses and inertia matrixes. Figure 3 presents the developed tool and also the input data used to calculate the main forces presented on Fig. 4. The same geometric parameters used in item 5.1 are used here.

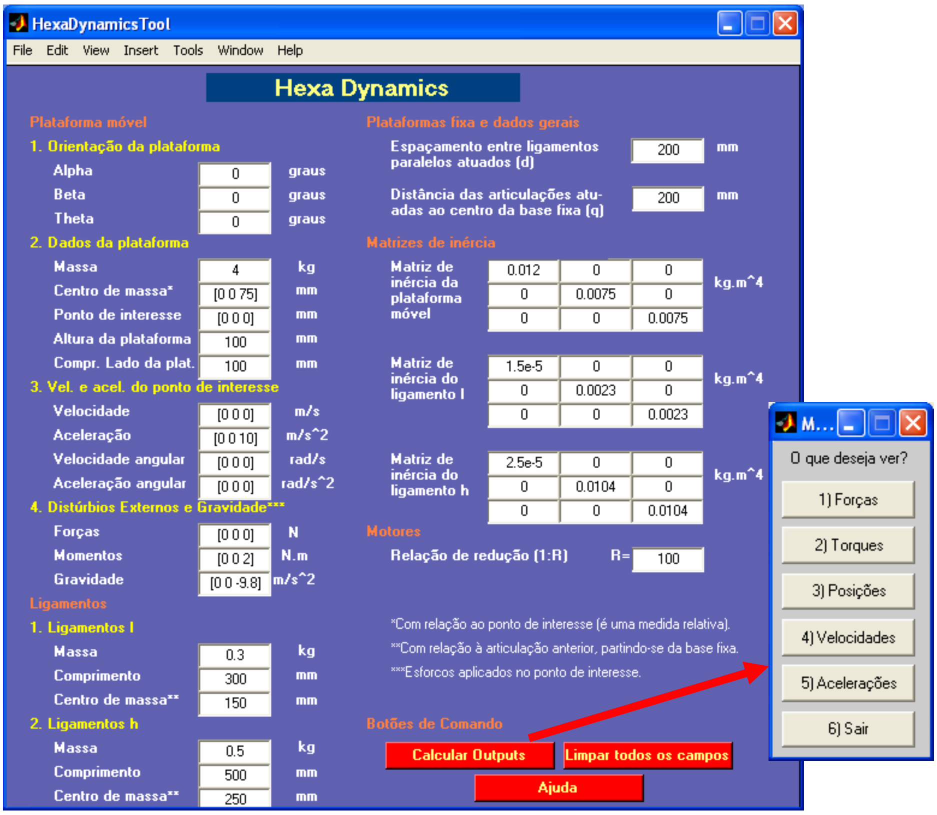

Figure 3. Dynamics Tool developed using Matlab. There are a few outputs that the user can request, such as forces, torques, velocities, accelerations.

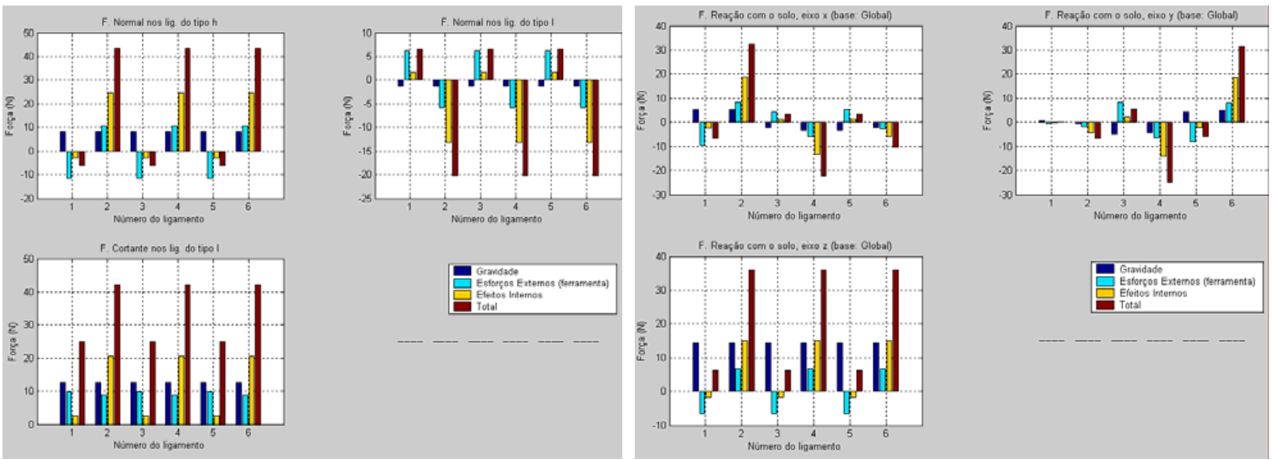

Figure 4. An example of the tool's outputs.

6. Conclusions

Both the Jacobian analysis and the dynamic modeling provide important tools for the synthesis and the optimization of new mechanisms. The Jacobian analysis provides strong information for singularity avoidance, even by project or during the operation and the performance measurement through the force transmission analysis. The dynamic model enables deeper studies for the machine operation and further knowledge of the forces and momentums involved during several cases of movement. The dynamic model developed here is not fast enough to be used in a real-time control scheme, being used for design purposes.

7. Next Steps

The next step in our work will the development of a Hexa's synthesis and optimization tool based on the studies described here. After the parameters definition, the mechanical project will be started, focusing specially in the joints (the most critical mechanical parts for a parallel machine). As for the optimization strategy, the goal will be finding Hexa's geometrical parameters that maximize the ellipsoid volumes inside its workspace and also makes the ellipsoids closer to a sphere. As constraints, a minimum workspace size will be demanded and a minimum size for the ellipsoid axes will be imposed.

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