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Mathematical Modeling of an Electropneumatic Pressure Regulator Servo-Valve

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Abstract. *This paper presents a mathematical modeling of an electropneumatic pressure regulator servo-valve, as an integrated part of a pneumatic positioning system. Both static and dynamic behavior are considered in the model. The theoretical analysis, that is based on concepts of fluid mechanics and laws of conservation of mass and energy, as well as on the control theory, is accomplished with experimental tests that validate the modeled behaviors. The determination of the gains and parameters intrinsic to the system is made based on experimental data, allowing to obtain the typical curves of these valves. Some simplifications of the model are suggested, that seem to be of high efficient for the proper characterization of this type of component. The specific valve in study is a Rexroth DS 50 R, controlled by a pilot valve, with integrated pressure sensor and controller. The obtained results shows that the developed mathematical model represent the real behavior very well, allowing to extend this model to other valves of pressure control.*

Keywords. *proportional pressure regulator servo-valve, electropneumatic, positioning control, mathematical model.*

1. Introduction

The electropneumatic pressure regulator servo-valves are employed to regulate the system's pressure at a desired value, proportional to an analogical control signal (voltage). They are used for limiting the maximal force in cylinders and pneumatic motors, and also to allow the positioning control of cylinders. The pneumatic components are the most common in industry. They are cheap and simple when compared with other electromechanical components of equal power density. However they are not competitive in applications where demands on accuracy are important. The main disadvantages of this type of valve is that they are inherently nonlinear, such fact associated to the air flow in its internal chambers and orifices, what requires a detailed modeling aiming the further analysis of the system.

A mathematical development, as well as some considerations and simplifications that gave origin to the model of the valve are presented in this paper. The equations are presented in their literal form.

Experimental tests and obtained results are represented through graphics and a table, allowing the visualization of the data and parameters used in the model, is shown at the end of this paper.

2. Electropneumatic Pressure Regulator Servo-Valve

Pinches and Callear (1996), shows that the proportional pressure regulator valve promotes the variation of the pneumatic pressure through a diaphragm, which position results from the equilibrium between the variable force, generated by a proportional solenoid, and the opposed force generated by the regulated pressure. The magnitude of the downstream pressure (regulated pressure) is considered proportional to the electromagnetic force and also to the solenoid current (reference signal). When the downstream pressure overcomes the adjusted value, the de-pressurization occurs through the exhaust port.

The combination of this proportional valve with an electronic amplifier and an integrated pressure sensor, forms a closed loop system. This system is industrially called Electropneumatic Pressure Regulator Servo-Valve. This valve is designed to control pressure and derived variables (for example: force) accurately, and the reference signal can be generated from potentiometers, computers and process controllers.

The technical data of the valve used in this study, an Electric Pressure Servo-Valve DS 50 R (Mannesmann Rexroth, [199-?]), does not include an internal view. Therefore, a general scheme of a pressure regulator valve, aiming the mathematical modeling of the servo-valve, is considered as shown in Fig. (1):

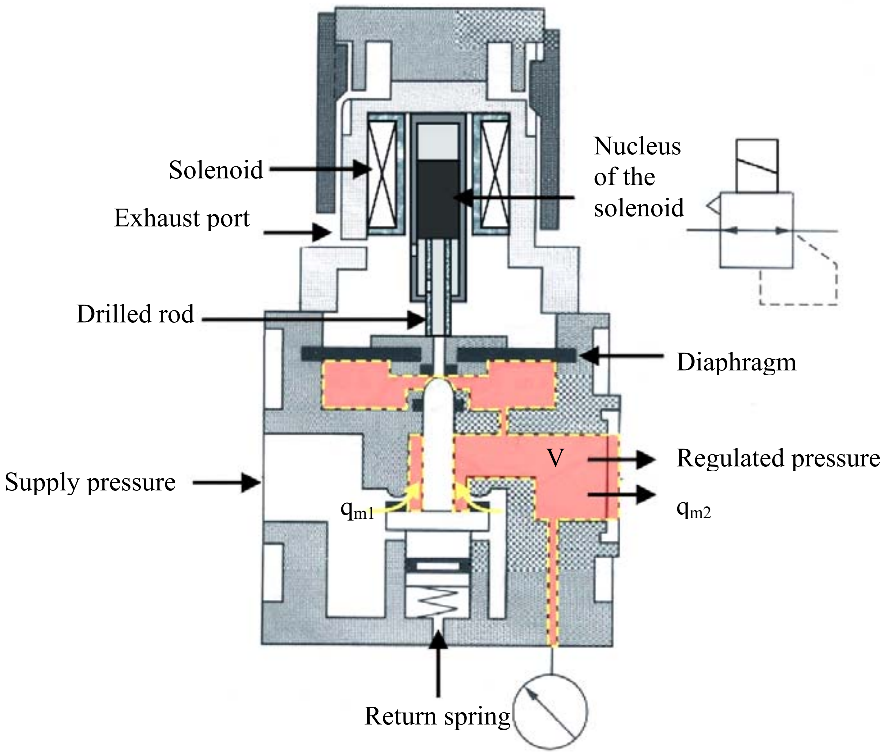

Figure 1. Pressure Regulator Valve.

3. Mathematical Model

This theoretical model is based on concepts of fluid mechanics including laws of conservation of mass and energy as described on Fox & McDonald (1981) and Streeter (1981).

3.1. Equation of continuity for a compressible flow:

This equation makes reference to the mass conservation through a chamber of the valve.

$$\int_{SC} \rho \cdot v \cdot dA + \frac{\partial}{\partial t} \int_{VC} \rho \cdot dV = 0 \tag{1}$$

where: "A" is the control surface, "V" is the volume of the considered chamber, surrounded by "A", "ρ" is the density of the fluid in "V" and "v" is the velocity of the fluid on "A".

Considering an isentropic process (Andersen, 1967, Fox & McDonald, 1981),

$$C_p \cdot dT = \frac{V}{m_f} \cdot dp_{reg} \tag{2}$$

where: "Cp" is the specific heat at constant pressure and "T", "m_f" and "p_{reg}" are, respectively, the absolute temperature, the mass of the fluid and the regulated absolute pressure, all them measured at "V".

Furthermore, to an ideal gas (Streeter, 1981),

$$C_p = \frac{\gamma \cdot R}{\gamma - 1} \tag{3}$$

where: "R" is the gas constant and "γ" is the ratio of specific heats,

results:

$$q_{m1} = q_{m2} + \frac{p_{reg}}{R \cdot T} \cdot \frac{dV}{dt} + \frac{V}{R \cdot T \cdot \gamma} \cdot \frac{dp_{reg}}{dt} \tag{4}$$

where: "q_{m1}" is the upstream mass flow rate and "q_{m2}" is the downstream mass flow rate of "A".

The volume of the valve's chamber can be considered invariable. Therefore, the Eq. (4) can be written as:

$$q_{m1} = q_{m2} + \frac{V}{R \cdot T \cdot \gamma} \cdot \frac{dp_{reg}}{dt} \tag{5}$$

or, in the Laplace domain (s-plane):

$$q_{m1} = q_{m2} + \frac{V}{R \cdot T \cdot \gamma} \cdot s \cdot p_{reg} \tag{6}$$

3.2. Mass flow rate equation for a compressible flow:

$$q_{m1} = \frac{A_o \cdot p_{\text{sup}}}{\sqrt{T_{\text{sup}}}} \cdot \left\{ \frac{2 \cdot \gamma}{(\gamma - 1) \cdot R} \cdot \left[\left(\frac{p_{reg}}{p_{\text{sup}}} \right)^{2/\gamma} - \left(\frac{p_{reg}}{p_{\text{sup}}} \right)^{(\gamma+1)/\gamma} \right] \right\}^{1/2} \tag{7}$$

where: "p_{sup}" is the supply absolute pressure (that corresponds to the upstream pressure or total pressure), "T_{sup}" is the absolute temperature measured at the input port of the valve (total temperature) and "A_o" is the cross-sectional area of the control orifice. The total pressure is defined as the pressure at the local isentropic stagnation state. This is the hypothetical state reached at any point in a flowing gas by an isentropic slowing of the gas to zero velocity.

Analysing the Eq. (7), it can be noted that the theoretical mass flow rate for compressible fluids is a function of the ratio between the downstream and upstream pressures taken at the control orifice, differently to the case for incompressible fluids, where the mass flow rate is a function of the difference between them ($\Delta p = p_{sup} - p_{reg}$).

Equation (7) is a relationship that is strictly valid at any point in a flowing gas. It is important to remember that this is not an approximation. A more detailed analysis with respect to Eq. (7) and derived variables, can be found in Andersen (1967).

It is seen in Fig. (2) that there is a maximum mass flow rate at which gas can flow through a given area for given values of total pressure and temperature. To obtain the pressure ratio corresponding to the maximum mass flow rate, one can differentiate the Eq. (7) and set the derivative equal to zero. This gives the critical pressure ratio,

$$\left(\frac{p_{reg}}{p_{\text{sup}}} \right)_{cr} = \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma}{(\gamma - 1)}} = 0.528 \tag{8}$$

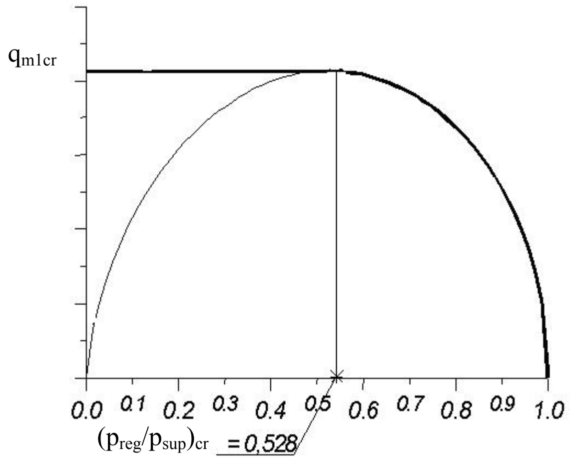

Figure 2. Relation of the mass flow rate to pressure ratio.

3.3. Motion equation (displacement of the spool):

The balance of the forces actuating on the spool of the valve is given by the sum of the diaphragm pressure force "$A_e.(p_{reg} - p_{atm})$", solenoid force "F_s", viscous damping force "$B.(dx_{vs}/dt)$" and linear spring force "$K_m.x_{vs}$", and this sum is equal to an inertial force "$m_{vs}.(d^2x_{vs}/dt^2)$" that has to be overcome,

$$-A_e \cdot (p_{reg} - p_{atm}) + F_s - B \cdot \frac{dx_{vs}}{dt} - K_m \cdot x_{vs} = m_{vs} \cdot \frac{d^2 x_{vs}}{dt^2} \tag{9}$$

where: "A_e" is the effective cross-sectional area of the diaphragm of the valve, "p_{atm}" is the atmospheric pressure, "B" is the viscous damping coefficient, "x_{vs}" is the spool displacement, "K_m" is the return spring stiffness and "m_{vs}" is the mass of the spool.

Another consideration can be made,

$$F_s = K_s \cdot U \tag{10}$$

where: "K_s" is the gain of the solenoid, once it is considered that the solenoid force is proportional to the control signal "U".

Neglecting "m_{vs}", isolating "x_{vs}", and working on the s-plane domain, results:

$$x_{vs} = \frac{K_s \cdot U - A_e \cdot (p_{reg} - p_{atm})}{B \cdot s + K_m} \tag{11}$$

Developing the terms that are shown inside brackets on Eq. (7) through two binomial series and neglecting the terms of power higher than three, results:

$$\begin{aligned} &\left(\frac{p_{reg}}{p_{\sup}}\right)^{2/\gamma} - \left(\frac{p_{reg}}{p_{\sup}}\right)^{(\gamma+1)/\gamma} = \left(1 - \frac{\Delta p}{p_{\sup}}\right)^{2/\gamma} - \left(1 - \frac{\Delta p}{p_{\sup}}\right)^{(\gamma+1)/\gamma} \\ &= \frac{(\gamma-1)}{\gamma} \cdot \frac{\Delta p}{p_{\sup}} \cdot \left[1 - \frac{3}{2\cdot\gamma} \cdot \frac{\Delta p}{p_{\sup}} - \frac{(3\cdot\gamma-5)}{2\cdot\gamma^2} \cdot \left(\frac{\Delta p}{p_{\sup}}\right)^2\right] \end{aligned} \tag{12}$$

where:

$$\Delta p = p_{\sup} - p_{reg} \tag{13}$$

Substituting Eq. (11) and Eq. (12) in Eq. (7), and considering

$$A_o = K_o \cdot x_{vs} \tag{14}$$

where: "K_o" is a constant of proportionality.

results:

$$q_{m1} = \frac{\left[K_s \cdot U - A_e \cdot \left(p_{reg} - p_{atm}\right)\right]}{(B \cdot s + K_m)} \cdot K_o \cdot \left\{ \frac{2 \cdot p_{\sup} \cdot \Delta p}{T_{\sup} \cdot R} \cdot \left[1 - \frac{3}{2 \cdot \gamma} \cdot \frac{\Delta p}{p_{\sup}} - \frac{(3 \cdot \gamma - 5)}{2 \cdot \gamma^2} \cdot \left(\frac{\Delta p}{p_{\sup}}\right)^2\right]\right\}^{1/2} \tag{15}$$

Remembering that:

$$\frac{p_{\sup}}{R \cdot T_{\sup}} = \rho_{\sup} = \frac{\rho}{\left(\frac{p_{reg}}{p_{\sup}}\right)^{1/\gamma}} = \frac{\rho}{\left(1 - \frac{\Delta p}{p_{\sup}}\right)^{1/\gamma}} \cong \frac{\rho}{\left[1 - \frac{1}{\gamma} \cdot \frac{\Delta p}{p_{\sup}} + \frac{(1 - \cdot \gamma)}{2 \cdot \gamma^2} \cdot \left(\frac{\Delta p}{p_{\sup}}\right)^2\right]} \tag{16}$$

results as final expression:

$$q_{m1} = \varepsilon \cdot K_o \cdot \frac{\left[K_s \cdot U - A_e \cdot \left(p_{reg} - p_{atm}\right)\right]}{(B \cdot s + K_m)} \cdot \sqrt{2 \cdot \rho \cdot (p_{\sup} - p_{reg})} \tag{17}$$

where, according to De Negri (2001):

$$\varepsilon = \left[\frac{1 - \frac{3}{2 \cdot \gamma} \cdot \frac{\Delta p}{p_{\sup}} - \frac{(3 \cdot \gamma - 5)}{2 \cdot \gamma^2} \cdot \left(\frac{\Delta p}{p_{\sup}}\right)^2}{1 - \frac{1}{\gamma} \cdot \frac{\Delta p}{p_{\sup}} + \frac{(1 - \gamma)}{2 \cdot \gamma^2} \cdot \left(\frac{\Delta p}{p_{\sup}}\right)^2}\right]^{1/2} \tag{18}$$

Substituting Eq. (6) in Eq. (17) results:

$$q_{m2} = \varepsilon \cdot K_o \cdot \frac{\left[K_s \cdot U - A_e \cdot \left(p_{reg} - p_{atm}\right)\right]}{(B \cdot s + K_m)} \cdot \sqrt{2 \cdot \rho \cdot (p_{\sup} - p_{reg})} - \frac{V}{R \cdot T \cdot \gamma} \cdot s \cdot p_{reg} \tag{19}$$

The same can be written as:

$$q_{m2} = \varepsilon \cdot \frac{\left[K_1 \cdot U - K_2 \cdot \left(p_{reg} - p_{atm}\right)\right]}{(\tau \cdot s + 1)} \cdot \sqrt{(p_{\sup} - p_{reg})} - \frac{V}{R \cdot T \cdot \gamma} \cdot s \cdot p_{reg} \tag{20}$$

where:

$$K_1 = \frac{K_o \cdot K_s}{K_m} \cdot \sqrt{2 \cdot \rho} \tag{21}$$

$$K_2 = \frac{K_o \cdot A_e}{K_m} \cdot \sqrt{2 \cdot \rho} \tag{22}$$

and the time constant "τ" is equal to,

$$\tau = \frac{B}{K_m} \tag{23}$$

4. Experimental Parameters

Experimental tests were made for verification of the time response of the valve. By closing its output port and applying a step input (Voltage), one can observe the behavior of the regulated manometric pressure as shown in Fig. (3). This dynamic response was considered as being of second order and it has been verified, through experimental tests, that the same is relativity fast when compared with the response obtained when the whole pneumatic servo positioning system, described in Fig. (4), is considered.

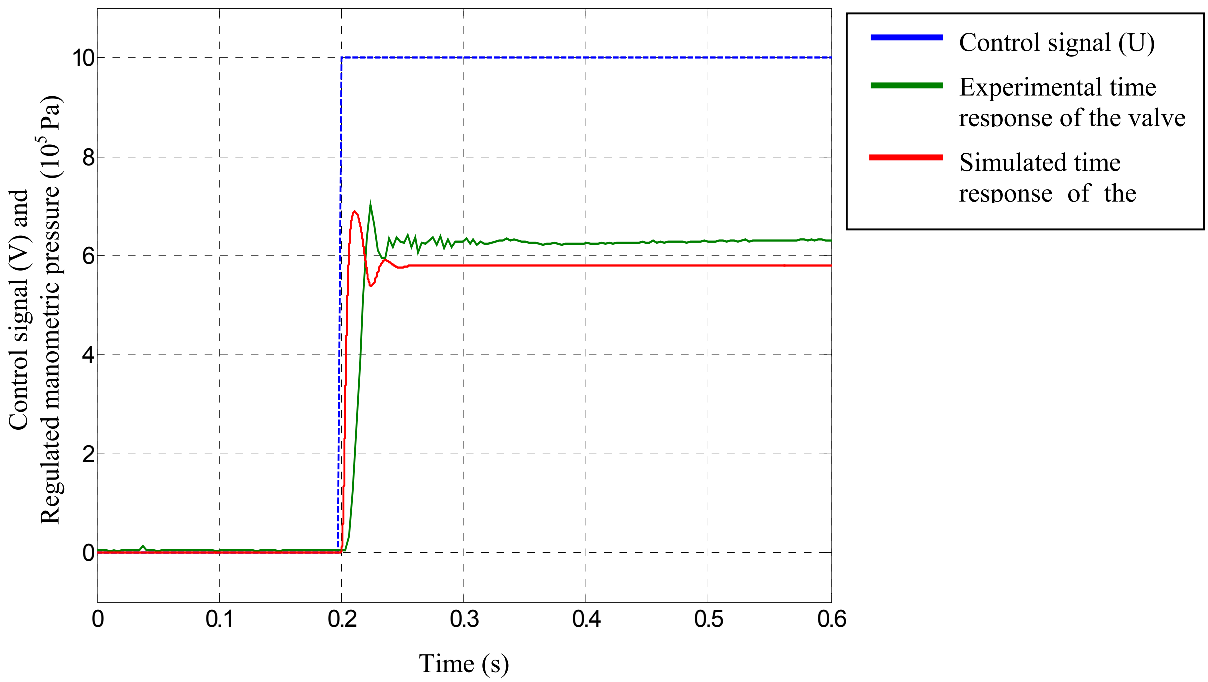

Figure 3. Time response of the valve.

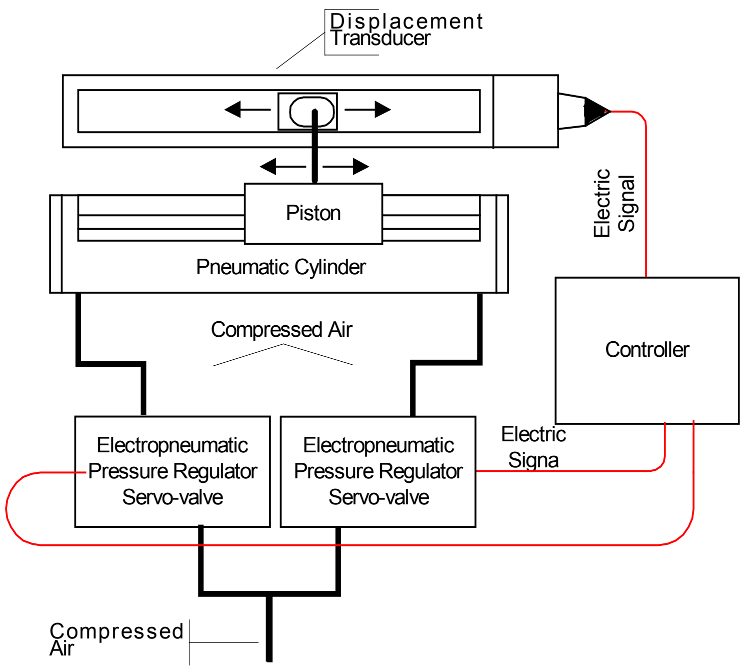

Figure 4. Pneumatic servo positioning system.

The response of the valve is strictly related with Eq. (20), where a nonlinear system of second order is presented.

In Figure (3), it can be seen the effects of these two poles, characterizing the curve's format as being composed by a damped sine wave. Other effects are due to the limit cycle.

To estimate the value of "τ", the equations that characterize the servo-valve were linearized, considering "q_{m2}" equal to zero, resulting in a final expression given by:

$$\left[\frac{V\cdot\tau}{R\cdot T\cdot\gamma\cdot\left(K_c+K_q\cdot\frac{A_e}{K_m}\right)}\cdot s^2+\frac{\frac{V}{R\cdot T\cdot\gamma}+K_c\cdot\tau}{K_c+K_q\cdot\frac{A_e}{K_m}}\cdot s+1\right]\cdot\delta p=\left(\frac{K_q\cdot\frac{K_s}{K_m}}{K_c+K_q\cdot\frac{A_e}{K_m}}\right)\cdot\delta U \tag{24}$$

where:

$$K_q=\left.\frac{\partial q_{m1}}{\partial x_{vs}}\right|_i=\varepsilon\cdot K_o\cdot\sqrt{2\cdot\rho\cdot(p_{\sup}-p_{reg}|_i)} \tag{25}$$

and

$$K_c=-\left.\frac{\partial q_{m1}}{\partial p_{reg}}\right|_i=\frac{\varepsilon\cdot K_o\cdot x_{vs}|_i\cdot\sqrt{2\cdot\rho}}{2\cdot\sqrt{p_{\sup}-p_{reg}|_i}} \tag{26}$$

To simplify Eq. (24), this one can be written as:

$$\left[\alpha\cdot\tau\cdot s^2+(\alpha+\beta\cdot\tau)\cdot s+1\right]\cdot\delta p=\eta\cdot\delta U \tag{27}$$

where:

$$\alpha=\frac{V}{R\cdot T\cdot\gamma\cdot\left(K_c+K_q\cdot\frac{A_e}{K_m}\right)} \tag{28}$$

$$\beta=\frac{K_c}{K_c+K_q\cdot\frac{A_e}{K_m}} \tag{29}$$

$$\eta=\frac{K_q\cdot\frac{K_s}{K_m}}{K_c+K_q\cdot\frac{A_e}{K_m}} \tag{30}$$

Considering that "K_c" has a low value when compared with the other parameters, "β" is very small and can be neglected. This form, the Eq. (27) becomes:

$$\left(\alpha\cdot\tau\cdot s^2+\alpha\cdot s+1\right)\cdot\delta p=\eta\cdot\delta U \tag{31}$$

The Laplace-transform shown in Eq. (31) can be written in a normalized format, according to Ogata (1998) and Franklin et al. (1994):

$$\left(\frac{1}{\omega_n{}^2}\cdot s^2+\frac{2\cdot\zeta}{\omega_n}\cdot s+1\right)\cdot p_{reg}=K_{RP}\cdot U \tag{32}$$

where: "ω_n" is the undamped natural frequency, "ζ" is the damping ratio and "K_{RP}" is the steady state gain.

Comparing Eq. (31) with Eq. (32), and remembering that for a second-order system the settling time "t_s", for 1% steady state error, is given by,

$$t_s = \frac{4.6}{\zeta \cdot \omega_n} \tag{33}$$

results on $\tau = 0.00489s$, for $t_s = 0.045s$ (see Fig. (3)) and $\zeta = 0.5$.

The values of "K_1" and "K_2" were experimentally obtained in steady state, considering the whole pneumatic servo positioning system (the pneumatic cylinder and the two electropneumatic pressure regulator servo-valves, Fig. (4)). With this purpose, Eq. (20) can be rewritten in steady state as:

$$K_1 \cdot U - K_2 \cdot \left(p_{reg} - p_{atm}\right) = \frac{q_{m2}}{\varepsilon \cdot \sqrt{(p_{\sup} - p_{reg})}} \tag{34}$$

It is noted that, for a specific steady state situation, the portion of the Eq. (34) located at the right side is constant at a specific operational condition. At the same way, the term "$(p_{reg} - p_{atm})$" also assumes constant values. Results of this analysis, an equation with known and constant terms, however with two variables (the "K's").

To overcome this problem, tests were done maintaining constant values of "U", and changing "q_{m2}" through the variation of the opposed control signal applied to the second valve. Dealing this way, resulted, for constant values of "U", several points on the "q_{m2} vs. p_{reg}" plane providing several possible combinations of "K's".

The curves generated, referent to the behavior of the regulated absolute pressure when occurs variation in the downstream mass flow rate, are shown in Fig. (5).

Selecting the extreme points of each one of the curves of constant "U", for effect of determination of the "K's", and, in sequence, substituting these ones in the Eq. (34) for different values of "U", once it is known, in accordance with Fig. (5), the variation band of the regulated absolute pressure, results a set of straight lines, for each one of the curves of constant "U", that provide an approximation of the real behavior of this valve. The set of straight lines that better described the real behavior of this valve was that resulted from the "K's" obtained from the curve of "U" equal to 4V. The same are shown in Fig. (5), together with the curves, already mentioned, referent to the real behavior of the valve.

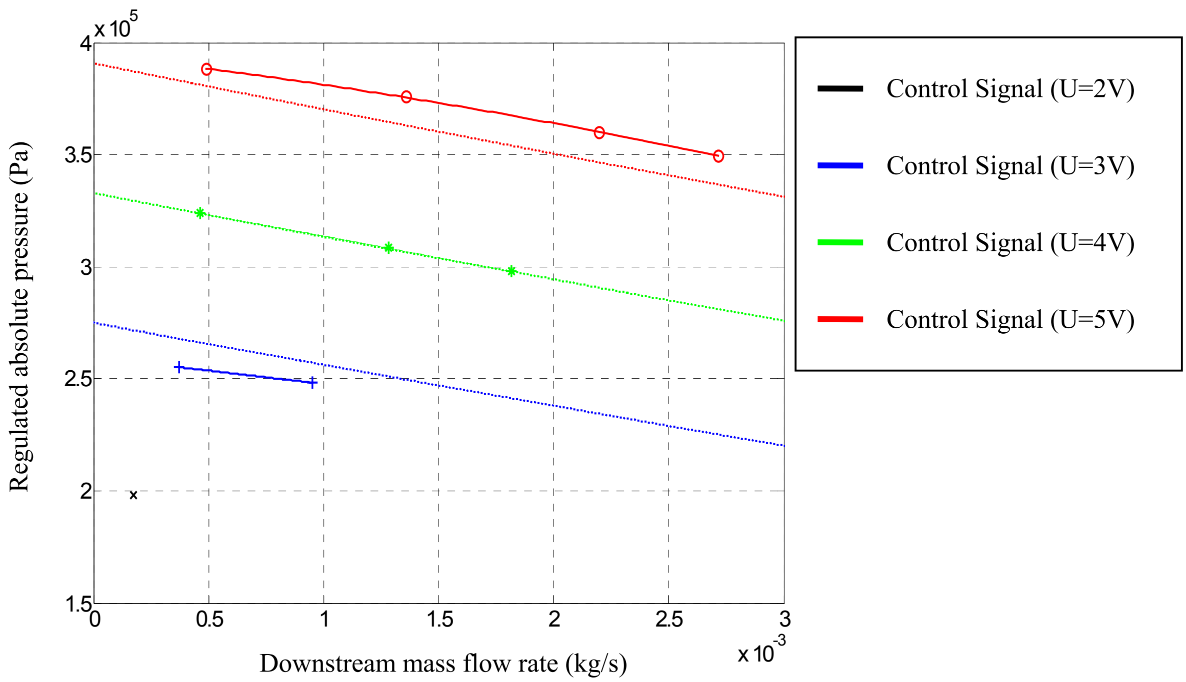

Figure 5. Graphic of the downstream mass flow rate vs. regulated absolute pressure.

Equation (20) represents the open-loop mathematical model of the electropneumatic pressure regulator servo-valve, where it has been considered the equations of continuity, mass flow rate and dynamic of the spool.

As a demonstration of the compatibility obtained between the response of the theoretical model and the real behavior of the electropneumatic pressure regulator servo-valve, it is presented in Fig. (6) the results of an experimental test accomplished with the simulated response (the input signal is a repeating sequence oscillating between 4.5V and 3V).

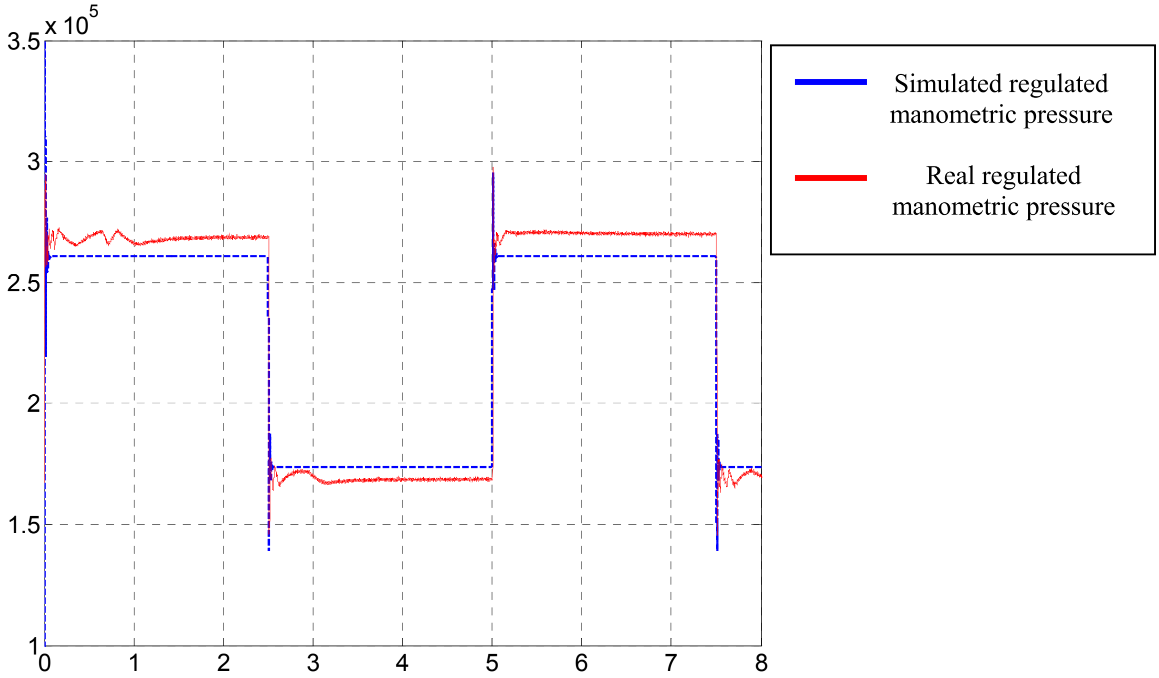

Figure 6. Comparison between the response of the mathematical model of the electropneumatic pressure regulator servo-valve and the behavior of the real system.

In Figure (7), it can be seen a block diagram representing the mathematical model of the electropneumatic pressure regulator servo-valve in study.

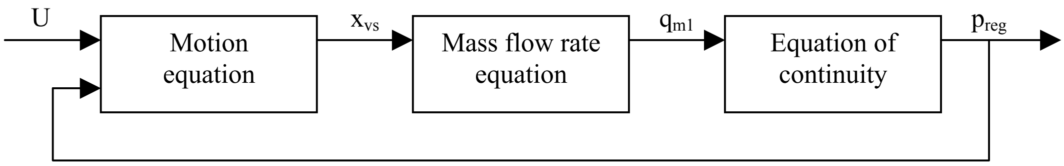

Figure 7. Block diagram representing the mathematical model of the electropneumatic pressure regulator servo-valve.

Table (1) shows the parameters used at the equations described above. Their description, values and respective units, are also presented.

Table 1. Parameters used at the equations.

Description and Measure Unit of the Parameter	Symbol	Value
Supply absolute pressure [Pa]	p_{sup}	8.013×10^5
Atmospheric pressure [Pa]	p_{atm}	1.013×10^5
Gas constant [J kg/K]	R	286.9
Ratio of specific heats (for the air, $\gamma = 1.40$)	γ	1.40
Control signal (repeating sequence) of the valve [V]	U	oscillating between 4.5 and 3.0
Volume of the output chamber [m^3]	V	8.5×10^{-6}
Absolute temperature measured at "V" [K]	T	296.5
Gain of the control signal [$(kg\ m)^{1/2}/V$]	K_1	$4.73230009451088 \times 10^{-6}$
Gain of the regulated manometric pressure [$m\ s^2\ (m/kg)^{1/2}$]	K_2	$8.17139272807425 \times 10^{-11}$
Time constant [s]	τ	0.00489

5. Conclusions

The obtained results shows that the developed mathematical model represent the real behavior very well, allowing to extend this model to other valves of pressure control.

The adopted simplifications of the model seem to be of high efficient for the proper characterization of this type of component.

6. References

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