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NONLINEAR PLANT IDENTIFICATION BY WAVELETS

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Abstract. *This article combines the wavelet theory with the basic concept of neural network to define a new mapping network called adaptive wavelet neural network (wavenet). It is an alternative to feedforward neural networks for approximating arbitrary nonlinear functions. The wavenet algorithm consist of self-construction of the network and the minimisation of the error. In the first step, the neural network structure is defined,the network gradually recruits hidden units to effectively and sufficiently cover the time-frequency region occupied by a given target. The network parameters are updated simultaneously preserving the network topology. In the second step, the instantaneous errors are minimised using an adaptation technique based on the LMS algorithm. The approach is applied to nonlinear plants identification, using simulated data.*

Keywords*. Neural Network, Wavelet, Identification*

1. Introduction

A great desire of people since the moment that the machines began to be developed have been the creation of a machine that can operate independently of the human control, the called autonomous, intelligent or cognitive machines. A machine of which the operating control is developed in agreement with its own learning capacity to interact with the environment and its uncertain, would respond to this expectation. One of the challenges for the developing of these machines has been the obtaining of techniques and mathematical models to represent the physical processes and phenomena observed in the nature reliably.

The model of a system can be represented by a mathematical model based in the physics laws that govern the problem or by using the experimental data measured in the own system, the called identification systems (Ljung,1987). In the first case, the model is defined from well known physical principles, which allow a well defined mathematical model. In the case of identification, the methods do not need previous knowledge of the system, so they are known as black-box identification process (Sjöberg et al. ,1996).

The modelling based on experimental data is an important issue in the area of identification and control. The technological advance verified in the last decades regarding the use of computers in the mains areas of engineering and sciences has been moving the focus of identification process. Some phenomena, usually, represented by linear models, actually have been substituted by their correspondent nonlinear, aiming at a better characterisation of the real phenomena. In some applications, the substitution of the linear models by their corresponding nonlinear models permits a better representation of given phenomena (Aguirre, 1996). This new focus is gain more importance in the area of identification and control, mainly in the case, where there is no sufficient information about of the model of the plant and it may present some nonlinearities. In this case, where the plant is unknown, it is necessary some techniques to identify the process with sufficient accuracy.

Various mathematical tools have been proposed in the literature for this purpose, showing that they can cope with significant unknown nonlinearities (Camargos e Khater, 2001). *Wavelet* neural network is one of these tools that more recently has been used for identification of unknown plant as well as for design of neural network controllers since it can cope with some nonlinearities present in the system (Kun e Wen-Kai, 2002).

In this context, the proposal of this work is to exploit the neural networks and *wavelet* transform, aiming at to define an adaptive *wavelet* neural network for identification of nonlinear plants. Once the plant is identified, it is possible to gather the necessary information to build a neuro-controller capable to manipulate the plant at a high cognitive level. The article presents the algorithm *wavenet*, showing the main details of its formulation, presents some example of identification and a discussing of the obtained results.

2. Wavenet

Wavelet transform combined with feedforward neural network have been discussed as an alternative to linear feedforward neural network for approximating arbitrary nonlinear functions in R^n (Lekutai, 1997; Pati and Krishnaprasad,1993). This combination leads to an adaptive *wavelet* neural network, called *wavenet*. It can be understood basically as a neural network that presents several daughters *wavelet*s acting as the activating functions.

The algorithm consists of a process of self construction of a neural network that gradually recruits the neurons (*wavelet windows*) to sufficiently cover the time-frequency region occupied by the signal to be identified (target). The parameters of the neural network are simultaneously updated through the minimisation of a sum of the mean square error. Figure (1) shows schematically the identification process of an unknown plant, by using the concept of *wavelet neural network*.

The signal *u(t)* represents the plant input that will also feed the neural network, producing an output $\hat{y}(t)$. Then the output of the network is compared with the output of the plant *y(t)* and the difference between the output of the neural network and the signal *y(t)* (target) is expressed by an error *e(t)*. The error $e(t)$ is minimised in relation to the parameters of the network, in iterative process, until the neural network output to be approximately equal to the plant output.

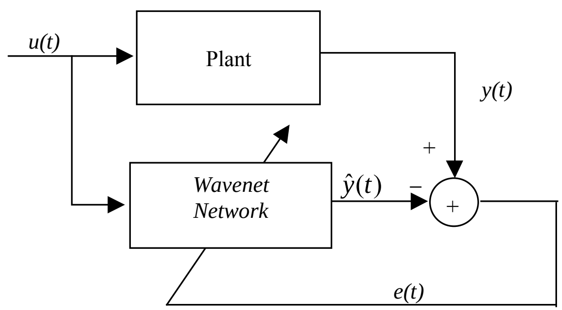
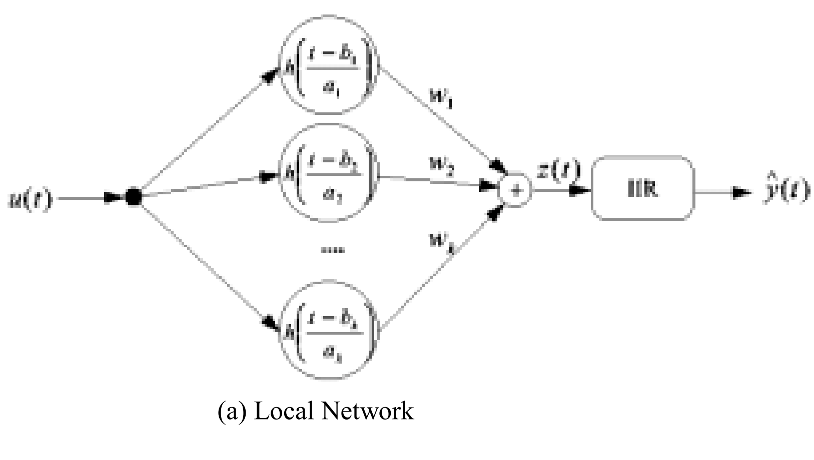


Figure1. Plant identification

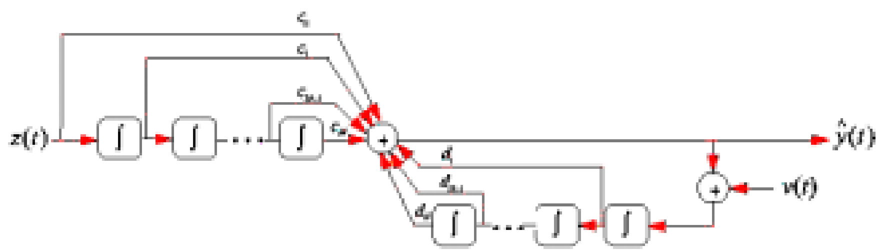
The neural network construction starts with the definition of the number of neurons (*wavelet window*), each *window* is defined by the parameters a>0 (scales) and b(shifting) and it is weighted by a coefficient w_k. These parameters, as well as, the coefficient w_k are all updated at each iteration. The neural network is fed by the input *u(t)* aiming to produce an output signal as close as possible to the output of the plant. Figure (2) details the architecture of the *wavenet* algorithm.

The input *u(t)* excites each neuron (*wavelet* window) of the neural network and the output $z(t)$ passes through a filter IIR (Infinite Impulse Response) and it produces the final neural network output $\hat{y}(t)$, as detailed in fig. 2(b). In this case, the signal is identified by a combination of a number of *daughter wavelets* $h_{a,b}(t)$, which are generated by dilation, *a*, and translation, *b*, of a *mother wavelet,* and the coefficients *c* and *d* of the filter (Narenda, 1990; Righeto, 2003). The *daughter wavelets* are defined by varying the parameters *a* and *b* of expression (1).

$$h_{a,b}(t)=h\left(\frac{t-b}{a}\right) \tag{1}$$



(a) Local Network



(b) IIR Model

Figure 2. Adaptive *Wavelet* Network Structure.

The approximated signal from the neural network $\hat{y}(t)$ is given by the expression (2).

$$\hat{y}(t)=\sum_{i=0}^{M} c_i z(t-i)u(t)+\sum_{j=1}^{N} d_j \hat{y}(t-j)\upsilon(t) \qquad (2)$$

where: $z(t)=\sum_{k=1}^{K} w_k h_{a_k,b_k}(t)$.

K is the number of *wavelet* windows, w_k is the weight of the kth window. *M* is the number of feedforward delays and c_i is the ith IIR filter coefficient. *N is* the number of feedback delays and d_j is the jth recursive filter coefficient. The signals u(t) and $\upsilon(t)$ are respectively the input and co-input of the system. The input $\upsilon(t)$ is usually kept small for feedback stability purposes. The parameters of the neural network $w_{k's}$, $a_{k's}$ and $b_{k's}$ and the filter coefficients $c_{i's}$ and $d_{j's}$ are optimised in the least mean square sense, by minimising a cost function, E , over all time *t*.

$$E=\frac{1}{2}\sum_{t=1}^{T} e^2(t) . \qquad (3)$$

The time-varying error function is defined by Eq. (4),

$$e(t)=y(t)-\hat{y}(t) . \qquad (4)$$

The parameters of the neural network are obtained through the minimisation of the function *E*, in this case, it is used the steepest descending gradient method. Hence, it is necessary the calculation of the gradient vector $\frac{\partial E}{\partial p}$, where *p* is a vector formed by the neural network parameters $w_{k's}$, $a_{k's}$ and $b_{k's}$ and also by the filter coefficients $c_{i's}$ and $d_{j's}$. The incremental change of each parameters is given the expression *5*.

$$\Delta p=-\frac{\partial E}{\partial p}. \qquad (5)$$

The parameters of the neural network are updated in accordance with the rule

$$p(n+1)=p(n)+\mu\Delta p , \qquad (6)$$

μ is a fixed learning rate for the nth iteration.

2.1. Identification Process

This section discusses the identification of a plant by using the adaptive *wavelet* neural network previously presented. The plant presents a nonlinear dynamic and it is excited by a linear input. The plant output *y(n)* (target) is given by the following nonlinear difference equation,

$$y(n) = \frac{y(n-1)}{1+y^2(n-1)} + u(n) \qquad (7)$$

where: $u(n) = 0.8\,\text{sen}(2\pi n/50)$, $n = 1,2,\dots$.

The neural network is defined based on the Polywog *wavelet* family (Lekutai, 1997). Several tests were accomplished, including different *wavelet* families, however, due to the space limitations, it will be presented in this article only the results of the Polywog family. The used *wavelet* is defined by eq. (8) and its shape is illustrated in the fig. (3). The number of neurons (*wavelet)* in the design of the neural network and its effect in the process of identification is also discussed.

$$Polywog = \left(3\tau^2 - \tau^4\right)\exp\left(-\frac{\tau^2}{2}\right) \qquad (8)$$

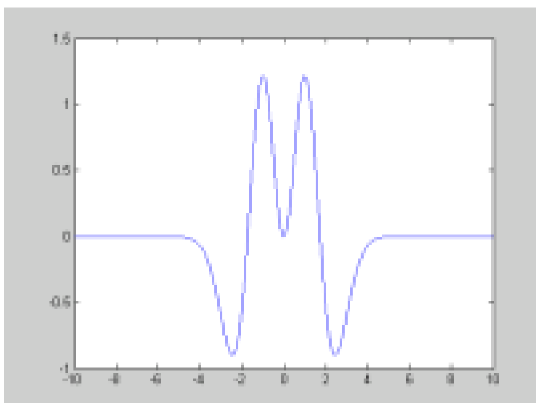
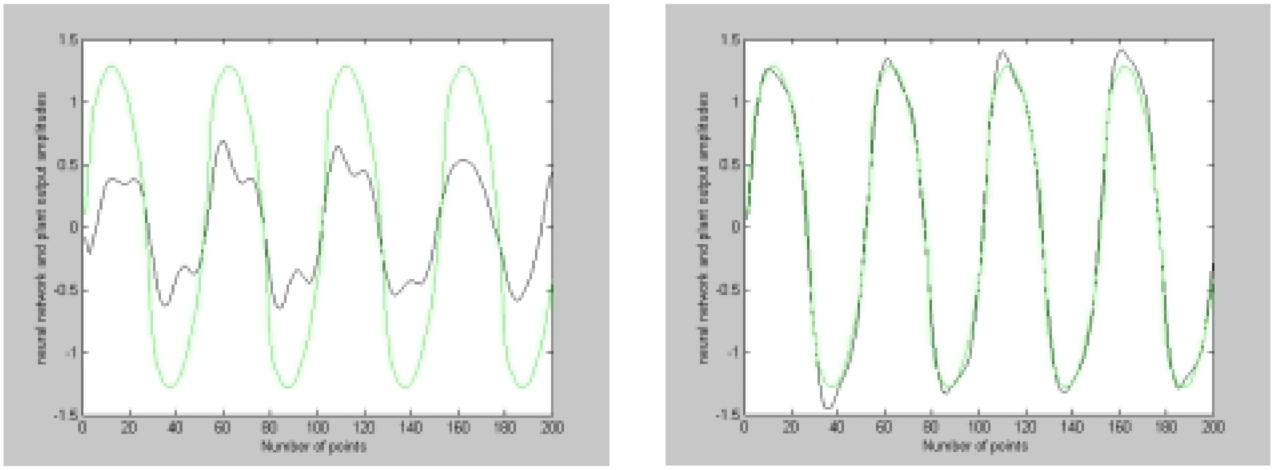


Figura 3. Polywog *Wavelet*

Initially, the plant identification was made using a neural network with 25 neurons and the *wavelets* were distributed along of the signal, uniformly spaced. The identification problem is solved iteratively and the used initial parameters are shown below.

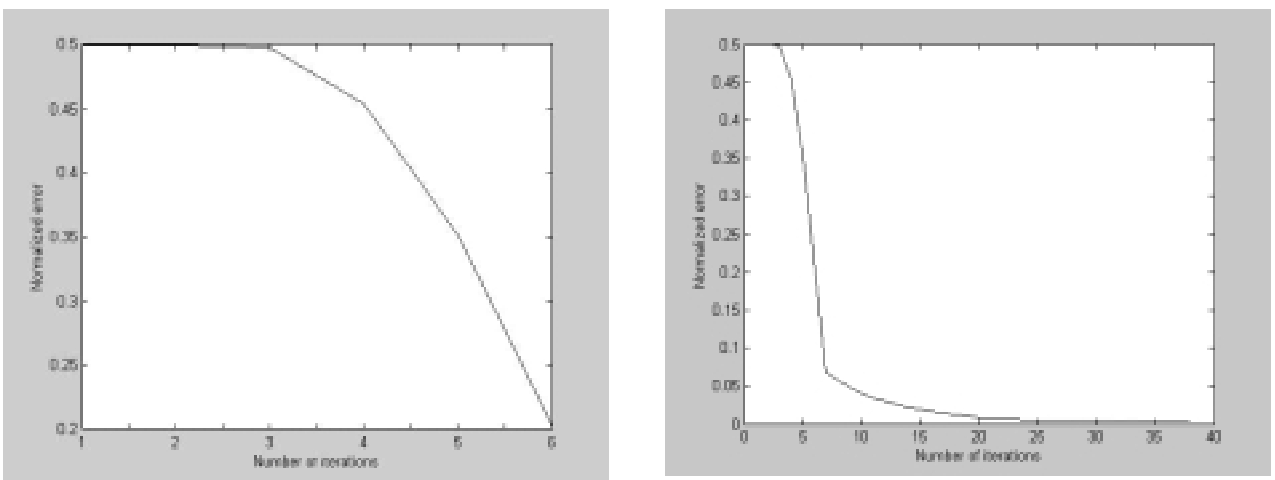
- ✓ scale $a_k = 5$, $k = 1,2,...,25$
- ✓ translation b_k = (number of points/number of neurons) x k = (200/25) x k
- ✓ weight $w_k = 1e-6$
- ✓ feedforward filter coefficients $c_1 = c_2 = c_3 = 0{,}01$
- ✓ feedback filter coefficients $d_1 = d_2 = d_3 = 0{,}01$

The figures (4.a) and (4.b) show the plant and the neural network output results of the identification process for the 6^{th} and 38^{th} iterations, respectively. The process converged and the observed error, at the 38^{th} iterations, it was 0,0018. The figures (5.a) and (5.b) show the evolution of the error.



(a) – Sixth iteration (b) thirty eighth iteration

Figure 4. Output of the neural network (black) output of the plant (green)



(a) (b)

Figure 5. Error obtained by the minimisation process

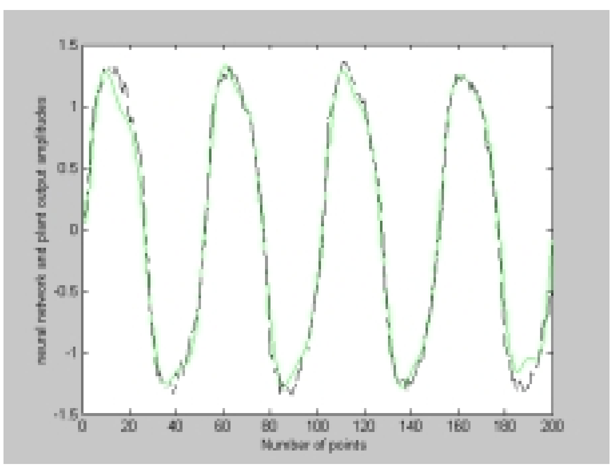
The specified parameters and conditions adopted in neural network shown appropriate for the identification of the plant. The number of neurons used provided satisfactory results. However, it should be pointed that it has a strong influences in the time iteration. Several tests were accomplished varying the number of neurons as well as the initial values of the parameters, aiming at studying their influence in the identification process. Six neural network architectures were studied using 5, 15, 20, 25, 30 and 35 neurons, the initial values of the scale were setted in 20, 10 and 5. All others parameters of the neural network (weights, learning rate, coefficients of the filters), they were maintained unaffected. Table (1) summarises the results obtained for each evaluated condition.

Table 1. Activation function (Polywog)

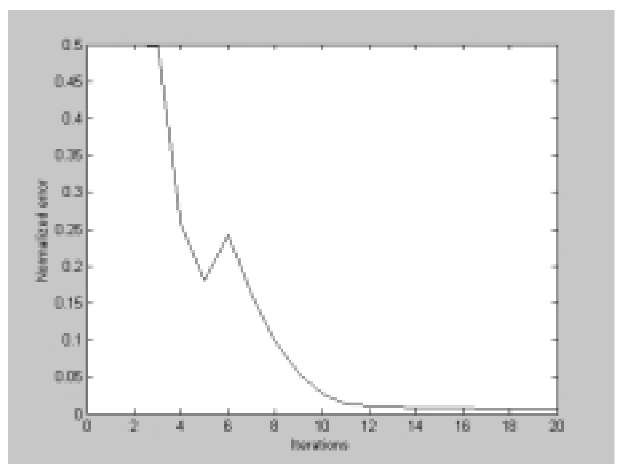
Numbers of neurons (K)	Scales (a)	Normalised Error	Iterations
5	20	0,0145	62
5	10	0,4999	4
5	5	0,1881	62
15	20	0,0143	51
15	10	0,0154	72
15	5	0,0207	122
20	20	0,0109	65
20	10	0,0038	65
20	5	0,0050	102
25	20	0,0155	52
25	10	0,0046	54
25	5	0,0018	38
30	20	0,3421	+335
30	10	0,4151	+201
30	5	0,0017	54
35	20	0,0212	38
35	10	0,0059	40
35	5	0,0024	46

The Tab. (1) shows that for this nonlinear plant, under the convergence point of view and smaller error, the number of neurons more appropriate to represent the plant is around 25 neurons. The normalised errors are of same order, however for an increasing of the number of neurons, the error increase and suffer a strong influence of the scale value. Under the point of view of time of computation, the results showed that it was not possible to establish a correlation between the increasing and the reduction of the number of neurons. In general, the results for K=5 and K=15 neurons show that these values are not recommended for analysis of the signal described by Eq. (7), due to the order of the error.

Additional tests were made to study the robustness of the identification process with relation to eventual disturbances in the plant. Some tests were accomplished using the original signal of the plant uncorrupted with noise (variance 0,05, uniformly distributed). Figure (6) displays the results obtained for a neural network with 25 neurons and initial scales $a_{k's}$ =5.



(a)



(b)

Figure 6. Results of the simulation done with noisy signal

The preliminary results, show that the network is few sensitive for noise/contamination of this order and that the final error after the convergence, is larger but it still stays in the same order of greatness. The fig. 6(a) displays the noisy plant output (black) and the neural network output (green). The fig. 6(b) illustrates the behaviour of the error. The algorithm converged after twenty iterations and the error reached the value 0,005. In the case of the plant not corrupted by noise and maintaining all the other information unaffected, the plant is identified with an error of 0,0018, as shown in the Tab. (1).

3. Conclusion

The neural networks activated by the *wavelets* were shown effectively in the identification processing of signals, particularly, in the identification of nonlinear plants, which could represent a good model approach for control adaptive proposal. The identification tasks discussed in this work showed that the success of the identification depends on the choice of the *wavelet* family used as activation function, as well as, the number of neurons, which should cover the whole extension of the signal efficiently (target). Those choices up to now are made in a heuristic way and the definition of an appropriated architecture of the neural network it is not trivial. However the great amount of simulations realised showed that *wavelets* are quite efficient in the process of identification. It needs some improvement in the method of optimisation to give better accuracy in the results, reducing the time and the number of iterations aiming at an application in adaptive control.

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