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HYBRID CONTROL TO MITIGATE EXCESSIVE VIBRATIONS CAUSED BY DYNAMIC LOADING WITH RANDOM PERTURBATIONS IN TALL BUILDINGS

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Abstract With the development of more efficient structural analysis tools and new construction techniques, modern buildings are becoming taller and more flexible in order to satisfy the requirement for more space. At the same time this type of structure becomes more vulnerable to excessive vibration caused by natural hazards, such as earthquakes and strong winds, which can cause damage and even the collapse of the structure. Structural control offers a promising alternative in protecting structures while maintaining desirable dynamic properties. Over the years passive control devices such as tuned mass dampers (TMD) have been successfully studied and installed on some civil engineering structures worldwide. Hybrid control strategies have been investigated by many researchers to exploit their potential to increase the overall reliability and efficiency of the controlled structure. A hybrid control system is typically defined as one that employs a combination of passive and active devices. In this work the hybrid control of a tall building subjected to an harmonic loading with random perturbations is studied. Adding to the considered harmonic load a random noise will make the analysis more realistic, since most of the civil engineering structures are subjected to random natural loadings. The chosen control mechanism is the so called hybrid mass damper (HMD), a combination of a tuned mass damper (TMD) and an active control actuator. The active control actuator force is calculated using the linear optimal control algorithm. The HMD behavior and efficiency compared to the passive mass damper is analyzed in detail. It is verified that the hybrid control system is more efficient in reducing vibration caused by harmonic loading with random perturbation than the passive mass damper, which is traditionally designed considering an harmonic load.

Keywords. *structural dynamics, structural control, hybrid control, tuned mass damper, hybrid mass damper*

1. Introduction

The protection of civil structures against undesirable vibrations, including their material contents and human occupants, is without doubt a worldwide priority. These high vibration levels can cause discomfort and, even more, compromise the structure safety and integrity. Events which cause the need of such protective measures are earthquakes, waves, winds, traffic, human occupation and even deliberate acts, among others.

An alternative to this problem studied by many researchers in the last years is the structural vibration control. It changes structural properties, enhancing damping, stiffness and strength, by installing external devices or applying external forces to the structure. There are basically three types of structural control: active, passive and hybrid.

Hybrid control systems combine active controllers with passive devices. The active portion of an hybrid system requires much less power than a similar active system, while providing better structural response than the passive system alone. This type of control has been extensively studied in recent years (see, for example, Tzan & Pantelides, 1994; Lee Glauser *et al*, 1997; Riley *et al.*, 1998; Spencer Jr. & Soong, 1999; Nishitani & Inoue, 2001; Avila, 2002; Avila & Gonçalves, 2002a).

A large number of studies on hybrid control analyzes its application for structure protection against earthquakes. Tzan & Pantelides (1994) combined viscoelastic dampers with active bracing systems, while Irshick *et al* (1998) and Riley *et al* (1998) applied hybrid control strategies to base isolation systems.

An hybrid control device already used in Japan and Taiwan buildings (Nishitani & Inoue, 2001) is the so called hybrid mass damper (HMD), a combination of a tuned mass damper (TMD) and an active control actuator (Nagashima *et al*, 2001; Fujinani *et al*, 2001; Avila & Gonçalves, 2002b).

In the present work a HMD is used to control the dynamical response of a tall building subjected to an harmonic load with random perturbations. Most of civil engineering structures are subjected to random natural loadings and even when deterministic loads such as harmonic loads, a random noise is usually present. The mathematical modeling and numerical methodology to obtain the non-deterministic force is described in Santee (1999). The active control actuator force is calculated using the linear optimal control algorithm (Meirovitch, 1990).

The TMD and HMD performances are compared. It is observed that the hybrid control system is more efficient in reducing vibration caused by loadings with random perturbations than the passive mass damper, which is traditionally designed considering an harmonic loading.

2. Equations of Motion

The equations of motion of a multi degree of freedom (MDOF) system with an HMD are similar to those of a structure controlled by a TMD. The difference is the inclusion of the control force $u(t)$, leading to

$$\boldsymbol{M}\,\ddot{\boldsymbol{y}}(t)+\boldsymbol{C}\,\dot{\boldsymbol{y}}(t)+\boldsymbol{K}\boldsymbol{y}(t)=\boldsymbol{F}(t)+\boldsymbol{D}p(t) \tag{1}$$

$$m\,\ddot{z}(t)+c\,\dot{z}(t)+kz(t)=-m\,\ddot{y}_N+u(t) \tag{2}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are the mass, damping and stiffness matrices respectively; $F(t)$ is the dynamic loading applied to the structure; $p(t) = c\,\dot{z}\,(t)+kz(t)-u(t)$; $y_i(t)$ is the displacement of the ith mass relative to the ground; $z(t)$ is the displacement of the HMD with respect to the floor where it has been installed; $\boldsymbol{D}$ represents the HMD location vector. The dth component of $\boldsymbol{D}$ is given by

$$d_j=\begin{cases}0, & j\neq i\\ 1, & j=i\end{cases} \tag{3}$$

where i identifies the floor on which the HMD is installed; when the HMD is on the top floor $i=N$.

In MDOF systems where the natural frequencies are well spaced, such as some high buildings, the structural response can be obtained by a reduced model using modal analysis (Soong & Dargush, 1997). The TMD effectiveness is greatest when the structure analyzed oscillates around a predominant mode, so the response vector $\boldsymbol{y}(t)$ can be approximately represented by a single coordinate y_N and a mode shape ϕ_1, i.e.,

$$\boldsymbol{y} = \boldsymbol{\phi_1}\, y_N. \tag{4}$$

Substituting Eq. (4) into Eq. (2) and pre-multiplying Eq. (2) by ϕ_1^T, equation (2) becomes

$$M_1^*\,\ddot{y}_N+C_1^*\,\dot{y}_N+K_1^*\,y_N=\left(c\,\dot{z}+kz\right)f_1^T+f(t) \tag{5}$$

where $M_1^*=f_1^T\boldsymbol{M}f_1$ is the modal mass; $C_1^*=M_1^*2x_1w_1$ and $K_1^*=M_1^*w_1^2$, ξ_1 and ω_1 being the damping ratio and natural frequency of the first mode of the structure; and $f(t)$ is the dynamic excitation.

The MDOF system modal representation is exactly the same as a single degree of freedom (SDOF) structure, except that the modal mass, stiffness and damping are employed instead of physical parameters. The mass ratio, μ, in this case, is defined as $\mu=m/M_1^*$.

3. Optimal Control Algorithm

The so-called state space approach is the most used in the formulation and solution of control problems. It describes structural systems by a set of simultaneous first-order differential equations of the form

$$\dot{\boldsymbol{z}}(t)=\boldsymbol{A}\boldsymbol{z}(t)+\boldsymbol{B}\boldsymbol{u}(t)+\boldsymbol{H}\boldsymbol{f}(t), \qquad \boldsymbol{z}(0)=\boldsymbol{z_0} \tag{6}$$

where

$$\boldsymbol{z}(t)=\begin{bmatrix}\boldsymbol{y}(t)\\ \dot{\boldsymbol{y}}(t)\end{bmatrix} \tag{7}$$

is the 2n-dimensional state vector,

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -\boldsymbol{M}^{-1}\boldsymbol{K} & -\boldsymbol{M}^{-1}\boldsymbol{C} \end{bmatrix} \tag{8}$$

is the 2n x 2n system matrix, and

$$\boldsymbol{B} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{M}^{-1}\boldsymbol{D} \end{bmatrix} \qquad \text{e} \qquad \boldsymbol{H} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{M}^{-1}\boldsymbol{E} \end{bmatrix} \tag{9}$$

are 2n x m and 2n x r location matrices specifying, respectively, the location of controllers and external excitations in the state-space.

In the active/hybrid control system development, one of the most important stages is the determination of an appropriate control law. This law is obtained through the use of a chosen control algorithm; in the present work the classical linear optimal control algorithm is used (Meirovitch, 1990).

The linear optimal control problem consist in finding the control vector $\boldsymbol{u}(t)$ that minimizes the performance index J subject to the constraint (6). In structural control, the performance index is usually chosen as a quadratic function in $\boldsymbol{z}(t)$ and $\boldsymbol{u}(t)$, as follows

$$J = \int_{t_0}^{t_f} \left[\boldsymbol{z}^{\boldsymbol{T}}(t)\boldsymbol{Q}\boldsymbol{z}(t) + \boldsymbol{u}^{\boldsymbol{T}}(t)\boldsymbol{R}\boldsymbol{u}(t) \right] dt \tag{10}$$

where $\boldsymbol{Q}$ is a 2n x 2n positive semi-definite matrix and $\boldsymbol{R}$ is a m x m positive definite matrix. The matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$ are referred to as weighting matrices, whose magnitudes are assigned according to the relative importance attached to the state variables and to the control forces in the minimization procedure. It is important to notice that HMD devices are single-input systems, where only one control force is necessary. In this case the location matrix $\boldsymbol{B}$ reduces to 2n x 1 matrix and the weighting matrix $\boldsymbol{R}$ reduces to a number.

The minimization problem leads to a Riccati differential equation system of the form

$$\begin{aligned} &\dot{\boldsymbol{P}}(t) + \boldsymbol{P}(t)\boldsymbol{A} - \frac{1}{2}\boldsymbol{P}(t)\boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{P}(t) + \boldsymbol{A}^{\boldsymbol{T}}\boldsymbol{P}(t) + 2\boldsymbol{Q} = \boldsymbol{0}, \\ &\boldsymbol{P}(t_f) = \boldsymbol{0} \end{aligned} \tag{11}$$

where $\boldsymbol{P}(t)$ is the Ricatti matrix.

The control vector $\boldsymbol{u}(t)$ is linear in $\boldsymbol{z}(t)$. In this case, the linear optimal control law is

$$\boldsymbol{u}(t) = \boldsymbol{G}(t)\boldsymbol{z}(t) = -\frac{1}{2}\boldsymbol{R}^{-1}\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{P}(t)\boldsymbol{z}(t) \tag{12}$$

where $\boldsymbol{G}(t) = -\frac{1}{2}\boldsymbol{R}^{-1}\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{P}(t)$ is the control gain. When $\boldsymbol{z}(t)$ is accessible through measurement, $\boldsymbol{u}(t)$ can be determined from Eq. (12). It is known that the feedback determined in this way generates a stable closed-loop system (Soong, 1990).

4. Non-deterministic Force

Consider that the applied load is composed of an harmonic deterministic portion plus a random term, such that

$$F_t(t) = F\cos(\Omega t) + G(t; F, \Omega) \tag{13}$$

where the random term $G(t; F, \Omega)$ depends on the deterministic parameters.

For the numerical simulation, following Santee (1999), the following hyothesis about G are adopted in the present work:

- A force that varies randomly in time is mathematically a stochastic process. A stochastic process is a random variable where the probability distribution depends on a parameter. If the parameter is continuous, the process is called continuous. In the present case, this parameter is time. If the statistics of the process (mean and variance) are time independent the process is called stationary.

- An *Ergodic Process* is a process where the statistics of the random variable $G(t;F,\Omega)$ are the same as the statistics of only a sample of the random process taken along time. An ergodic process is always stationary, but a stationary process may not be ergodic. In this work it is assumed that the random portion $G(t;F,\Omega)$ is an ergodic process and, consequently stationary.

Another hypothesis is that *G* has expected value zero, that is:

$$E[G(t;F,\Omega)] = 0 \tag{14}$$

The description of a stochastic process is usually made in the frequency domain, the hypothesis about the random term is that it has a spectral density function given by

$$\Phi_{GG}(w) = \frac{s_{GG}^2}{2w_l} \text{ for } \Omega - \frac{w_l}{2} < w < \Omega + \frac{w_l}{2} \tag{15}$$

where s_{GG}^2 is the variance of the random force amplitude and ω_l is the random frequency bandwidth.

Figure (1) shows the spectral density function of the random portion. Additionally, it is considered that the standard deviation of the random force amplitude is proportional to the deterministic force amplitude, thus

$$\sigma_{GG} = aF \tag{16}$$

where *a* is the proportion value. It is important to emphasize that the random force depends on frequency and amplitude of the deterministic term.

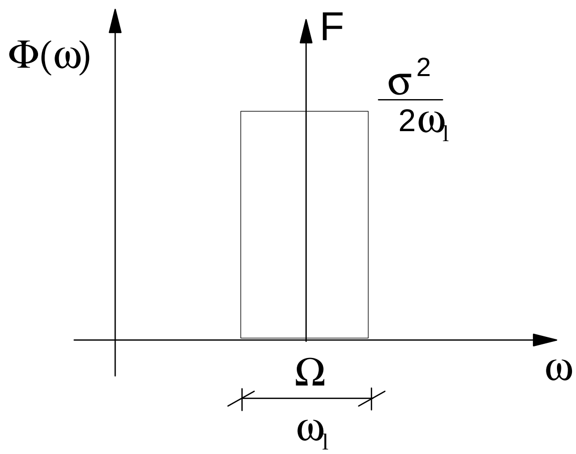

Figure 1. Spectral density function of the random term

Physically, the random term is a noise that increases with an increase in the applied force. Another point to be emphasized is that the random term depends on two prescribed parameters: the standard deviation proportion *a* with respect to the deterministic force amplitude *F*, and the frequency bandwidth w_l around the a forcing frequency Ω.

5. Stochastic ergodic process simulation, with expected value zero and specified spectral density function

In the following the theoretical fundaments and methodology used to generate the random force in time domain is presented (Santee, 1999; Avila, 2002).

The idea of an algorithm to the generate a stochastic process sample *G*(*t*) comes from the expression of the process variance in terms of the spectral density function

$$s_{GG}^{2} = \int_{-\infty}^{\infty} \Phi_{GG}(w)dw \tag{17}$$

Assuming that the process is ergodic, the variance can also be calculated in time domain as

$$s_{GG}^{2} = \underset{T_0 \to \infty}{l.i.m} \frac{1}{T_0} \int_{0}^{T_0} g^2(t)dt \cong \frac{1}{T_0} \int_{0}^{T_0} g^2(t)dt \tag{18}$$

where T_0 is the force duration and $g(t)$ is a sample of the stochastic process $G(t)$.

Based on relations (17) and (18), the variance expressions can be calculated by two different approachs and equaled to obtain a relation between the time function $g(t)$ and the spectral density function, as follows

$$\frac{1}{T_0} \int_{0}^{T_0} g^2(t)dt \cong \int_{-\infty}^{\infty} \Phi_{GG}(w)dw \tag{19}$$

Discretizing Eq. (18), one obtains

$$\frac{1}{N} \sum_{m=0}^{N-1} g^2(m\Delta t) \cong 2 \sum_{k=1}^{N/2} \Phi_{GG}(kw_0)w_0 \tag{20}$$

where $\Delta t = T_0 / N$ and $\Delta w = w_0 = 2p / T_0$

Parceval Theorem (Pneumont, 1994), which relates the amplitude of a stochastic process in time with the process amplitude on frequency domain, states that

$$\frac{1}{N} \sum_{m=0}^{N-1} g^2(m\Delta t) \cong \sum_{k=0}^{N-1} \left| C_g(k\Delta w) \right|^2 \tag{21}$$

where $C_g(w)$ is the Discrete Fourier Transform (DFT) coefficient of the process sample $g(t)$.

Substituting (21) on the right hand side of Eq. (20) and remembering that for $g(t)$ to be real it is necessary that $C_g(N/2 + i) = C_g^*(N/2 - i)$, Eq. (20) can be rewritten as

$$2 \sum_{k=1}^{N/2} \left| C_g(kw_0) \right|^2 \cong 2 \sum_{k=1}^{N/2} \Phi_{GG}(kw_0)w_0 \tag{22}$$

The above expression is true if

$$\left| C_g(kw_0) \right| = \sqrt{\Phi_{GG}(kw_0)w_0}, \qquad k = 1, \ldots, N/2 \tag{23}$$

This expression allows to determine the modulus value of C_g coefficients of a Discrete Fourier Transform sample of the stochastic process $G(t)$, in a way that it has a specified spectral density function. Finally, each DFT coefficient of $g(t)$ can be calculated from

$$C_g(kw_0) = \left| C_g(kw_0) \right| \cos(q_k) + i \left| C_g(kw_0) \right| \mathrm{sen}(q_k) \tag{24}$$

where the phase angles θ_k are random variables with constant distribution between 0 and 2π. Samples of the random variables can be obtained using a random number generator.

The basis of a random force generation methodology is Eq. (24), to use this expression the following initial values are necessary:

- T_0 Random process duration
- N Number of points analyzed on the process
- $\Phi_{GG}(\omega)$ Specified spectral density function

6. Numerical Example

The ten-storey shear building, described in Villaverde & Koyama (1993), is considered in this study. The mass, stiffness and damping properties of the building reduced model are, M_1 = 589.1 t; K_1 = 5.94 x 10^3 kN/m e C_1 = 74.8 KNs/m. A mass damper is connected to the top floor. Admitting a mass relation μ = 0.05, the mass damper structural properties obtained by Den Hartog´s expressions (Den Hartog, 1956) are: mass m_d = 29455.0 Kg; stiffness k_d = 269150.1 N/m and damping coefficient c_d = 23796.5 Ns/m.

Initially consider the mass damper acting passively as a traditional TMD. The dynamic response of the structure subjected to an harmonic loading $F = F_0$ sen Ωt (F_0 = 1 kN; Ω = 3.174 rad/s) and to a non-deterministic force (Ω = 3.174 rad/s; ω_l = 0.5 and a= 0.3), illustrated in Fig. (2), is analyzed. The equations of motion are solved by the fourth-order Runge-Kutta numerical integration technique.

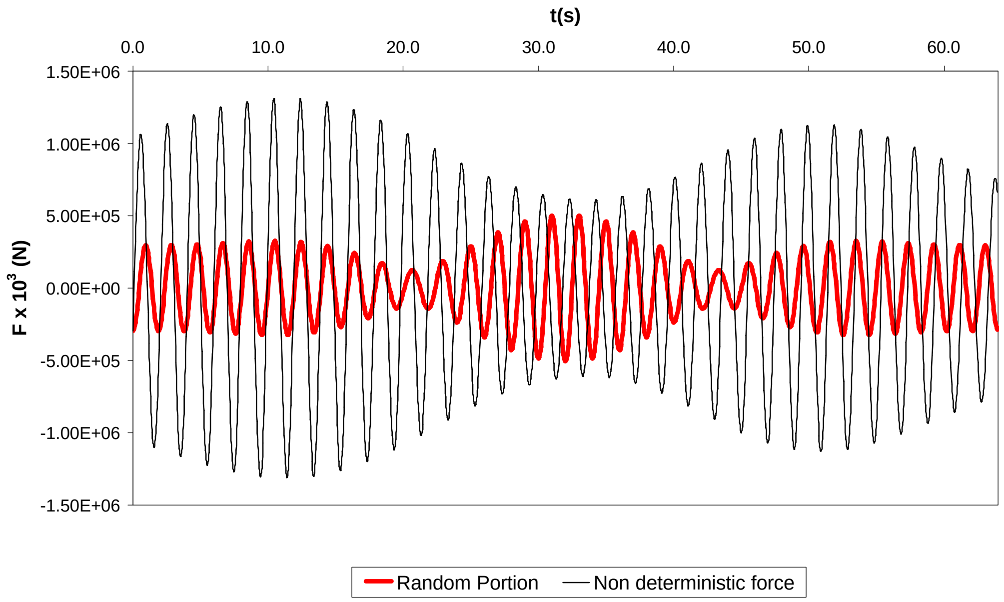

Figure 2. Random force sample (Ω = 3.174; ω_l = 0.5 and a = 0.3)

Fig. (3) shows the time evolution of the displacement of the top floor for both loading cases. The maximum and root mean square (*rms)* values for the top floor displacement are summarized in Tab. (1). It can be observed that the presence of a random disturbance greatly reduces the TMD effectiveness.

Table 1 – Maximum and *rms* displacements of the main structure with TMD

	u_{max} (mm)	u_{rms} (mm)
Deterministic Force	0.8704	0.5377
Non-deterministic Force	1.0560	0.6625
Δ(%)	21.3	23.2

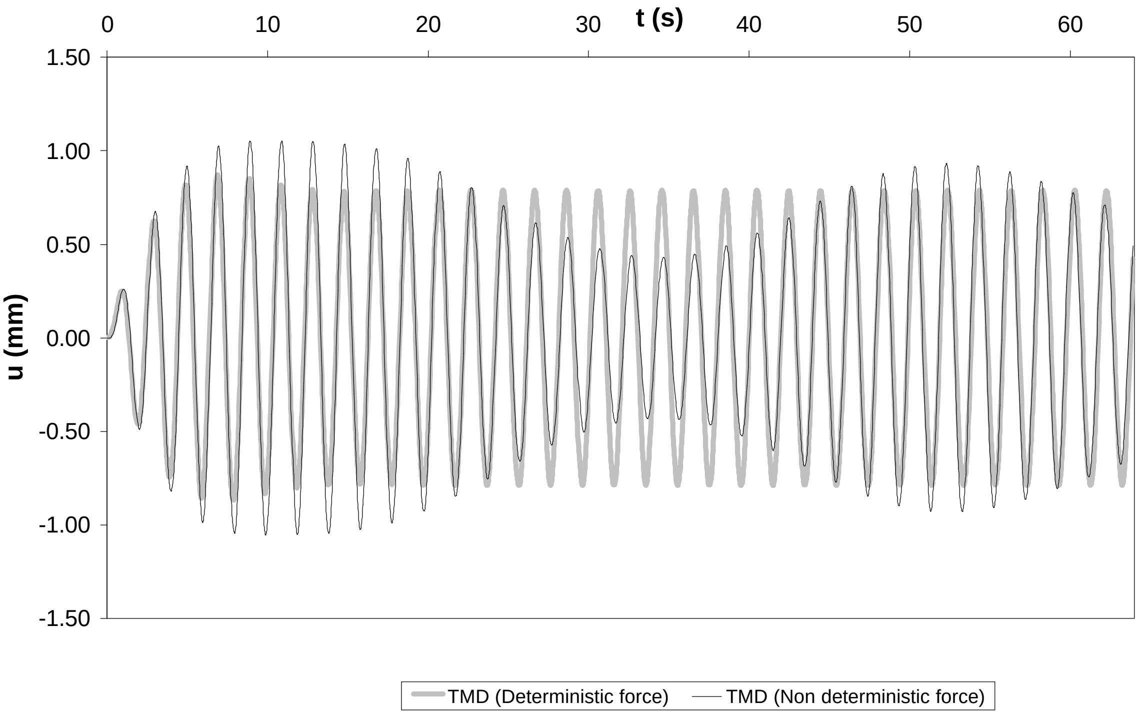

Figure 3. Displacement time evolution of main structure with TMD

Subsequently, the dynamical response is calculated considering the structure protected by an HMD. The control force $u(t)$ is obtained using the following weighting matrices

$$\boldsymbol{Q} = \begin{bmatrix} 10^6 & 0 & 0 & 0 \\ 0 & 10^3 & 0 & 0 \\ 0 & 0 & 10^6 & 0 \\ 0 & 0 & 0 & 10^3 \end{bmatrix} \text{ and } R = 10^{-7}$$

Figure (4) compares the top floor displacement time evolution using a TMD and an HMD, when the structure is subjected to the harmonic load with random perturbation. It can be observed that the HMD is more efficient, leading to maximum and *rms* displacement of about 75% less than those obtained with the passive mechanism only (TMD), as summarized in Tab. 2.

Table 2 – Maximum and *rms* displacements of the main structure with TMD or HMD, subjected to non- deterministic loading

	u_{max} (mm)	u_{rms} (mm)
TMD	1.0560	0.6625
HMD	0.2568	0.16949
Δ(%)	-75.5	-74.4

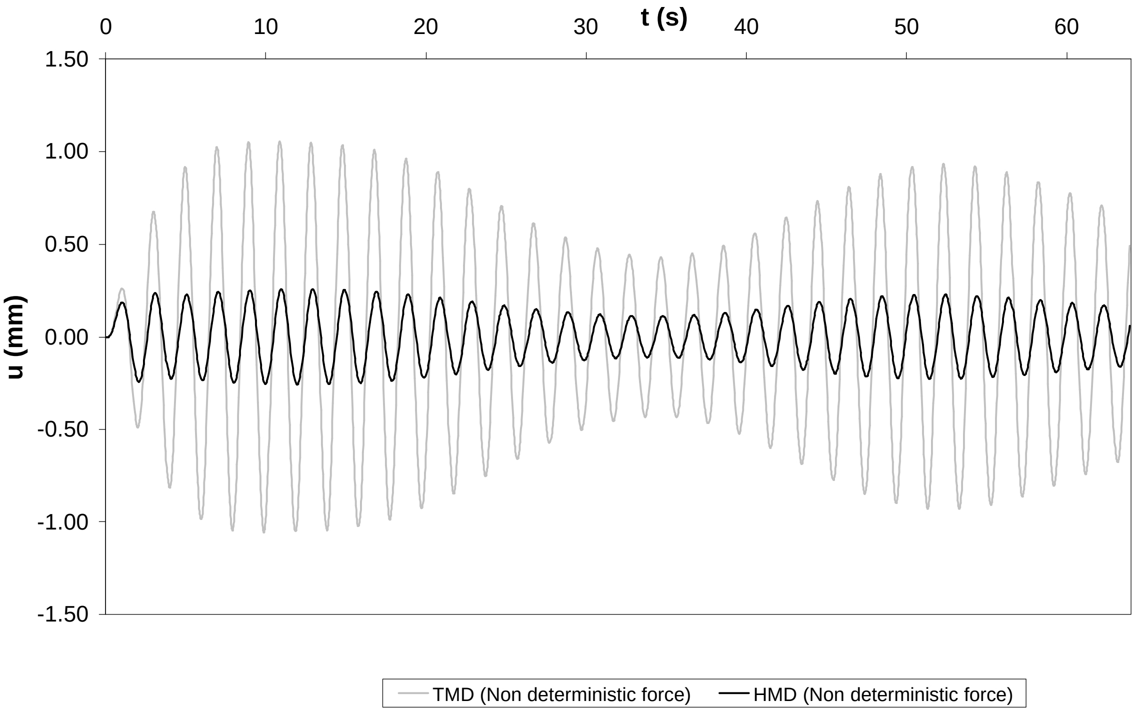

Figure 4 – Displacement time evolution of main structure (TMD x HMD)

The maximum and *rms* forces calculated for the HMD actuator under non-deterministic load are respectively $F_{máx}$ = 703.4 kN and F_{rms} = 692.6 kN. The maximum value is approximately only 2% higher than the necessary to control the same structure subjected to an harmonic load only. The increase in the *rms* force is about 15%.

The control force in the HMD actuator improves the effectiveness of the mass damper, makes it more robust in relation to changes that can probably occur in the harmonic loading considered in the design of the control system.

7. Conclusion

In this paper the HMD performance on controlling a high building was compared to the corresponding passive mechanism, the TMD. The structure was excited by an harmonic load with random perturbations, this type of load makes the analysis more realistic, since civil engineeering structures are often subjected to random natural loadings. It was verified that, in this case, the hybrid control is more efficient than the passive one, because the control force makes it more robust to changes on the design load. The results also show that the control force does not have a significant increase in its maximum magnitude comparing the structure with an HMD subjected to deterministic and non-deterministic loads.

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