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Optimal Control Approach to the Vibrations of a Flexible One-Link Manipulator Carrying Moving Sliders

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Abstract: *Practical operation of many mechanical machines makes use of several internal motions to accomplish complex tasks. The standard procedures for mechanical design assume independent element motions to avoid undesirable induced vibrations and couplings. In most cases, the resulting design leads to heavy structures and slow motions for the machines. Based on the examples of rotating cranes and rotative/prismatic joint robots, this work explores simultaneous rotating/translational motions to minimize vibrations on a light one-link manipulator that performs large rotational maneuvers. In this way, a complex dynamic model for a flexible arm carrying moving sliders is obtained through the Extended Hamilton's Principle. Using substructuring techniques, simpler models can be achieved, comprising any number of structural modes and any number of sliders moving over the arm. This paper considers the case where the flexible link carries two sliders and the first two modes describe the elastic dynamics. An Optimal Control approach is then applied to the resulting non-linear problem of minimizing arm tip vibrations. Several cases were simulated and analyzed using different cost functions, constraints and initial conditions. Results show the benefits achieved when using coupled motions to attenuate manipulator vibrations.*

Keywords*: couplings, light one-link, sliders, Optimal Control, non-linear problem*

1. Introduction

Mechanical systems require low vibration levels to achieve design performance specifications. From the other side, the dynamic characteristics of modern machines demand increasing larger velocities, accelerations and reduce weights and inertias. For many applications, structural flexibilities must be considered since the early design stages in order to assure good vibration attenuation.

This paper address the active control of vibrations on a flexible slowing arm trough the independent motion of sliding masses over the arm. This problem is inspired on cranes and prismatic joint robots, where large angular manouvers are usually employed. The basic question we intended to investigate is how the motion of independent parts may contribute to reduce the vibration levels of the whole system. Although restricted to simple systems, the results achieved are encouraging to extend the research to other more complex applications.

In this paper, the objective are the achievement of suitable system models and the synthesis of optimal controllers using the torque applied to the hub where the flexible arm is fixed and the forces applied to the sliding masses as control variable. The case of a single sliding mass has been presented in the work of Oliveira (2000), Fleury et al. (2002) and Fleury and Oliveira (2003). Here the investigation is extended to include any number of sliders and structural modes Terceiro (2002). The dynamics model of the structural system has been derived through the Extended Hamilton's Principle resulting in a set of coupled integro differential non linear equations where system parameters are time and space variants due to changes in the inertia terms. Using substructuring techniques, arm and sliders motions have been separated and systems responses have expanded in products of spatial and time functions. Several cases where the sliders move according to pre specified trajectories while the flexible arm performs large angular manoeuvers have been simulated and analyzed Oliveira (2000); Terceiro (2002). The control mode is still time variant, although linear, thus allowing the application of "adiabatic approximations" Friendland et al (1990) to design liner quadratic (LQ) control laws. These cases are helpful to understand the very influence of the composed torque sliders position controls on the elastic vibrations but many questions about design parameters remain inconclusive.

A second approach, where slider trajectories are control variables, leads to Optimal Control Problems (OCP) and is the focus of this paper. The resulting models are non linear and time variant and optimal arm and slider trajectories are investigated through the use of RIOTS'95 Schwartz et al (1997), a computational package based on the Consistent Approximation Theory Schwartz et Polak (1996).

2. Continuous Model

The concept design of the entire mechanism is shown in Figure 1. Hub motion is driven by a DC motor and monitored by an encoder. Two sliders run on each side of the flexible arm, driven by independendent motors inside the hub.

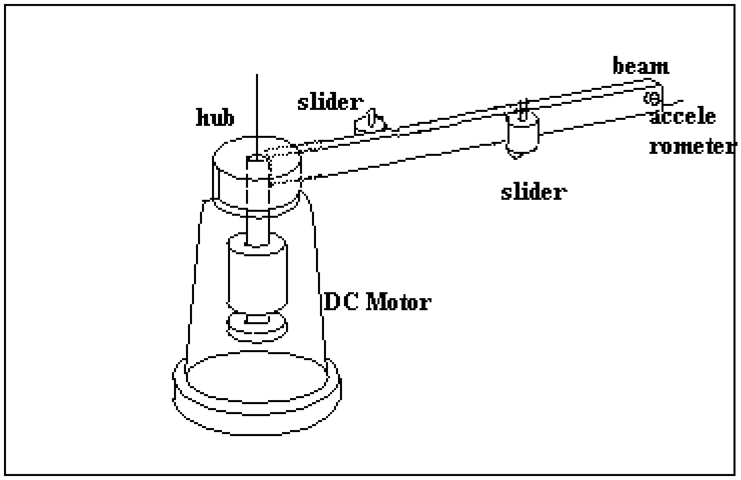

Figure 1 Concept design of the flexible mechanism

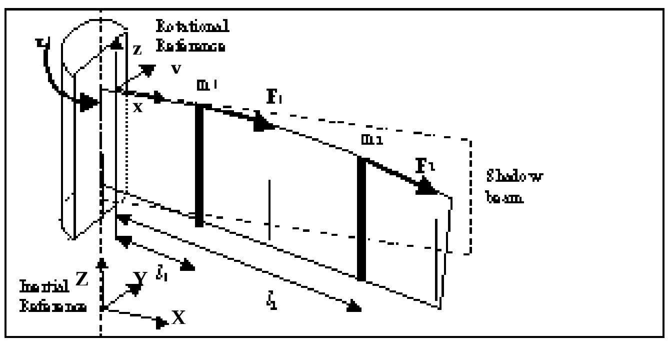

Figure 2 Deformed and Shadow Beam corresponding to the Flexible Arm

The motion of the flexible arm is supposed to occur in a horizontal plane (Oxy), in such a way that gravitational forces can be disregarded. In order to apply the Extended Hamilton's Principle and get the corrresponding model one has to consider the kinetic energies of the arm, of the two sliders and of the hub, the elastic potential energy of the arm and the virtual work of the non conservative forces on the system, namely the torque of the hub (τ) and the force on the each slider (F_1 and F_2). The resulting model is given by (Terceiro, 2002):

$$\int_0^L \rho\left(-x\ddot{e}-\left(x^2+e^2\right)\ddot{\theta}\right)dx + M_1\int_0^L\left(e_1\ddot{l}_1 - l_1\ddot{e}_1 - e_1^2\ddot{\theta}\right)\Delta l_1 dx +$$

$$+ M_2\int_0^L\left(e_2\ddot{l}_2 - l_2\ddot{e}_2 - e_2^2\ddot{\theta}\right)\Delta l_2 dx - \left(M_1 l_1^2 + M_2 l_2^2 + J_C\right)\ddot{\theta} + \tau = 0$$

$$\int_0^L \rho\left(-\ddot{e} - x\ddot{\theta} + e\dot{\theta}^2\right)dx - \int_0^L EI_V \frac{\partial^4 e}{\partial x^4}dx = 0$$

$$M_1\int_0^L\left(-\ddot{e}_1 + e_1\dot{\theta}^2 - \dot{l}_1\dot{\theta}\right)\Delta l_1 dx = 0$$

$$M_2\int_0^L\left(-\ddot{e}_2 + e_2\dot{\theta}^2 - \dot{l}_2\dot{\theta}\right)\Delta l_2 dx = 0$$

$$M_1\int_0^L\left[\left(e_1\ddot{\theta} + \dot{e}_1\dot{\theta}^2\right)\right]\Delta l_1 dx + M_1\left(l_1\dot{\theta}^2 - \ddot{l}_1\right) + F_1 = 0$$

$$M_2 \int_0^L \left[\left(e_2 \ddot{\theta} + \dot{e}_2 \dot{\theta}^2 \right) \right] \Delta l_2 \, dx + M_2 \left(l_2 \dot{\theta}^2 - \ddot{l}_2 \right) + F_2 = 0 \tag{3.1a}$$

with boundary conditions given by:

$$\begin{aligned} &e\big|_{x=0} = 0; \quad \left.\frac{\partial^2 e}{\partial x^2}\right|_{x=L} = 0 \\ &\left.\frac{\partial e}{\partial x}\right|_{x=0} = 0; \quad \left.\frac{\partial^3 e}{\partial x^3}\right|_{x=L} = 0 \end{aligned} \tag{3.1b}$$

3. Substructuring of the model

Due to the mathematical difficulties on the treatment of the set of integro-partial distributed parameter equations above, a simpler model is required. This model is derived through substructuring techniques, when a structure is divided in smaller substructures and the interactions between them are considered. In this case, the substructures are the hub-flexible arm and each of the sliders. Interactions occur at the contact points where punctual forces $F_e(l_i)$ appear. Then, for the hub/flexible arm substructure the external efforts are the torque t used to drive the rotation maneuver and the forces $F_e(l_i)$ normal to the arm and applied at the mass positions l_1 and l_2. Use of the Extended Hamilton's Principle results in:

$$\int_0^L \rho \left(-x\ddot{e} - x^2 \ddot{\theta} - e^2 \ddot{\theta} \right) dx - J_C \ddot{\theta} + \tau = 0 \tag{4.1}$$

$$\int_0^L \rho \left(-\ddot{e} - x\ddot{\theta} + e\dot{\theta}^2 \right) dx - \int_0^L EI_V \frac{\partial^4 e}{\partial x^4} dx = 0 \tag{4.2}$$

with the same boundary conditions of Eq.(3.1b).

Considering the three degrees of freedom, motion synchronization and normalized modes, the motions of the three substructures is given by (Terceiro, 2002):

$$\ddot{\eta}_r + \omega_r^2 \eta_r = \int_o^L F_E(l_i) \phi_r \, dx + \tau \phi_r' \big|_{x=0} \qquad \text{r=1,….,p} \tag{4.3}$$

where p corresponds to the number of vibration modes considered for the arm.

After algebraic manipulations, Eq. (4.3) can be written, for the rth mode, as:

$$\ddot{\eta}_r + \omega_r^2 \eta_r = -\int_o^L M_i \sum_{s=1}^{\infty} \left[\phi_s \big|_{x=l_i} \ddot{\eta}_s \right] \phi_r \, dx - \int_o^L M_i \, 2\dot{l}_i \dot{\theta} \phi_r \, dx + \tau \phi_r' \big|_{x=0} \tag{4.4}$$

This equation describes the vibration of the flexible arm expanded in normal modes. In matrix form

$$\{\ddot{\eta}_r\}_{px1} = [T]_{pxp}^{-1} \left(-[W]_{pxp} \{\eta_r\}_{px1} - 2M_i \dot{l}_i \dot{\theta} \left\{ \int_0^L \phi_r \, dx \right\}_{px1} + \tau \left\{ \phi_r' \big|_{x=0} \right\}_{px1} \right) \tag{4.5}$$

with $[S]_{pxp} = [T]_{pxp}^{-1}$ given by:

$$S_{rs}=\begin{cases}\dfrac{M_i+M_i^{\,2}\displaystyle\sum_{k=1,k\neq r}^{p}\phi_k\big|_{x=l_i}\int_o^L\phi_k\,dx}{1+\displaystyle\sum_{k=1}^{p}M_i\,\phi_k\big|_{x=l_i}\int_o^L\phi_k\,dx} & \text{for r = s}\\[2ex] \dfrac{-M_i^{\,2}\phi_r\big|_{x=l_i}\displaystyle\int_o^L\phi_s\,dx}{1+\displaystyle\sum_{k=1}^{p}M_i\,\phi_k\big|_{x=l_i}\int_o^L\phi_k\,dx} & \text{for r} \neq \text{s}\end{cases} \tag{4.6}$$

Newton's Law applied of the rotational motion is given by:

$$\left(J_B+J_C+M_i l_i^{\,2}\right)\ddot{\theta}=\tau+F_{Ej}\left(l_i\right)l_i \tag{4.7}$$

In consequence, the angular acceleration can be written as:

$$\ddot{\theta}=\frac{\displaystyle\sum_{r=1}^{p}\sum_{s=1}^{p}S_{rs}\left(\omega_s^2\eta_s\right)\phi_r\big|_{l_i}}{J_B+J_C+M_i l_i^{\,2}}+\frac{2M_i\dot{l}_i\dot{\theta}\left(\displaystyle\sum_{r=1}^{p}\sum_{s=1}^{p}S_{rs}\left(\int_0^L\phi_s\,dx\right)\phi_r\big|_{l_i}-1\right)}{J_B+J_C+M_i l_i^{\,2}}+\frac{\left(1-\displaystyle\sum_{r=1}^{p}\sum_{s=1}^{p}S_{rs}\left(\phi_r'\big|_{x=0}\right)\phi_r\big|_{l_i}\right)\tau}{J_B+J_C+M_i l_i^{\,2}} \tag{4.8}$$

Equations (4.5) and (4.8) describe the angular motions of the arm and of the sliders (Terceiro, 2002).

4. Optimal Control Problem

Given the motion of the mechanism elements, one can propose an Optimal Control Problem based on the model above and where the control variables are the external forces that drive the slider movements, besides the torque applied to the hub. The state space system is highly non-linear and written as:

$$\dot{x}_{2r-1}=x_{2r} \tag{5.1 a}$$

$$\dot{x}_{2r}=-\frac{\displaystyle\sum_{s=1}^{p}\omega_s^2 S_{rs}x_{2s-1}}{M_1+M_2}-\frac{2\left(M_1x_{2p+4}+M_2x_{2p+6}\right)x_{2p+2}\displaystyle\sum_{s=1}^{p}S_{rs}\int_0^L\phi_s\,dx}{M_1+M_2}+$$
$$+\frac{\displaystyle\sum_{s=1}^{p}S_{rs}\,\phi_s'\big|_{x=0}}{M_1+M_2}u_i \tag{5.1 b}$$

$$\dot{x}_{2p+1}=x_{2p+2} \tag{5.1 c}$$

$$\dot{x}_{2p+2}=\ddot{\theta}=\frac{\displaystyle\sum_{r=1}^{p}\sum_{s=1}^{p}\omega_s^2\phi_r\left(l_i\right)S_{rs}x_{2s-1s}}{J_{BC}+M_1x_{2p+3}^2+M_2x_{2p+5}^2}+\frac{2\left(M_1x_{2p+4}+M_2x_{2p+6}\right)x_{2p+2}\left(\displaystyle\sum_{r=1}^{p}\sum_{s=1}^{p}\left[\phi_r\left(l_i\right)S_{rs}\int_0^L\phi_s\,dx\right]-1\right)}{J_{BC}+M_1x_{2p+3}^2+M_2x_{2p+5}^2}+$$

$$+\frac{1-\sum_{r=1}^{p}\sum_{s=1}^{p} S_{rs}\,\phi_r'\big|_{x=0}\phi_r(l_i)}{J_{BC}+M_1x_{2p+3}^2+M_2x_{2p+5}^2}u_1 \qquad (5.1\ d)$$

$$\dot{x}_{2p+3}=x_{2p+4} \qquad (5.1\ e)$$

$$\dot{x}_{2p+4}=\frac{u_2}{M_1}+x_{2p+3}x_{2p+2}^2 \qquad (5.1\ f)$$

$$\dot{x}_{2p+5}=x_{2p+6} \qquad (5.1\ g)$$

$$\dot{x}_{2p+6}=\frac{u_3}{M_2}+x_{2p+5}x_{2p+2}^2 \qquad (5.1\ h)$$

The elements of the matrix S are written according to Eq. (4.7) with the necessary adaptations.

5. Simulation results

For solution to the Optimal Control Problem (OCP), numerical simulations have been performed using the RIOTS_95 package (Schwartz et al, 1997). One has considered as validated cases the simulated problems that satisfy the rigid convergence criterium of the RIOTS algorithms. Four of them have been selected to discussion in this paper according to the importance of the motion conditions they represent. In the first two cases the sliders should return to their initial assigned initial positions and the arm displacement should be null at the end of the manoeuver. For the other two cases (3 and 4), the second slider should stop its motion close to the hub while the same null condition is imposed to the arm tip at the end of the movement.

The physical parameters of the mechanism elements are resumed in Table 1. Table 2 shows more details of the 4 cases.

Young Module for aluminum	$E=7.1\mathrm{x}10^{10}$ PA	Arm length	L = 0.7 m
Arm thickness	h = 0,001 m	Arm width	b=0.0254 m
Arm mass	m= ρ_B *L*b*h=0,0482 kg	Arm linear mass density	ρ_o=m/L
Aluminum density	ρ_B = 2710 kg/m^3 (Al)	Mass moment of inertia of the hub	J_c=1.35x10^{-4}
Area moment of inertia of the arm	I_v=b*h^3/12	Slider mass 1	M_1 = 0.05*m
Mass moment of inertia of the arm	J_v=ρ_o*L^3/3	Slider mass 2	M_2 = 0.05*m

Table1 Physical parameters of the Optimal Control Problem

	Cases 1 and 2	Cases 3 and 4
Slider 2 initial position	0.7m (free tip o the arm)	0.7m (free tip o the arm)
Index of the performance	$\int_0^T\left(4x_1^2(t)+x_2^2(t)\right)dt$	$\int_0^T\left(4x_1^2(t)+x_2^2(t)\right)dt$
Slider 1 final position	Equal to initial position	Equal to initial position
Slider 2 final position	Equal toinitial position	Close to hub
Arm tip displacement at final time	null	null
Maneuver time (T)	4s (Case 1) and 2s (Case 2)	4s (Case3) and 3s (Case 4)
Number of sucessful convergence trials	5 (Case 1) and 7 (Case 2)	6 (Case 3) and 8 (Case 4)

Table 2 Details of the four cases.

Figures 3 to 8 below show the time evolutions of the main variables for Cases 1 and 2. Figure 3 shows the behavior of the arm tip during the manoeuver, synthetizing the vibration levels to which the flexible mechanism is submited. A Very smooth, low vibration levels can be observed, even for a quite fast 2s manoeuver of a extremelly thin arm (Figure 3, up).

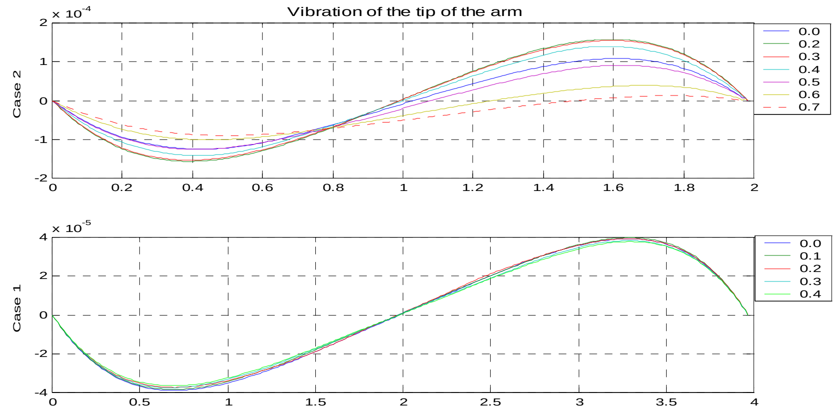

Figure 3 Arm tip displacements: Case 2 up; Case 1 down

Figures 4, 5 and 6 show the angular displacements and velocities of the arm, slider trajectories and slider velocities, respectively, for Cases 1 and 2.

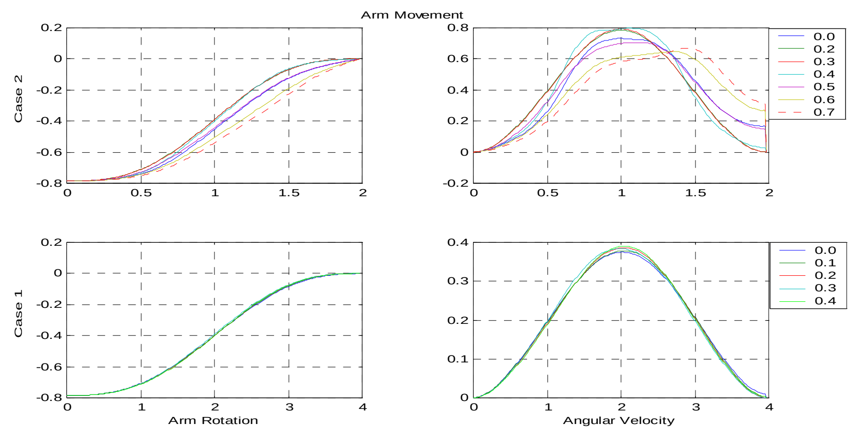

Figure 4 Angular trajectories and velocities of the arm: Case 2 up; Case 1 down

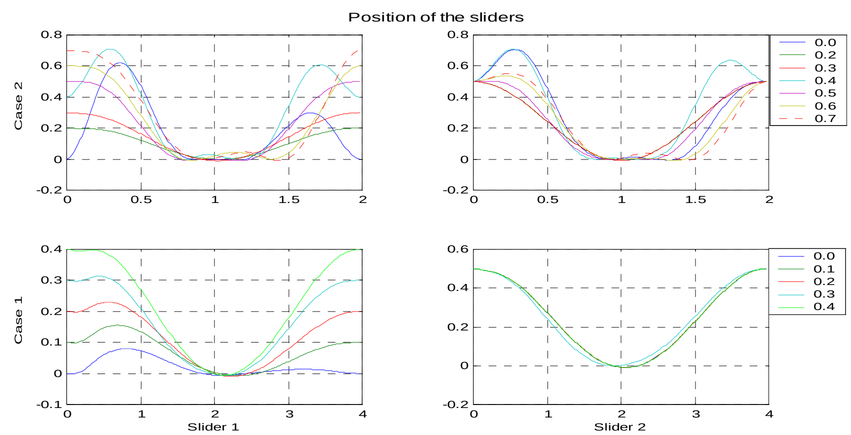

Figure 5 Mass slider optimal trajectories: Case 2 up; Case 1 down

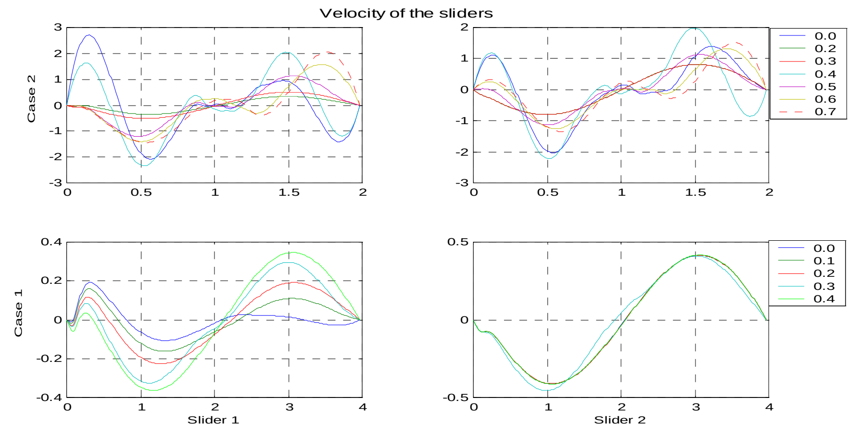

Figure 6 Mass sliders velocities: Case 2 up; Case 1 down

Figure 7 represent the torque to rotate the flexible arm and the forces to move the sliders and Figure 8 show the ratio between the forces on the sliders.

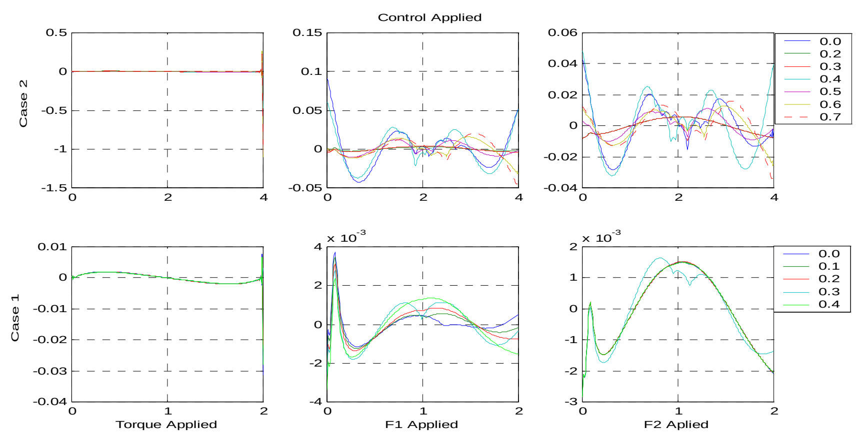

Figure 7 Torque applied to hub and forces to sliders: Case 2 up; Case 1 down.

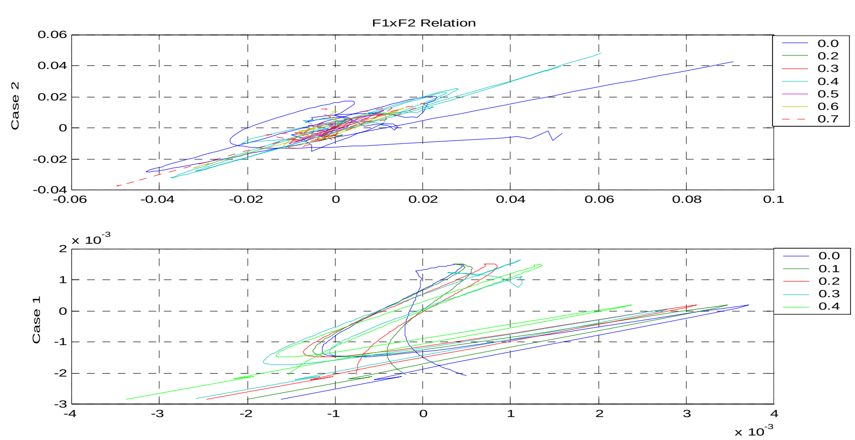

Figure 8 Ratio between the forces applied to the sliders: Case 2 up; Case 1 down

Some comments should be addressed concerning the results shown in these figures. When comparing cases 1 and 2 from Figures 3 and 4, one can observe that for smaller tip displacements, the tip stays longer back of the “shadow beam”. Appearently, the reason for this behavior is related to the manoeuver time, since a small time requires a large control effort to stop the angular motion. Longer manoeuver times lead to smoother motions. From Figure 8, it is possible to observe that a 4s manoeuver time (Case 1) leads to similar shapes for the ratio between the control forces, even for quite different initial conditions. The same is not true for case 2. This fact is reinforced by the different shapes of the slider velocities as can be seen in Figure 6(up). The optimal solutions seem to be strongly dependent on the initial conditions when the manoeuver time is reduced. The same behavior has been detected in many other cases studied by Terceiro (2002) and not presented here.

Figures 9 to 14 show the corresponding results for cases 3 and 4.

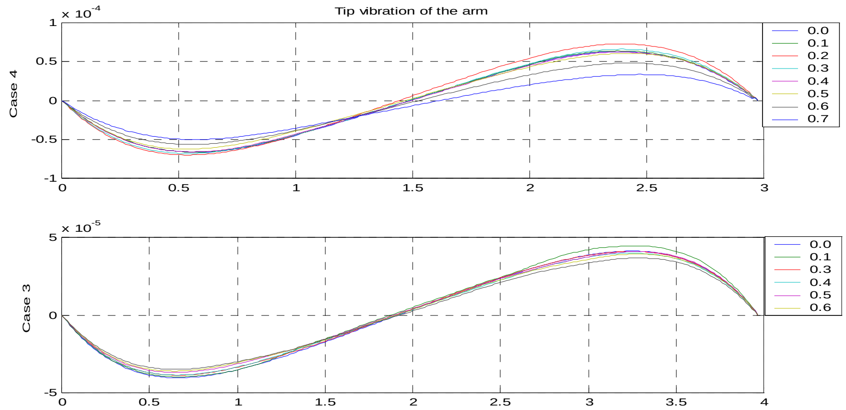

Figure 9 Arm tip displacements: Case 4 up; Case 3 down

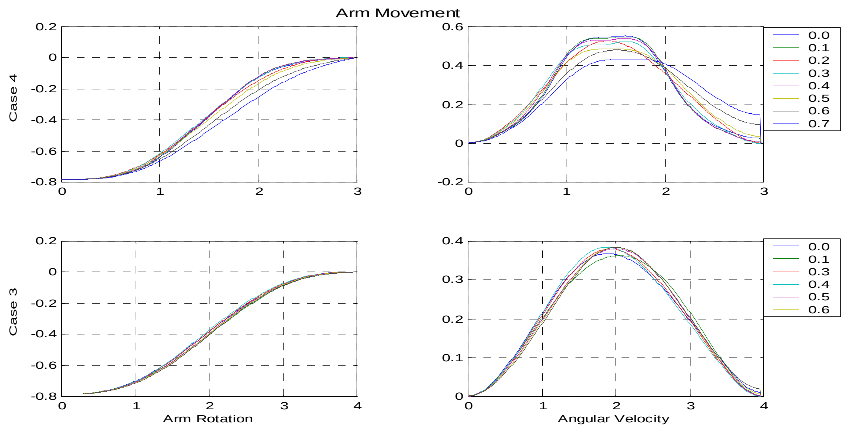

Figure 10 Angular trajectories and velocities of the arm: Case 4 up; Case 3 down

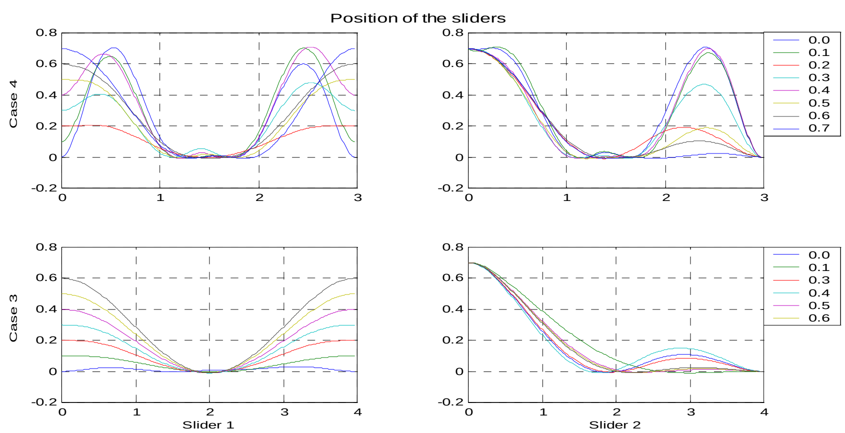

Figure 11 Mass slider optimal trajectories: Case 4 up; Case 3 down

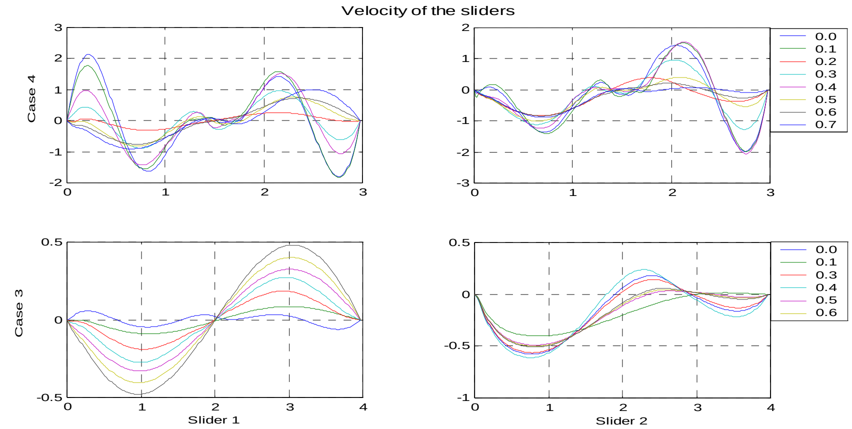

Figure 12 Mass sliders velocities: Case 4 up; Case 3 down

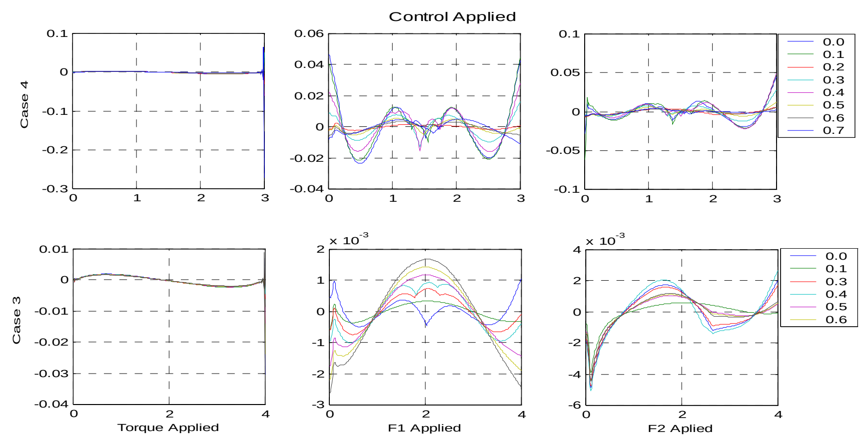

Figure 13 Torque applied to hub and forces to sliders: Case 4 up; Case 3 down

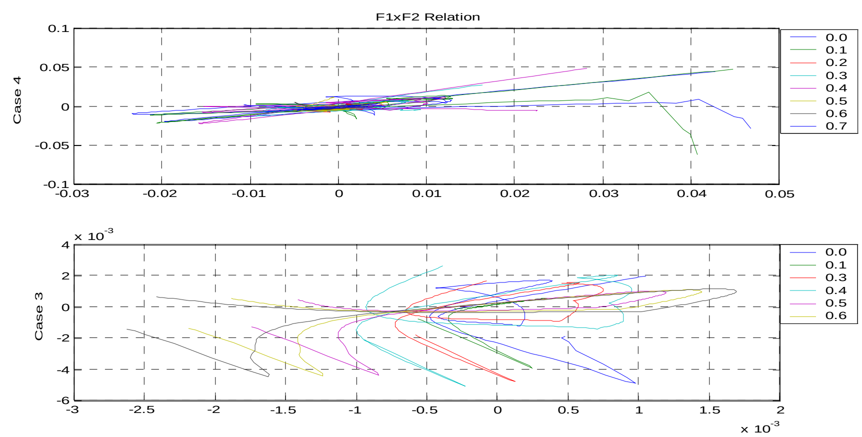

Figure 14 Ratio between the forces applied to the sliders: Case 4 up; Case 3 down

From the above figures, one can observe that optimal solutions for Cases 3 and 4 are also dependent on manoeuver times and initial slider positions, although results for 3s seem smoother than for 2s manoeuver times. For some initial conditions, the control efforts to stop slider and arm motions (see Figure 10, for example). This fact reflects on the forces ratio displayed in Figure 14, where the force shapes for a 4s manoeuver (Case 3) are quite similar, while for a shorter time manoeuver (Case 4) these shapes do not present a regular behavior, since the control efforts at the end show abrupt changes.

6. Concluding Remarks

Results shown in the previous section allows to conclude that coupling of different element motions in a multibody system should be used for attenuation of induced vibrations in mechanical devices. For the system under study, even in case of simple manoeuvers, the dynamic effects generated by slider motions may constitute an efficient vibration control method. Author's efforts are now focused in a deeper analysis of the achieved results and in the use of similar approaches to other flexible mechanisms.

7. Acknowledgements

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8. References

Fleury, A.T.; Oliveira, F.R.F.; Lima, R.G.; Terceiro, E., 2002, "On the Use of Moving Sliders to the Optimal Control of a Flexible Rotating Arm", Proceedings of CBA 2002/14 th Brazilian Congress of Automatics, Natal, RN, Brazil, paper 590 (CD Rom).

Fleury,A.T.; Oliveira, F.R.F., 2003,"Dynamics and Control of a Flexible Rotating Arm through the Movement of a Sliding Mass". In: Udwadia,F.;Weber,H.; Leitmann,G.,Eds. *Dynamical Systems and Contro*l, Taylor & Francis, ISBN 0415309972, Chapter 17.

Oliveira, F.R.F., 2000, "Vibration Control of a Flexible Rotating Arm through the Movement of a Sliding mass over the arm" (In Portuguese), MSc. Dissertation, University of São Paulo, Brazil, 140p.

Oliveira, F.R.F., Fleury, A.T., 2001, "Vibration Control of a Flexible Rotating Arm through the Movement of a Sliding Mass" , Proceedings of the 16th Brazilian Congress Mechanical Engineering, paper TRB2396.

Ozguner, E.; Barbieri, U., 1988 , "Unconstrained and Constrained Mode Expansions for a Flexible Slewing Link", Transactions of the ASME, Vol. 110, pp. 416-421.

Schwartz, A. L., Polak, E., Chen Y. , 1997, "RIOTS -Recursive Integration Optimal Control Trajectory Solver. A MATLAB toolbox for solving optimal control problems. Version 1.0", Dept. of E1ectrica1 Engineering and Computer Science, University of California at Berk1ey.

Schwartz, A. L., "Theory and Implementation of Numerical Methods Based on Runge-Kutta Integration for Solving Optimal Control Problems", Ph.D. dissertation, University of California at Berkley, 1996

Terceiro, E., 2002, "Use of the slider masses to the optimal control of flexible rotating arm vibrations: Generalization, Simulation and Results." (In Portuguese), MSc. Dissertation, University of São Paulo, Brazil, 139p.