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APPLICATION OF DIRECT ADAPTIVE GENERALIZED PREDICTIVE CONTROL (GPCAD) TO A ROBOTIC JOINT

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Abstract – *The number of robots working in industry significantly increases due to their capacity to realize operations that require flexibility, rapidity and accuracy. However, as quick flexible manipulators are essential to achieve this performance leading to a minor production time and small energy consumption, more resourceful control algorithms must be implemented, which can cope with important parameters variations, such as inertia.*
On the other side, even if predictive control has proved to be an efficient control strategy in industry, the maintenance of a high level of performances may be impossible to reach with a fixed predictive controller in case of important parameters variations. A solution is then to develop an adaptive version of the predictive controller for systems with parametric disturbances.
This paper presents a direct version of adaptive generalized predictive control. The algorithm is rewritten in an original form minimizing a performance index, using a least-squares type strategy for the controller parameters on line identification and including a conditional updating test in the adaptation loop. An application of this structure to a robotic joint is finally developed, and a comparison between fixed predictive control and adaptive predictive control strategies stresses the advantages of adaptation in case of important inertia variations.

Keywords *Generalized Predictive Control (GPC), Adaptive control, robotic joints and flexible manipulators.*

1 INTRODUCTION

The progress of technology and the close relationships among several sciences, such as micro-electronic, software engineering and telecommunications, open space for a great development in the robotics area and process automation.

The number of robots working in industry increases progressively because of their capacity to realize operations that demand flexibility, rapidity and accuracy (David and Rosario, 1998). But for many operations, the operator defines the tasks of the controller with respect to a coordinate system that is fixed to the end-effectors of the robot (in the Cartesian space). But the desired movements (expressed in angular coordinates) and the control laws are in different coordinate systems, demanding the implementation of fast algorithms for the inversion of the geometrical model and generation of the reference trajectory (see Figure 1).

Moreover, as quick flexible manipulators are required to development rapidity and accuracy for a minor production time and small energy consumption (Pimenta, *et al*., 2001), more resourceful control algorithms must be implemented, which can cope with important parameters variations (e.g. inertia).

This paper presents in Section 2 the dynamic model of such a robot, then Section 3 develops the basic ideas of generalized predictive control. Section 4 examines the direct adaptive predictive control strategy. Finally some experimental results of a one-degree of freedom robotic joint are given in Section 5, with both GPC and GPCAD controllers for comparison purposes.

2 MODELLING OF THE ROBOT

The position control of a manipulator can be implemented through the control of each isolated joint (Craig, 1989). Each joint model is thus necessary, but all the joints must be coordinated as shown on Figure 1, so that the dynamic model of the structure must also be defined.

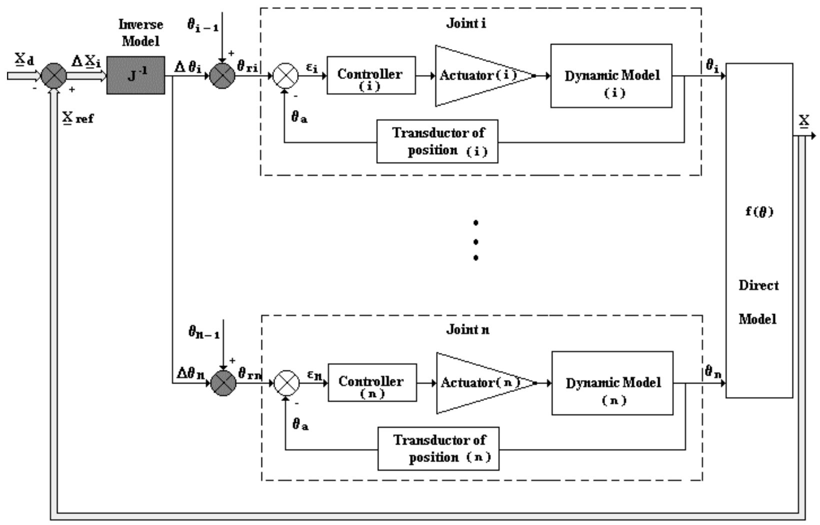


Figure 1: Inverse Kinematic Model (J Jacobian Matrix, $f(\theta)$ Direct Kinematic Model).

2.1 Dynamic model of a manipulator

The dynamic control involves the determination of the inputs so that the drive of each joint moves its links for the position values with desired speed. The dynamic model of a robotic joint can be derived through the Euler-Lagrange formulation that expresses the generalized torque (Craig, 1989):

$$\tau(t) = J(\theta(t))\,\ddot{\theta}(t) + C(\theta(t),\dot{\theta}(t)) + Q(\theta(t)) \tag{1}$$

Where $\tau(t)$ is the generalized torque vector, $\theta(t)$ the generalized frame vector (joints), $J(t)$ the inertial matrix, $C(\theta,\dot{\theta})$ the non-linear forces (for example centrifugal) matrix, $Q(\theta)$ the gravity force matrix.

2.2 Actuator model

Each robotic joint commonly includes a d.c. motor, a gear and an encoder. Considering the d.c. motor, the three classical equations are the following:

$$\begin{aligned} u(t) &= L\,\dot{i}(t) + R\,i(t) + K_v\dot{\theta}(t) \\ T(t) &= J_m\,\ddot{\theta}(t) + C_m\,\dot{\theta}(t) \\ T(t) &= K_T\,i(t) \end{aligned} \tag{2}$$

Where $T(t)$ is the motor torque, $\theta(t)$ the angular position of the motor, $i(t)$ the motor current, L, R, J_m respectively the inductance, resistance and inertia of the motor, leading to the block diagram of Figure 2.

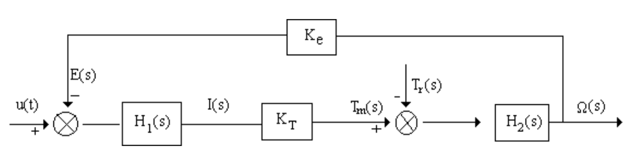


Figure 2: Block Diagram of the d.c. Motor.

Where $H_1(s)$ and $H_2(s)$ are transfer functions respectively corresponding to the electrical and mechanical time constants. This model coupled with the previous dynamic one enables the representation of a complete isolated joint.

3 GENERALIZED PREDICTIVE CONTROL

Predictive Control philosophy, aiming at creating an anticipative effect using the explicit knowledge of the trajectory in the future, can be summarized as follows (Boucher and Dumur, 1995):

- Definition of a numerical model of the system, to predict the future system behavior,
- Minimization of a quadratic cost function over a finite future horizon, using future predicted errors,
- Elaboration of a sequence of future control values, only the first value is applied both on the system and on the model,
- Repetition of the whole procedure at the next sampling period according to the receding horizon strategy.

This part reminds the main developments of Generalized Predictive Control (GPC) (Clarke, *et al.*, 1987).

3.1 Model definition

The predictive control law uses an external input-output representation form, given by the polynomial relation:

$$A(q^{-1})y(t) = B(q^{-1})u(t-1) + \frac{\xi(t)}{\Delta(q^{-1})} \tag{3}$$

where u, y are the input and output of the system, $\Delta(q^{-1}) = 1 - q^{-1}$ the difference operator, A and B polynomials in the backward shift operator q^{-1}, of respective order n_a and n_b.

3.2 Prediction equation

The predictive methodology requires the definition of an optimal j-step ahead predictor which enables to anticipate the behavior of the process in the future over a finite horizon. From the input-output model Eq. 3, a polynomial predictor is designed under the following form:

$$\hat{y}(t+j) = \underbrace{F_j(q^{-1})y(t) + H_j(q^{-1})\Delta u(t-1)}_{\text{free response}} + \underbrace{G_j(q^{-1})\Delta u(t+j-1) + J_j(q^{-1})\xi(t+j)}_{\text{forced response}} \tag{4}$$

Unknown polynomials F_j, G_j, H_j and J_j corresponding to the expression of the past and of the future, are derived solving Diophantine equations, with unique solutions.

3.3 Cost function

The GPC strategy minimizes a weighted sum of square predicted future errors and square control signal increments:

$$J = \sum_{j=N_1}^{N_2} \left(\hat{y}(t+j) - w(t+j)\right)^2 + \lambda \sum_{j=1}^{N_u} \Delta u(t+j-1)^2 \tag{5}$$

3.4 Cost function minimization

The optimal j-step ahead predictor Eq. 4 is rewritten in a matrix form:

$$\hat{\mathbf{y}} = \mathbf{G}\,\tilde{\mathbf{u}} + \mathbf{if}(q^{-1})\,y(t) + \mathbf{ih}(q^{-1})\,\Delta u(t-1) \tag{6}$$

with:
$$\mathbf{if}(q^{-1}) = \left[F_{N_1}(q^{-1}) \quad \cdots \quad F_{N_2}(q^{-1})\right]'$$
$$\mathbf{ih}(q^{-1}) = \left[H_{N_1}(q^{-1}) \quad \cdots \quad H_{N_2}(q^{-1})\right]'$$
$$\tilde{\mathbf{u}} = \left[\Delta u(t) \quad \cdots \quad \Delta u(t+N_u-1)\right]'$$
$$\hat{\mathbf{y}} = \left[\hat{y}(t+N_1) \quad \cdots \quad \hat{y}(t+N_2))\right]'$$
$$\mathbf{G} = \begin{bmatrix} g_{N_1}^{N_1} & g_{N_1-1}^{N_1} & \cdots & \cdots \\ g_{N_1+1}^{N_1+1} & g_{N_1}^{N_1+1} & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ g_{N_2}^{N_2} & g_{N_2-1}^{N_2} & \cdots & g_{N_2-N_u+1}^{N_2} \end{bmatrix}$$

With $\mathbf{w} = \left[w(t+N_1) \quad \cdots \quad w(t+N_2)\right]'$, the future control sequence is obtained minimizing the criterion Eq. 5:

$$\tilde{\mathbf{u}} = \mathbf{M}\left[\mathbf{w} - \mathbf{if}(q^{-1})\, y(t) - \mathbf{ih}(q^{-1})\, \Delta u(t-1)\right] \tag{7}$$

with: $\mathbf{M} = \mathbf{Q}\ \mathbf{G}'$, $N_u \times (N_2 - N_1 + 1)$ matrix
$$\mathbf{Q} = \left[\mathbf{G}'\mathbf{G} + \lambda\, \mathbf{I}_{N_u}\right]^{-1}$$

Only the first value of Eq. 5 is finally applied to the system according to the receding horizon strategy. The equivalent RST controller is computed through a difference equation (with $\mathbf{m}_1$ first line of the $\mathbf{M}$ matrix):

$$\Delta u(t) = T(q)\, w(t) - R(q^{-1})\, y(t) - S^*(q^{-1})\, \Delta u(t) \tag{8}$$

with:
$$S^*(q^{-1}) = \mathbf{m_1}\ \mathbf{ih}(q^{-1})\, q^{-1} \qquad \text{degree}\left[S^*\right] = \text{degree}\left[B\right]$$
$$R(q^{-1}) = \mathbf{m_1}\ \mathbf{if}(q^{-1}) \qquad \text{degree}\left[R\right] = \text{degree}\left[A\right]$$
$$T(q) = \mathbf{m_1}\left[q^{N_1} \quad \cdots \quad q^{N_2}\right] \qquad \text{degree}\left[T\right] = N_2$$

4 DIRECT ADAPTIVE GPC

In the time-varying parameters case, the previous controller must be included within an adaptive structure. A first possibility is to build an indirect scheme (Söderström and Stoica, 1989), updating the plant parameters, then calculating the controller with the new set of estimated plant polynomials. This approach is however time-consuming with predictive structures, because all predictors must be re-computed on-line for the design of the new RST controller.

Consequently, it is proposed here an original approach, which directly updates the controller parameters. This strategy imposes first to reformulate the polynomial controller under an adequate form, without changing the structure of the control law, then to develop an identification algorithm which may update the controller parameters vector.

4.1 Reformulation of the control law

With θ the controller parameters matrix of dimension $N_u \times (n_a + n_b + N_u + 1)$:

$$\theta = \left[\mathbf{M\,if} \quad \mathbf{I}_{N_u} \quad \mathbf{M\,ih}\right]' \tag{9}$$

where $\mathbf{M\ if}$ and $\mathbf{M\ ih}$ are matrices formed with the coefficients of the $\mathbf{M\ if}(q^{-1})$ and $\mathbf{M\ ih}(q^{-1})$ matrices.

With the following matrix $\Phi(t)$ called 'regressor', of dimension $(n_a + n_b + N_u + 1)$:

$$\Phi(t) = \begin{bmatrix} y(t) & \cdots & y(t-n_a) & \tilde{\mathbf{u}} & \Delta u(t-1) & \cdots & \Delta u(t-n_b) \end{bmatrix}' \tag{10}$$

The control law Eq. 7 is equivalent to:

$$\mathbf{Mw} = \theta'\Phi(t) \tag{11}$$

This particular form appears to be more adequate for the coming adaptive structure.

4.2 Performance error

It can be shown (Ramond, *et al.*, 1998) that, considering the nominal model, the fixed GPC control law explicitly cancels the following performance error:

$$\mathbf{e}_f(t+N_2) = \mathbf{iP}(t+N_2) - \mathbf{iPw}(t+N_2) \tag{12}$$

with:

$$\mathbf{iP}(t+N_2) = \begin{bmatrix} \mathbf{M} & \lambda\mathbf{Q} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{y}} \\ \tilde{\mathbf{u}} \end{bmatrix} = \mathbf{M}\,\hat{\mathbf{y}} + \lambda\mathbf{Q}\tilde{\mathbf{u}}$$

$$\mathbf{iPw}(t+N_2) = \begin{bmatrix} \mathbf{M} & \lambda\mathbf{Q} \end{bmatrix} \begin{bmatrix} \mathbf{w} \\ \mathbf{0} \end{bmatrix} = \mathbf{M}\,\mathbf{w} = \theta'\Phi(t)$$

With these definitions, $\mathbf{iP}(t+N_2)$ appears as an indication of the measured performances and $\mathbf{iPw}(t+N_2)$ as an evaluation of the expected performances, considering the fact that the output vector $\hat{\mathbf{y}}$ has to converge to the reference vector $\mathbf{w}$ and at the same time the control signal $\tilde{\mathbf{u}}$ has to tend to zero.

The objective in the adaptive case is to minimize this performance error at each step, to reach asymptotically and without plant parameter knowledge:

$$\lim_{t\to+\infty} \mathbf{e}_f(t+1) = 0 \tag{13}$$

Including the RST structure and the performance index, the DAGPC algorithm is given Figure 3.

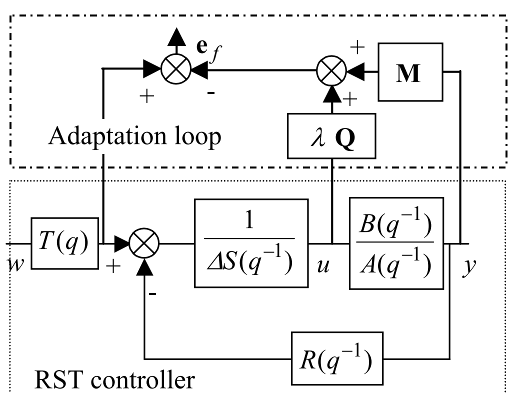


Figure 3: Equivalent structure of the DAGPC.

4.3 Recursive parameters estimation

If the time-varying parameters case is now studied, the fixed controller parameters matrix θ must be moved to an estimated matrix $\hat{\theta}(t)$ to insure the minimization of the performance error. The minimization process Eq. 12 is consequently performed with respect to this $\hat{\theta}(t)$ matrix:

$$\mathbf{e}_f(t) = \mathbf{iP}(t) - \hat{\boldsymbol{\theta}}(t-1)'\Phi(t-N_2) \tag{14}$$

The controller parameters matrix will be updated according to the least squares-type method (Iserman, *et al*., 1992), where the performance error Eq. 14 has to be minimized using the least squares estimator.

To avoid a matrix inversion quite difficult to achieve in a real time context, a recursive least-squares identification is examined (Aström and Wittenmark, 1989), providing on-line the controller parameters matrix $\hat{\boldsymbol{\theta}}(t+N_2)$ as a function of $\hat{\boldsymbol{\theta}}(t+N_2-1)$ under the following form:

$$\hat{\theta}(t+N_2) = \hat{\theta}(t+N_2-1) + \Gamma(t)\Phi(t)\mathbf{e}_f(t+N_2)' \tag{15}$$

$$\Gamma(t) = \Gamma(t-1) - \frac{\Gamma(t-1)\Phi(t)\Phi(t)'\Gamma(t-1)}{1+\Phi(t)'\Gamma(t-1)\Phi(t)} \tag{16}$$

4.4 Conditional updating

The method specificity has consisted since the beginning in the comparison between the measured performances index $\mathbf{iP}(t+N_2)$ and the expected performances index $\mathbf{iPw}(t+N_2)$. Within this context, the idea is to maintain the performances at the same level as they were when the algorithm started, which was supposed to be the nominal point. Consequently, the adaptation must be set off considering the following conditional test:

$$\mathbf{iPw}(t+N_2) > \mathbf{iPw}_{\mathrm{nom}} \tag{17}$$

4.5 Stability of DAGPC

Taking into account the following assumptions:

(A1): Degrees of polynomials *A* et *B* of the CARIMA model are known

(A2): The first N_2 coefficients of the step response are known, so that the G matrix and the polynomials appearing in the optimal predictor are known

(A3): The characteristic polynomial:

$$D(q^{-1}) = M(q^{-1})B(q^{-1}) + q^{-N_2+N_u}\lambda Q(q^{-1})\Delta A(q^{-1})$$

is stable, i.e. $D(q) \neq 0$ for $|q| > 1$.

According to this, the adaptive algorithm gives using the Goodwin lemma (Goodwin and Sin, 1984) :

(S1): The sequences $\{y(t)\}$ and $\{\Delta u(t)\}$ are bounded,

(S2): It leads:

$$\lim_{k\to\infty}\left| \begin{matrix} M(q^{-1})y(k) - M(q^{-1})w(k) \\ \qquad + \lambda Q(q^{-1})\Delta u(k+N_u-N_2-1) \end{matrix} \right| = 0 .$$

These results show that, according to the choice of the tuning parameters achieved on the initial configuration, the adaptive control law stabilizes the system.

4.6 Design methodology

The first aspect of the algorithm is the design on the 'nominal' system at time $t=0$ of a 'fixed' RST GPC controller, which will initialize the whole procedure. The second aspect considers both the control loop and the recursive adaptation loop.

For the next detailed structure, it will be considered now that, as only the first value of the future control sequence is applied to the system, N_u is equal to 1. With that choice, the $\mathbf{M}$ and $\mathbf{Q}$ matrices respectively reduce to a vector $\mathbf{m}_1$ and a scalar q_1, the controller parameters matrix $\hat{\theta}(t)$ becomes a vector.

At time t, the different steps are the following:

1. Measurement of the output value $y(t)$ and computation of the measured performances:

$$iP(t) = \mathbf{m}_1 \; \mathbf{y} + \lambda \, q_1 \, \Delta u(t - N_2)$$

with: $$\mathbf{y} = \left[y(t + N_1 - N_2) \quad \cdots \quad y(t) \right]'$$

2. Computation of the performance error:

$$e_f(t) = iP(t) - \hat{\theta}(t-1)' \Phi(t - N_2)$$

3. Computation of the controller parameters vector:

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \Gamma(t_0)\Phi(t_0)e_f(t)' \qquad (t_0 = t - N_2)$$

$$\Gamma(t_0) = \Gamma(t_0 - 1) - \frac{\Gamma(t_0 - 1)\Phi(t_0)\Phi(t_0)'\Gamma(t_0 - 1)}{1 + \Phi(t_0)'\Gamma(t_0 - 1)\Phi(t_0)}$$

4. Controller parameters vector update. In the direct adaptive control framework, the coefficients of R are the first $n_a + 1$ terms of the $\hat{\theta}(t)$ vector, the coefficients of S^* are the $n_a + N_u + 2$ terms to the $n_a + n_b + N_u + 1$ terms of the $\hat{\theta}(t)$ vector.
5. Application of the control value $u(t)$ to the system, Eq. 8.

5 APPLICATION ON ROBOTIC JOINT

The purpose of this application is to test the GPC and GPCAD algorithms on a one-degree of freedom robot. The considered system (Pimenta, 2003), developed at Unicamp – Brazil**,** is built with a d.c. motor, two disks of inertia and incremental encoders, used for supervision and control (see Figure 4), with the respective parameters (see Table 1):

J_m	Inertia (Kg m^2)	4.802 e-6
B_m	Viscous Friction (Nm/rad/s)	6.74 e-5
τ_m	Mechanical time constant (ms)	2.3
K_T	Torque constant. (Nm/A)	4.090 e-2
K_E	Velocity constant (V/rad/s)	4.11 e-2
R	Resistance (Ω)	0.85
L	Inductance (mH)	< 0.1

Table 1: System parameters.

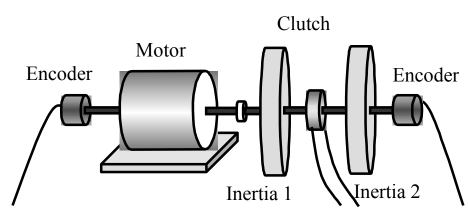


Figure 4: Model of a Robotic Joint.

The purpose of following experiments is to control the motor position. An initial GPC controller has been designed for a sampling period of 20 ms (this value has been chosen to allow real time implementation due to the computation burden of the adaptive part), satisfying stability and robustness features (Boucher and Dumur, 1996), with the tuning parameters:

$$N_1 = 1; N_2 = 5; N_u = 1; \lambda = 32 \tag{18}$$

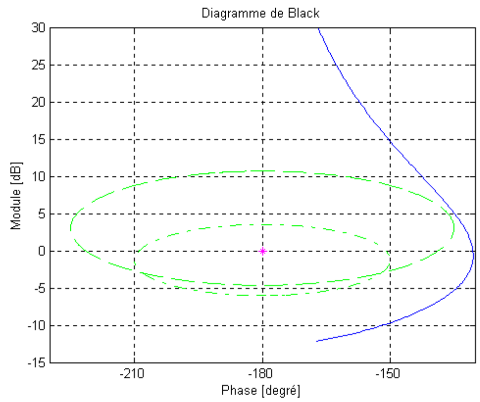
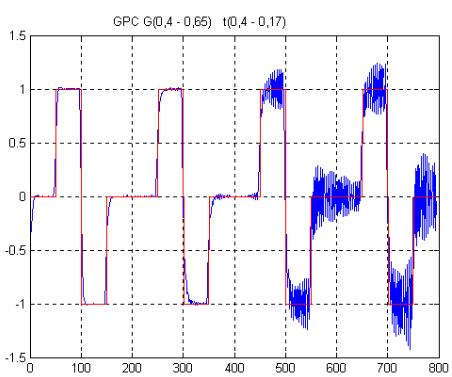


Fig.5 Black diagram of the controlled open loop

This tuning provides the required stability margins, as illustrated in the Black diagram Fig. 5. This diagram also shows the direct and complementary sensitivity functions, which area is avoided by the controlled system frequency response, thus providing good robustness towards disturbance (e.g. influence of a measurement noise on the control signal, influence of an output disturbance on the output (Boucher and Dumur, 1996).

During the experiments, some variable inertia is connected to the system, thus changing the initial configuration, mainly the mechanical time constant. The dc gain of the mechanical chain can also be changed during the experiment.



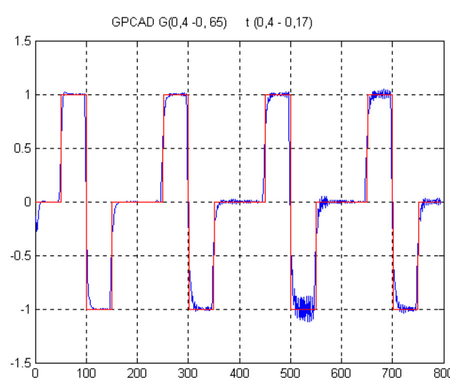


Figure 6: Simulation with the same parameters (GPC and GPCAD).

In the figure 6, it can be observed the comparative results between GPC and GPCAD, the same variation of parameters was applied, where increased Gain (G) more than 50% and decrease of more than 50% of the mechanical constant. These results prove the positive effect of adaptation, stabilizing the system after a short transient, compared to the fixed GPC which becomes unstable.

6 CONCLUSIONS AND FUTURE WORKS

This paper has presented a direct version of adaptive Generalized Predictive Control, for which the GPC controller is rewritten in an adequate form for adaptation. The recursive adaptation loop has allowed updating the controller parameters vector according to a least square-type strategy, including a particular conditional updating test.

An application to a one-degree of freedom robot has been examined. It has proved that, even with important variations of the parameters, the developed adaptive structure maintains a high level of performances, in terms of rapidity and overshoot, cancellation of oscillations.

This adaptive control law is very easy to implement and considering the small number of identified parameters (due to small degrees of the GPC *R* and *S* polynomials) can be integrated in real-time context without any difficulty. Further work should focus on stability proof of this structure as a function of the different tuning parameters and explicit consideration of constraints in the cost function.

7 AKNOWLEDGEMENT

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