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APPLICATION OF WAVELET TRANSFORM TO DETECT FAULTS IN ROTATING MACHINERY

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Abstract. The field of fault diagnostic in rotating machinery is vast, including the diagnosis of items such as rotating shafts, rolling element bearings, couplings, gears and so on. The different types of faults that are observed in these areas and the methods of their diagnosis are accordingly great, including vibration analysis, model-based techniques, statistical analysis and artificial intelligence techniques. However, they have difficulties with certain applications whose behavior is non-stationary and transient nature. In the present study, a rotor system model capable of describing the theoretical dynamic behavior resulting from shaft misaligned and unbalanced rotor is developed during run-up motion. A comparison between experimental and numerical results clearly indicates that validity of the theoretical model was successfully verified for fault misalignment. The results show that the fault mechanical looseness and the effect of the evolution of fault misalignment can be monitored and detected during the machine run-up without passing by critical speed. Extensive numerical and experimental results show that ability and feasibility of the application of wavelet analysis in the diagnostic of faults inserted in the experimental set-up is very suitable to non-stationary signal analysis. Results show that the sensitivity and efficiency in the fault diagnostic using transient response during start-up is higher than steady state response of rotating machinery.

Keywords: *Wavelet transform, Condition monitoring, Fault detection, Rotating machinery, Transient response.*

1. Introduction

Using vibration analysis on rotating machinery enables the early detection of faults before breakdown. This will reduce economical losses to production and equipment, saving industry millions of dollars in machine down time. The evaluation of the changes in vibration response, critical speeds and stability of a machine have become an important part of most maintenance predictive programs. This will enable the condition monitoring and diagnostic of a machine; therefore repairs can be planned and performed economically. Vibration signal analysis has been extensively used in the fault detection and condition monitoring of rotating machinery. Many schemes predictive maintenance and machinery diagnostic systems use the condition machine to identify and classify faults through the analysis of vibration signals.

The vibration in rotating machinery is mostly caused by unbalance, misalignment, mechanical looseness, rubs, shaft crack, and others malfunctions. The majority of the studies available in the literature have paid attention to diagnostic on these faults by analysing the steady state vibrations (Wauer, 1990; Xu and Marangoni, 1994; Sekhar and Prabhu, 1995 and Hamzaqui et all, 1998). But it is investigated in earlier research (Imam, 1989; Prabhakar et all, 2001) that it is easier to detect cracks using transient response during run-up or shut-down of a machine. Other works include references (Smalley, 1989; Gasch, 1993; Al-Bedoor, 2000 e Adewusi, 2001) on vibration monitoring to detect faults using transient response during passage by a critical speed.

The vibration signals during machine run-up or shut-down are non-stationary (frequency changes along time) in nature. Conventional techniques, such as spectral and time series analysis, are effective tools for feature extraction for a broad range of faults in machines. However, they have difficulties with certain applications whose behavior is non-stationary and transient nature. To deal with non-stationary signals, several time-frequency and time-scale technique analysis were developed. Among them, can relate Short-Time Fourier Transform (STFT), Wigner-Ville Distribution (WVD) and Wavelet Transforms (WT). The STFT is computationally efficient, but has the drawback that the choice of the window length simultaneously affects both frequency and time resolution: for a good frequency resolution a high window length has to be chosen, but this choice detrimentally affects time resolution. The WVD introduces cross-terms when signals with multifrequency components are analyzed. In recent years, wavelet analysis (WT) has been applied with great success to various signal and image processing areas. Fourier transform gives the spectral content of the signal, but it gives no information regarding where in time those spectral components appear. On the other hand, wavelets analysis provides time-scale information of a signal, enabling the extraction of features of the signal effectively. This property makes the wavelet analysis an ideal tool for analysing signals of a transient or non-stationary nature (Sekhar, 2003 e Peng et all, 2003).

This work shows the ability and feasibility of the application of Continuous Wavelet Transform (CWT) in the diagnostic of faults inserted in the rotating machinery using the vibrations signals during machine run-up. In the experimental set-up are inserted the following faults: unbalance, misalignment and mechanic looseness. Run-up vibrations signatures on these faults are analysed using Morlet mother wavelet and the results extracted the time-scale features for fault diagnosis are presented in the form 2-D and 3-D graphs by means of the wavelet scalogram. The applications of wavelet analysis using real data, as well as its theoretical and practical aspects of implementation with Matlab software are discussed.

In the present study, a rotor system model capable of describing the theoretical dynamic behavior resulting from shaft misaligned and unbalanced rotor is developed during run-up motion. A comparison between experimental and numerical results clearly indicates that validity of the theoretical model was successfully verified for fault misalignment. The results show that the fault mechanical looseness and the effect of the evolution of fault misalignment can be monitored and detected during the machine run-up without passing by critical speed. Extensive numerical and experimental results show the ability and feasibility of the application of wavelet analysis in the diagnostic of faults inserted in the experimental set-up is very suitable to non-stationary signal analysis. Finally, results show that the sensivity and efficiency in the fault diagnostic using transient response during run-up is higher than steady state response for rotating machinery.

2. Wavelet analysis

It is well known that an energy limited signal (i.e. a square integrable signal), $f(t)$, can be decomposed by its Fourier transform $F(w)$ as:

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F(w).e^{iwt}dw \tag{1}$$

where,

$$F(w) = \int_{-\infty}^{+\infty} f(t).e^{-iwt}dt \tag{2}$$

Note that $F(w)$ and $f(t)$ constitue a pair of Fourier transforms. Equation (2) is called the Fourier transform of $f(t)$ and Eq. (1) is called inverse the Fourier transform. From a mathematical point of view, Eq. (1) implies that the signal $f(t)$ can be decomposed into a family of harmonics e^{iwt} and the weighting coefficients $F(w)$ represent the amplitudes of the harmonics in $f(t)$.

The wavelet transform is defined in a similar manner. However, instead of using the harmonics e^{iwt}, the wavelet transform uses wavelet basis function $\psi_{a,b}(t)$:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right),\ a \succ 0,\ b \in \Re \tag{3}$$

where a represents the scale parameter, b represents the translation in time, and $\psi(t)$ is called a mother wavelet function. The daughter wavelets $\psi_{a,b}(t)$ are a family of short-duration high frequency and long duration low frequency functions. The factor $1/\sqrt{a}$ is used to ensure energy preservation. Accordingly, a signal $f(t)$ can be decomposed into (Daubechies, 1988; Mallat, 1989):

$$f(t) = \frac{1}{C_\psi}\int_{-\infty}^{+\infty}\int_{0}^{+\infty} W(a,b)\frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right)da\,db \tag{4}$$

where, C_ψ is a constant depending on the base function, and $W(a,b)$ is the wavelet transform defined by:

$$W(a,b) = \frac{1}{\sqrt{a}}\int_{-\infty}^{+\infty} f(t)\,\psi^*\left(\frac{t-b}{a}\right)dt \tag{5}$$

Similar to the Fourier transform, $W(a,b)$ and $f(t)$ constitute a pair of wavelet transforms. The Equation (5) is called the wavelet transform of $f(t)$.

Analogy to the Fourier transform, Eq. (4) implies that wavelet transform can be considered to be signal decomposition. It decomposes a signal $f(t)$ onto a family of wavelets bases, and the weighting coefficients, $W(a,b)$, represent the amplitudes at given location b and scale a. The scale factor a is related (inversely) to the radian frequency w. Similarly, with wavelet analysis, the scale is related to the frequency of the signal. Compared to the Fourier transform, the wavelet transform is a time-frequency function that describes the information on $f(t)$ in various time window and frequency bands. It forms a three-dimensional figure against the time-frequency plane (spectogram). This lead us to define the wavelet spectogram, or scalogram, as the squared modulus of the $W(a,b)$ (Rioul and Vetterli, 1991). It is a distribution of the energy of the signal in the time-scale plane. On the other hand, the Fourier transform $F(w)$ depends only on frequency and hence, forms a two-dimensional curve against the frequency axis. As the result, the wavelet transform is capable of capturing non-stationary information such as frequency variation and magnitude undulation but the Fourier transform cannot.

In this work a Morlet analysing wavelet has been used and it is expressed as follows:

$$\psi(t) = e^{-t^2/2}\cos(5t) \tag{6}$$

It is a complex wavelet function, so its CWT (Continuous Wavelet Transform) coefficients, $W(a,b)$, is also complex. The coefficient can then be divided into real part, $R\{W(a,b)\}$, and the imaginary part , $I\{W(a,b)\}$, or amplitude, $|W(a,b)|$, and phase, $\tan^{-1}\left[I\{w(a,b)\}/R\{W(a,b)\}\right]$. Finally, one can define the wavelet power spectrum as $|W(a,b)|^2$. In fact, it is a two-dimensional matrix, which has the number of rows and columns equal to that of the scales and the sampling points, respectively. It reflects the distribution of energy of wavelet power spectrum in the direction of scale or frequency. The scale factor a is related (inversely) to the radian frequency w. Similarly, with wavelet analysis, the scale is related to the frequency of the signal (Misiti et all, 1997 and Zheng et all, 2002).

3. Transient response

In the present study, a rotor system model capable of describing the dynamic behavior resulting from shaft misaligned and unbalanced rotor is developed during start-up motion. It is now more and more important to know what happens when a rotor starts-up, stops or goes through a critical speed, effects known as transient motions.

The experimental set-up is illustrated in Fig. 1(a). It consists an electrical motor, a flexible coupling and two disks mounted on the rotating shaft supported by two identical ball bearings.

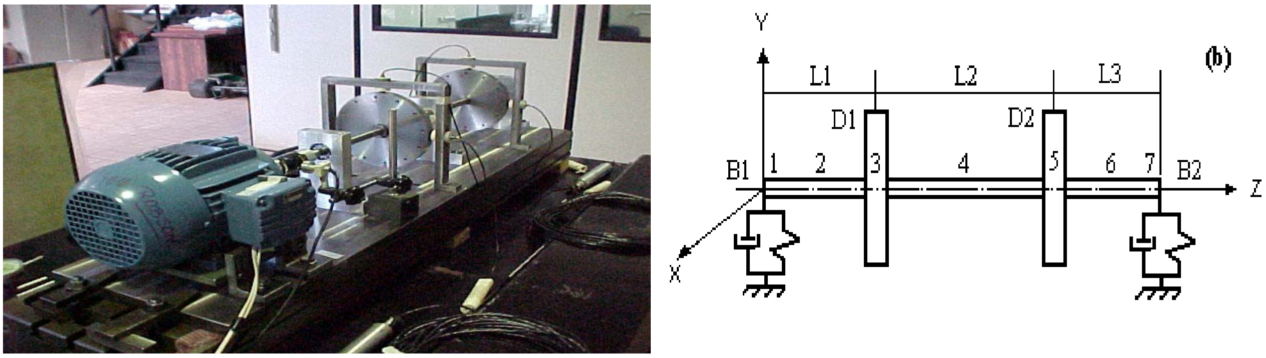

Figure 1 – (a) Experimental set-up and (b) Rotor system model.

In this modeling, Finite Element Method (FEM) is preferred. Figure 1(b) present the rotor systems model of discretization containing 7 finite elements (1, 7 – Bearing elements; 2, 4, 6 – Shaft elements and 3, 5 – Disks elements). The shaft was modeled as three finite element beams with a constant circular cross-section. The finite element used has two nodes, including four displacements and four slopes. Combining the effect of shaft, bearings and disks element models, and including all degrees of freedom of the rotor systems, the general differential equation of transient motion can be written as

$$[M]\ddot{q}(t)+[C1+\dot{\phi}C2]\dot{q}(t)+[K1+\ddot{\phi}K2]\,q(t) = F(t) \tag{7}$$

Where $[M]$ is the classical mass matrix and includes the influence of the secondary effect of rotatory inertia of the shaft, mass and diametral moments of the disk. The matrices $[C1]$ and $[C2]$ gives the bearings damping and gyroscopic effect, respectively. The matrices $[K1]$ and $[K2]$ includes the bearings stiffness and stiffness of the shaft elements, respectively. More details about the individual matrices of Eq. (7) are given in (Lalanne and Ferraris, 1998). The $q(t)$ is the vector of generalized co-ordinates containing all the nodal displacements and $F(t)$ is the vector of excitation forces used to model the faults, such as unbalance forces and misalignment forces. Depending on the type of faults, the vector $F(t)$ changes.

To investigate the effects of misalignment on the rotor dynamic characteristics is considered a dynamic model for coupling angular misalignment (where the shafts centerlines of the two shafts meet at an angle) and parallel misalignment (the shafts centerlines are parallel but displaced from one another). In this study, the reaction forces and moments of shaft developed due to angular and parallel misalignment given in (Sekhar and Prabhu, 1995; Lee and Lee, 1999) have been used in the numerical analysis. Several references (Xu and Marangoni, 1994; Sekhar and Prabhu, 1995; Lee and Lee, 1999) showed that an increase in angular and parallel misalignment had caused increase in the second harmonic of the radial vibration response. Then, the misalignment forces are assumed in this work to be periodic at the twice the rotational frequency of the shaft and the unbalance forces is equal a rotational frequency or fundamental frequency. Therefore, the reaction forces due to shaft misalignment and unbalance rotors are then incorporated into the excitation force $F(t)$ in the Eq. (7) of transient motion at the corresponding degrees of freedom.

Table 1 – Parameters used in the experimental and numerical analysis

Shaft	
Diameter, D	$17\text{x}10^{-3}$ m
Length, L_1, L_2, L_3	$180\text{x}10^{-3}$ m, $360\text{x}10^{-3}$ m, $180\text{x}10^{-3}$ m
Bearings	
Stiffness, $Kxx1 = Kyy1$	$5.64\text{x}10^{5}$ N/m
Stiffness, $Kxx2 = Kyy2$	$9.95\text{x}10^{5}$ N/m
Damping, $Cxx1 = Cyy1$	43.6 Ns/m
Damping, $Cxx2 = Cyy2$	5.37 Ns/m
Disks	
Mass, m	4 kg
Polar moment of inertia, I_p	0.0162 kg m^2
Diametral moment of inertia, I_d	0.0081 kg m^2
Unbalance eccentricity, m_u	$2\text{x}10^{-4}$ Kg
Inbalance eccentricity, e	0.080 m

4. Numerical analysis

The transient response due to excitation forces that characterize faults such as misalignment and unbalance is obtained by integrating of Eq. (7), using the Runge-Kutta integration method with a step size of 1 ms . In this analysis the characteristics of the bearings will be assumed to be constant and the angular velocity is not constant and is a function of time The rotor systems model data used in the numerical analysis given in Tab. (1). The analysis has been carried out considering the effect of angular misalignment forces actuating in the first bearing (near electrical motor) and the effect of unbalance forces actuating in the two disks, both in the x direction axis (horizontal) and y direction axis (vertical) as showed in Fig. 1(b). The angular misalignment value used in the simulation was of 1 mm.

Figure 2(a) shows time run-up of unbalance rotor obtained for first disk in x direction. The response shows that the rotor only passes by the first critical speed simulated (29 Hz). This is due to maximum rotational speed of electrical motor is of 60 Hz and the second critical speed simulated is equal to (72 Hz). To the implementation of CWT analysis, the scale parameter varied from 1 to 48. Are presented the results of the CWT coefficients at scale 28, because it characterizes or evidence better the coefficient peaks related to critical speed and sub-harmonic critical speed during run-up rotor. This will be better understood in the experimental analysis observing the general scalogram of the run-up.

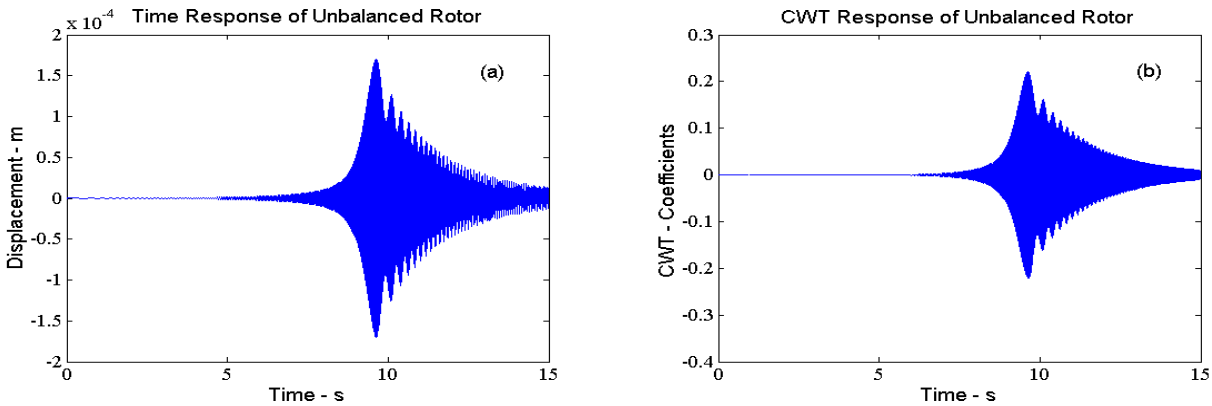

Figure 2 – Transient response of unbalanced rotor; (a) Time domain; (b) Wavelet domain.

Figures 2(a) and 2(b) show the run-up or start-up of unbalance rotor and the corresponding CWT coefficient plots, respectively, passing by first critical speed with an acceleration of 20 rad/s^2 . Similarly, Figs. 3(a) and 3(b) show the time run-up and the corresponding CWT coefficient plots for misalignment fault. It can be seen that the misalignment fault has excited the second sub-harmonic of the critical speed. The presence of the peak related the second sub-critical speed appears clearly evident in the CWT response and the time response. For small misalignment, the CWT response is more sensitive as compared to the only time response. It will be seen in the experimental analysis. Then, from the practical point of view, the application of the wavelet analysis is very important in the field of condition monitoring and fault diagnostic, in despite the early detection of faults before breakdown of a machine.

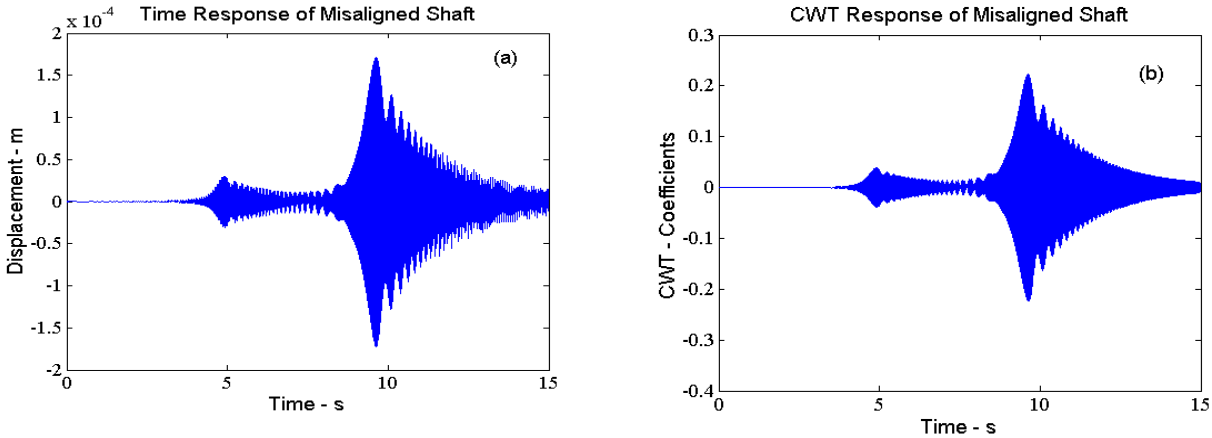

Figure 3 - Transient response of misaligned shaft; (a) Time domain; (b) Wavelet domain.

5. Experimental analysis

In this part, the wavelet analysis is applicable to the experimental signals obtained for different fault conditions using the vibration signals during machine run-up. The faults conditions inserted in the experimental set-up are normal condition (unbalanced rotor), angular misalignment and mechanical looseness.

The rotor shaft is driven by an electrical motor (three-phase asynchronous motor 220-380 V, power 3 CV). A inverter frequency (Newtronic FUJI FVR-E7S-EX) is used to vary the rotating speed continuously increasing or decreasing from 0 to 2400 rpm (40 Hz). The instrumentation used in the experiment includes three non-contacting displacement transducers or proximity probes used for displacement measurements. The three probes measure directly the rotor displacement in the horizontal and vertical directions. The third displacement probes, mounted in the vicinity of the motor's output shaft, was used as a tachometer signal to control the motor angular speed and acceleration. Two levels of angular misalignment (0.5 mm and 1 mm) were introduced in the first bearing by inserting steel plates of 0.5 mm and 1 mm of thickness, respectively, in the bearing base. The mechanical looseness was inserted by loose of four bolts at the first bearing base. Other real parameters used in the experimental analysis are given in Tab. (1).

In all signals vibration signals acquisition during machine run-up was used 12000 points of sampling, sampling frequency of 1000Hz and rotating speed varying from 0 to 2400 rpm. The measuring device was based on a Pentium II/300 MHz computer, equipped with a PCMCIA DAQCard-1200 data acquisition card from National Instruments. This is an 8-channel software-configurable 12-bit data acquisition card, with a total sampling rate capacity of 100 KHz. The code of the algorithm that was used in the data acquisition procedure has been developed under the LabVIEW programming environment of National Instruments. The Morlet mother wavelet has been chosen to obtain the CWT coefficients. To CWT analysis implementation, the scale parameter varied from 1 to 48. Next, are presented the results of the CWT coefficients at scale 31, because it characterizes or evidences better the coefficient peaks related with critical speed and sub-harmonic critical speed during run-up rotor.

In this section, the theoretical model for misaligned rotor system is experimentally verified, and the effect of misalignment and mechanical looseness faults on the transient response such as increasing amplitudes in the critical and sub-critical speed and sensitivity to accomplish the fault evolution are investigated. The run-up response has been analyzed at various rotor acceleration ($20\,rad/s^2$, $38\,rad/s^2$ *and* $75\,rad/s^2$). The results presented show run-up response for different faults conditions inserted in the experimental set-up; all obtained in the first disk (near electrical motor) in x direction (horizontal).

Figure (4) shows the time run-up and the corresponding CWT coefficients to normal condition (unbalanced rotor) for an angular acceleration of $20\,rad/s^2$. The both plots shows clearly the amplitude peaks related to unbalanced rotor during passing by critical speed. However, the CWT run-up enables evident and identifies the presence of faults or non-stationary events related to sub-harmonic critical speed, which are embedded in time run-up. Figure (5) shows the frequency analysis or steady state for rotational speed of 40 Hz and CWT run-up map to evidence the distribution of energy of wavelet power spectrum in the direction of scale or frequency. This map enables a comparison between the different faults inserted in the experimental and the choice of scale that better evidences the presence of fault.

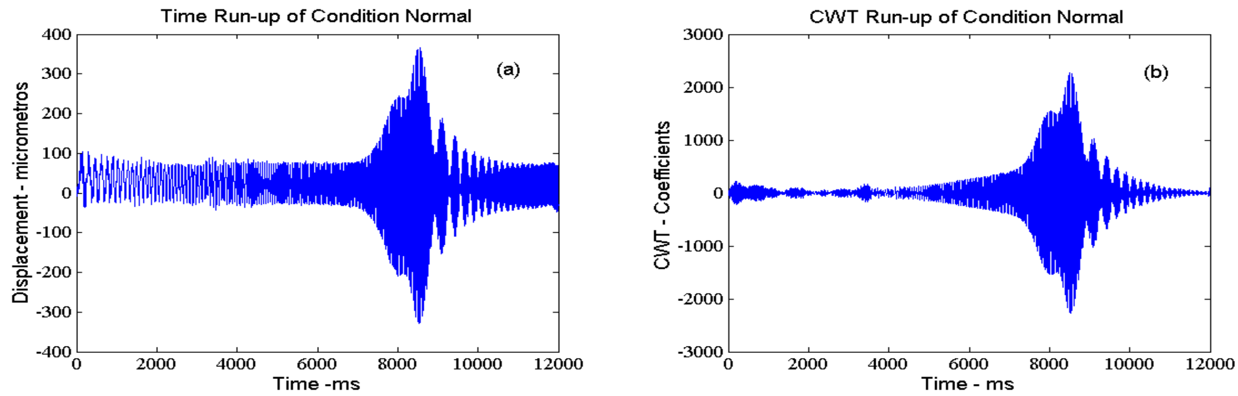

Figure 4 – Time run-up and CWT run-up for condition normal.

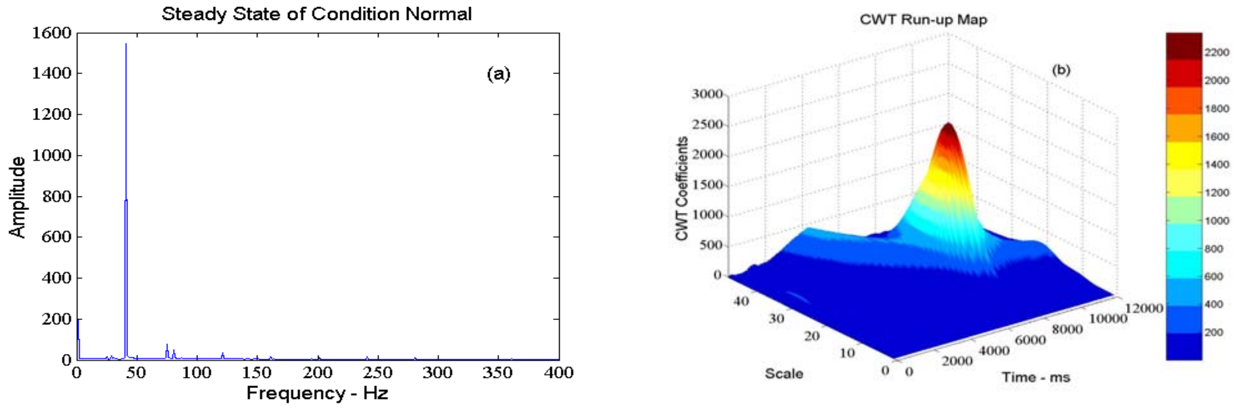

Figure (5) – Frequency analysis and CWT run-up map for normal condition.

The Fig. (6) shows the time run-up and the corresponding CWT coefficients to angular misalignment of 0.5 mm inserted in the first bearing base for an angular acceleration of $20\,rad\,/\,s^2$. The presence of the peaks related the second sub-critical speed appear more clearly evident in the CWT run-up than the time run-up due effect of misalignment.

Figure (7) shows that the frequency analysis there is practically no fluctuations in the amplitude related to second harmonic due misalignment when compared the frequency response of condition without misalignment showed in Fig. (5). On other hand, the CWT run-up map shows clearly the presence of peak related with second sub-critical speed due misalignment, while the CWT run-up map showed in Fig (5) does not evidence the presence of the peak to condition without misalignment. Then, for small misalignment the CWT run-up is more sensitive if compared to the only time run-up. However, from the practical point of view, the application of the wavelet analysis is very important in the field of monitoring condition and fault diagnostic.

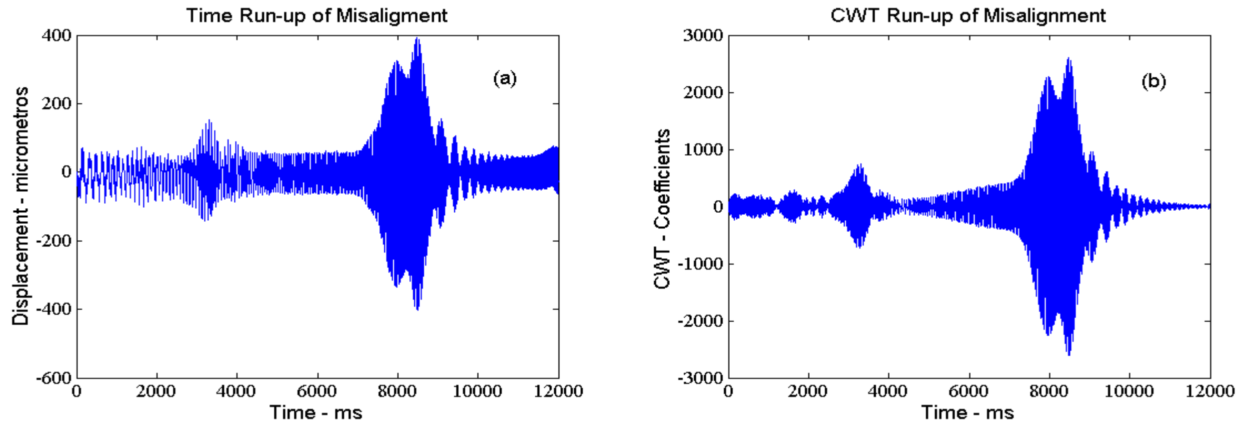

Figure 6 - Time run-up and CWT run-up for angular misalignment of 0.5 mm.

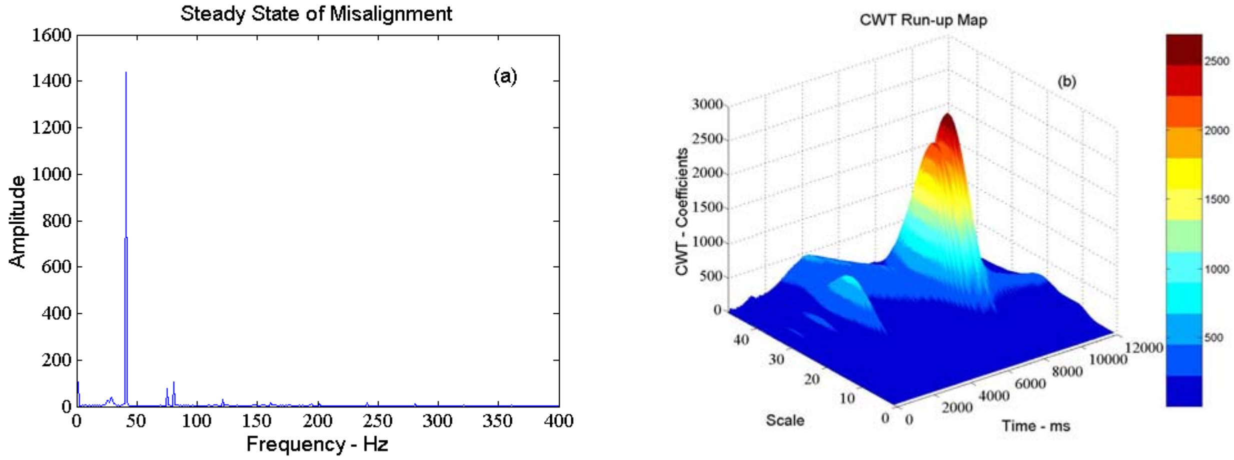

Figure 7 - Frequency analysis and CWT run-up map for angular misalignment of 0.5 mm.

Similarly, Fig. (8) shows the time run-up and the corresponding CWT coefficients to angular misalignment of 1 mm inserted in the first bearing base for an angular acceleration $75\, rad / s^2$. The presence of the peaks related to the second sub-critical speed appear more clearly evident in the CWT run-up than the time run-up due effect of misalignment fault. This result suggests that the wavelet analysis using the transient response can be used for fault diagnostic, particularly for higher accelerations. On the other hand, the presence of peak related with second sub-critical speed due misalignment, enables to examine the validity of the theoretical model derived to misalignment. Then, a comparison between experimental and numerical results clearly indicates that validity of the theoretical model was successfully verified for fault misalignment.

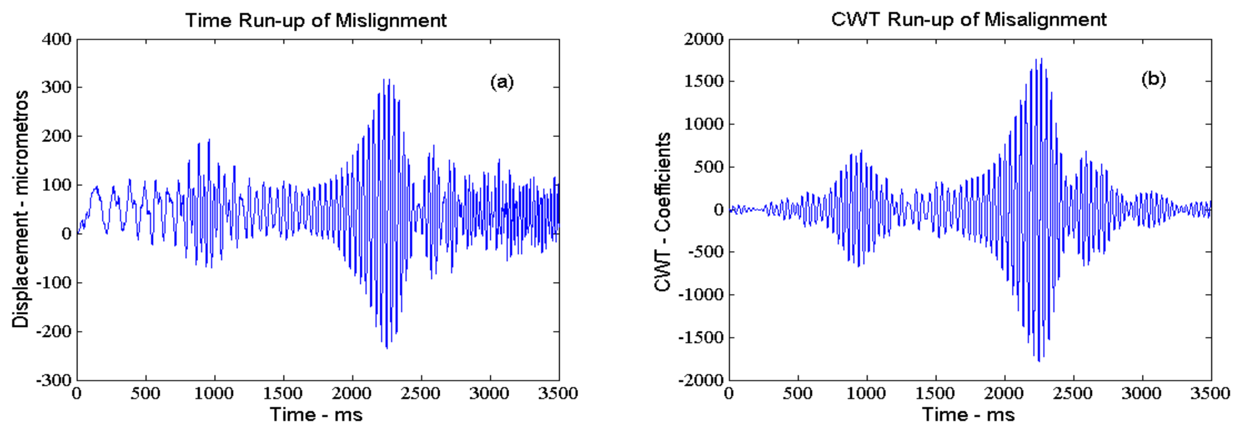

Figure 8 - Time run-up and CWT run-up for angular misalignment of 1 mm.

Figure (9) shows the time run-up and the corresponding CWT coefficients to mechanical looseness inserted in the first bearing base for an angular acceleration $38\, rad / s^2$. In this fault, it has been observed the presence of a peak related to the second sub-critical speed and a decreasing of the amplitude related with critical speed, when compared to Fig. (4) for condition without mechanical looseness. This effect is more clearly evident in the CWT run-up than the time run-up due effect of mechanical looseness.

Figure (10) shows that the frequency analysis there is practically no fluctuations in the amplitude related with first and second harmonic due mechanical looseness when compared the frequency response for condition without mechanical looseness showed in Fig. (5). On the other hand, the CWT run-up map in Fig. (10) has been showed clearly the presence of peak related to second sub-critical speed due mechanical looseness, while the CWT run-up map showed in Fig (5) does not evidence the presence of the peak for condition without mechanical looseness.

The transient response, on the other hand, shows a clear change in the pattern of the time run-up and CWT run-up. But, the CWT run-up enables evident and localize the presence of faults or non-stationary events related to sub-harmonic critical speed, which are embedded in time run-up. However, for small misalignment the CWT run-up is more sensitive when compared to the only time run-up.

This results presented show no clear difference between frequency analysis for normal condition, fault angular misalignment and mechanical looseness. In this paper, extensive numerical and experimental results are used to confirm and evidence that the wavelet analysis is very suitable to non-stationary signal analysis. Experimental analysis using the vibration signals during machine shut-down or coast-down was carried out and the results obtained showed that the sensitivity and efficiency in the fault diagnostic using CWT analysis is higher than frequency analysis.

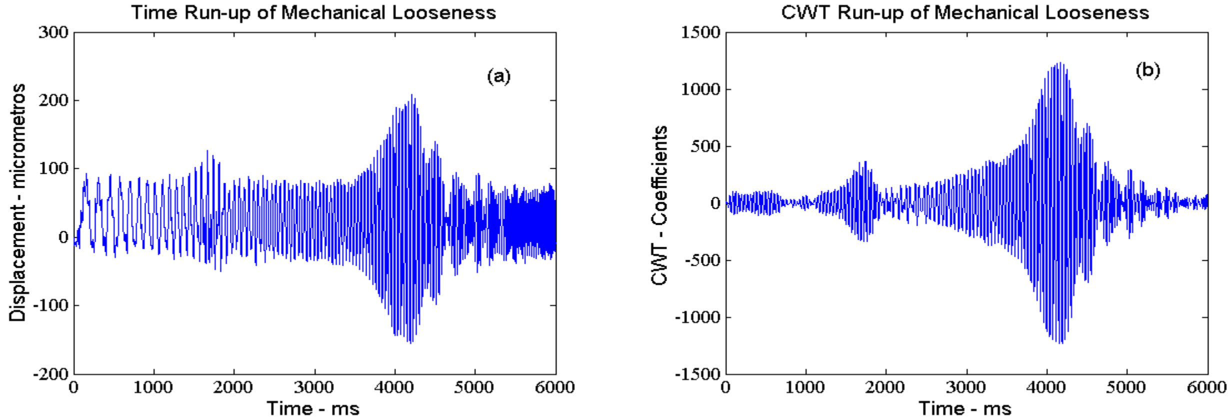

Figure 9 - Time run-up and CWT run-up for mechanical looseness.

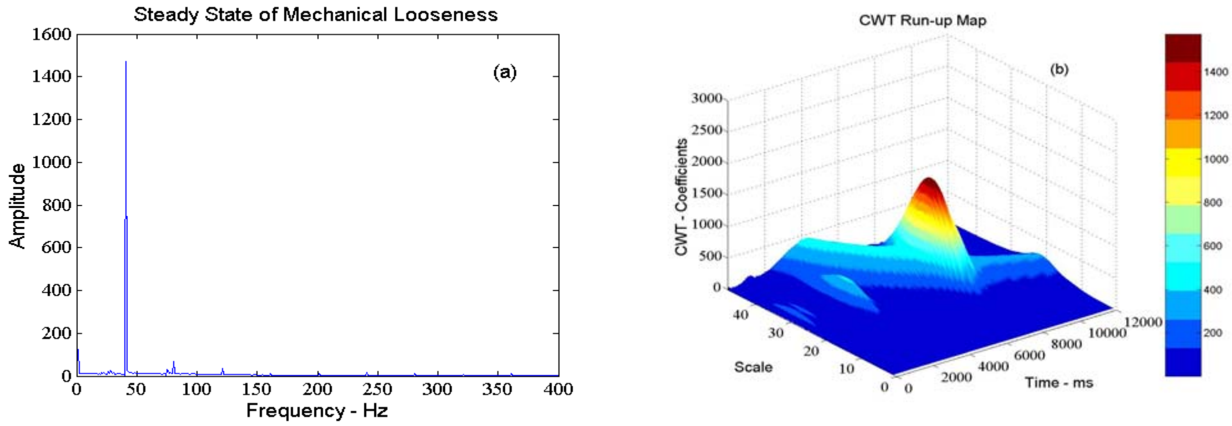

Figure 10 - Frequency analysis and CWT run-up map for mechanical looseness

6. Conclusions

The vibration in rotating machinery is mostly caused by unbalance, misalignment, mechanical looseness, rubs, shaft crack, and other malfunctions. Most studies available in the literature have paid attention to diagnostic on these faults by analyzing the steady state vibrations. But it is proved in earlier research that it is easier to detect cracks using transient response during run-up or shut-down of a machine. In the present study, a rotor system model capable of describing the dynamic behavior resulting from shaft misaligned and unbalanced rotor is developed during run-up motion. It is now more and more important to know what happens when a rotor starts-up or shuts-down through a critical speed, effects known as transient motions.

The presence of the peak related the second sub-critical speed appears more clearly evident in the CWT analysis than the time response. For small misalignment the CWT analysis is more sensitive as compared to the only time response. Then, from the practical point of view, the application of the wavelet analysis is very important in the field of condition monitoring and fault diagnostic, in despite of the early detection of faults before breakdown of a machine.

In this study, a comparison between experimental and numerical results clearly indicates that validity of the theoretical model was successfully verified for fault misalignment. The results show that the mechanical looseness fault and the effect of the evolution of misalignment fault can be monitored and detected during the machine run-up without passing by critical speed.

This results presented show practically no clear difference between frequency analysis for normal condition, angular misalignment and mechanical looseness fault. Finally, extensive numerical and experimental results show that ability and feasibility of the application of wavelet analysis in the diagnostic of faults inserted in the experimental set-up is very suitable to non-stationary signal analysis and that run-up signal can be used to detect incipient faults in rotating machinery.

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