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AUTO-TUNING OF PID CONTROLLERS BASED ON THE RELAY FEEDBACK TEST FOR MULTI-MODE LOW DAMPING SYSTEMS

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Abstract: *The strong presence of PID (Proportional-Integral-Derivative) controllers in the industry and the difficulties of tuning these controllers have motivated the development of several auto-tuning techniques. The relay feedback methodology is the most important of these techniques. In the present literature, although contemplating a great number of systems, it does not contemplate the auto-tuning of PIDs for oscillatory systems with low damping and several vibration modes. This constitutes the main motivation of this research work. This work presents a new methodology of automatic tuning of PIDs based on the relay feedback test: the complete method. The methodology adds to the relay test a compensator and an automatic and variable reference signal. The frequency response function of the system is estimated with confidence at low frequencies (DC) and close to the first natural frequency of the system. The PID gains are obtained by fitting the frequency response of the dynamic system (mechanical system + controller) to a reference response. The method is evaluated numerically for various dynamical systems and experimentally on a cantilever beam with piezoelectrics actuators. The work concludes that the proposed methodologies are efficient for a wide spectrum of systems, especially for oscillatory systems with low damping and several vibration modes in the band of interest.*

Keywords: *PID controller, relay feedback, auto-tuning.*

1. Introduction

Nowadays a technologically modern society is not built without the presence of automatic control systems. In the control systems stand out those whose actions control (system inputs) depends of the responses (outputs) of the system to be controlled. Such controllers are denominated feedback controllers. The feedback controllers more spread in the technological society are the Proportional-Integral-Derivative type (PID).

The simple architecture of the PID controllers make of these a good tool in the systems control, being its responsible nowadays for more than 90% of all the closed-loop controllers in the industry (Huang, 2000). Such use and versatility has wakened great attention of researchers in the last years (Preface, 2001). Essentially, PID controller's project requires the choice of three parameters: the gains proportional, integral and derivative. It is denominated tuning of a PID the task or methodology used to find such parameters. The tuning can be made on-line or off-line, in way automatic or manual.

In the industry, in general, the tuning of PID is accomplished individually for each process, improving the performance and the robustness of the group of processes. When the gains of PID are found in a manual way, the tuning task becomes monotonous and slow, and the controller's performance depends on experience and on knowledge that the operators have about the process. It is recognized that in practice many industrial controllers are tuned poorly. For this reason, the researchers and control engineers have destined attention more and more to the techniques of automatic tuning of controllers. The auto-tuning allows finding the gains of the controller in an automatic way, starting from a requirement put by an operator, (Hagglund and Aström, 1988, Aström et al, 1993 and Cardoso, 2002). Industrial experiences indicate that the auto-tuning controllers are more advantageous in the performance and in the time involved in the search of the controller's parameters (Hang et al, 2002).

Aström and Hagglund (2001) discusses the PID controllers' use in the current times, and concludes for the longevity of the same ones, observing that such controllers will be part of the industrial parks for many years, even with the appearance of new control techniques.

The method more known of PID controllers' tuning was proposed by Ziegler and Nichols in 1942 (Ziegler and Nichols, 1942). They proposed two tuning alternatives: one for systems that present growing responses when subjects to step input and other for systems that present unstable behavior for high gain in unitary feedback loop. This methodology, although of simple conception, it is revealed, in practice, imprecise for many systems (Huang, 2000).

Astrom and Hagglund, recognizing the limitations of the Ziegler and Nichols method, proposed the use of a relay in the feedback loop of the system to be tuned, creating the "relay method for tuning of PID" (Astrom and Hagglund, 1984; Hagglund and Aström, 1988; Huang, 2000; Hang et al, 2002 and Preface, 2001).

In the relay test appear oscillations in the output of the system in a very close frequency to the critical frequency, (frequency where the phase of the output is delayed of -180 degrees of the input), and once known this oscillation point (gain and period critics) is derived expressions for the PID'S gains. This methodology was one of the first ones to be marketed and your success is due to your simplicity and robustness (Hagglund and Aström, 1988; Hang and Sin, 1991 and Hagglund and Aström, 1991). Several researchers developed methodologies that modify the relay feedback test in some way, making possible your use in a larger range of processes.

In Cardoso (2002) two new methodologies of automatic tuning of the PID controllers, based in the relay feedback test, are presented: the complete method and the simplified method. In the methods proposed by Cardoso (2002) stand out the high automation level of these methodologies and the expressive generality, being applicable to several system types, besides to oscillatory systems with low damping and several natural frequencies, what cannot be found in most of the methods presents in the current literature. The simplified method is detailed in Cardoso (2002), and the complete method is presented in this work.

This work is organized like this: firstly a short introduction is accomplished about the relay feedback test and latter the complete method is detailed. Soon afterwards the proposed method is applied numerically to several systems and experimentally to a continuous system constituted by a cantilever beam with piezoelectrics actuators. Finally, are presented the conclusions.

2. The Relay Feedback Test

The automatic tuning of the PID controllers remounts to the year of 1942, when Ziegler and Nichols proposed a methodology capable to find PID controller's gain starting from identified characteristics of the system to be controlled. Basically they are three the methodologies proposed by Ziegler and Nichols: the Step Response Method, the Frequency Response Method and the Modified Method of Ziegler-Nichols. Larger details of the methodologies proposed by Ziegler and Nichols can be found in Cardoso (2002), Ziegler and Nichols (1942), Hagglund and Aström (1988) and Wittenmark and Aström (1989).

The development of the relay feedback test proposed by Astrom and Hagglund (1984) left, initially, of the Ziegler-Nichols Frequency Response Method limitations on identifying the critical point (critical gain and critical period where the phase of the system is -180^o). Although the experiment proposed by Ziegler and Nichols in the Frequency Response Method is simple in the characterization of the system and tuning of PID, it is of difficult automation, once the oscillation amplitude must be maintained under control, since the operation of systems close to the unstable area is dangerous. Besides this limitation, the accurate determination of the critical gain is an arduous work in practical conditions.

Aström and co-workers, recognizing the limitations of the Ziegler-Nichols Frequency Response Method, modified the procedure of determination of the critical point's parameters, adding an element of the type relay in the feedback loop of the system (Aström and Hagglund, 1984 and Hagglund and Aström, 1988). This procedure promotes stationary oscillations very close to the critical frequency for a great range of processes (Huang, 2000).

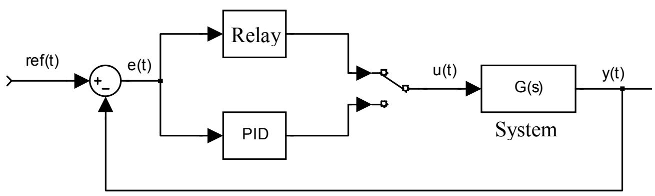

Figure 1. Scheme of the relay feedback test.

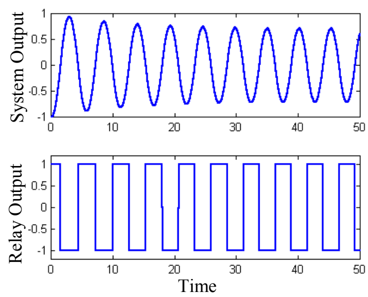

Figure 2. System output signal and output of relay feedback.

The Figure 1 shows the relay feedback test diagram. The switch can alternate among the relay test - when the PID tuning is accomplished - and the control of the system with PID previously tuned in.

The relay feedback test, Fig. 1, produces in stead state a square wave type in the critical frequency of the system, for most of the processes, and the system responses in a sinusoidal way, as show the Fig. 2. These signals, showed in Fig. 2, are in phase opposition and the oscillation amplitude of the system is proportional to the amplitude of the relay.

It is desired to obtain the value of the critical gain (*Ku*) and of the critical period (*Tu*), like in the Ziegler-Nichols Frequency Response Method.

Expanding the stationary relay output signal, that is a square wave with frequency w_u and amplitude h, in Fourier series and, assuming that the system attenuates the superior order harmonic effects, and still admitting that the oscillation of the system output, *y(t)*, have amplitude *a* and that this signal is in phase opposition with the input signal, *u(t)*, it can be determined the critical gain by the equation (Wittenmark and Aström, 1989):

$$Ku = \frac{1}{|G(j \cdot w_u)|} = \frac{4h}{\pi a} \tag{1}$$

Determined the values of the critical gain and of the critical period, it can be used the rules of the Ziegler-Nichols Frequency Response Method (Astrom and Hagglund, 1984; Hagglund and Aström, 1988; Cardoso, 2002) to find PID controller's gains. With the tuned controller the switch of the scheme of the Fig. 1 can be commuted, putting the system on the PID action.

This method, also called of ideal relay method, produces satisfactory results for a wide spectrum of processes, however it presents two important limitations

- the representation of the relay output signal (that is a square wave) for the first term of the Fourier series it is an approach that, for many processes (systems of high order, with significant delays, etc.), it compromise the tuned controller's performance (Wang et al, 1997).
- the method makes possible the identification of just a point of the system frequency response function, what can be insufficient to describe the process satisfactorily, resulting in errors of the controller's tuning.

Several methodologies of auto-tuning of PID exist derived of the relay feedback test proposed by Astrom and Hagglund (1984). However, such methodologies are not presented in this work. The main methodologies presents in the current literature are presented in Cardoso (2002).

3. The Complete Method

The methodology proposed here possesses a different configuration for the relay feedback test (Fig. 3). In this test stand out the simple relay, the compensator, *Q(s)*, the reference signal, *Ref(t)* and the system to be controlled, $G_p(s)$.

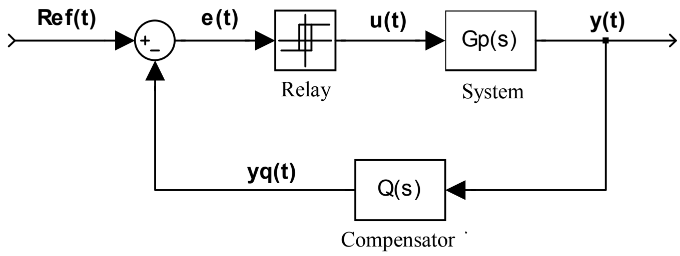

Figure 3. Scheme of the proposed identification test.

The compensator *Q(s)* inserted in the feedback loop of the system will must delay the input signal of the relay, delaying the output signal of system of -90°. Besides this function the compensator still has the property of rejecting high frequencies, protecting the test against noisy interferences. To be reached these specifications proposes that the compensator is a pure integrator or a first order low-pass filter with cut frequency very inferior to the first natural frequency of the system.

The reference signal, *Ref(t)*, must be different from zero to promote an asymmetry in the relay output so that the output signal has a level DC. This reference signal is variable with the time and it depends, in the magnitude and in the frequency, of the output signal of the compensator. It proposes to calculate the reference signal in the following way:

$$Ref(t) = nref \cdot \left[pc(t) - vl(t) \right] + vl(t) \qquad (2)$$

The values *pc(t)* and *vl(t)* are the maximum and minimum of the output signal of the compensator, *yq(t)*, identified in the previous period and *nref* (reference level) assumes values between *0.6* and *0.9*.When *nref* is *0.5* the value of the reference signal is same that the medium value of the oscillation signal of the compensator, with the relay oscillating in a symmetrical way, not inserting DC signal on the system. When *nref* is *1.0* the input signal of the relay have little oscillations or, is saturated in the *pc(t)* value at the stationary oscillations, not permitting the relay oscillations and inserting just DC signal on the system. Thus, choosing *nref* value between *0.6* and *0.9* be assure asymmetric relay oscillations, inserting on the system a DC signal + square wave in the first natural frequency.

With the configuration proposed above for the relay test, it is assured that the oscillation frequency is very close of the first natural frequency of the system and, due to the introduction of the reference signal different from zero, it is had two very defined areas in the frequency response function of the system: the region of the first natural frequency and the region of static level (low frequencies).

3.1. Identification of the system's FRF

Assuming that the system is initially in steady state, the test begins and are acquired, for a known interval of time, the signals of the relay output (*u(t)*) and of the system output (*y(t)*), (see Fig. 3). The signals *y(t)* and *u(t)* are multiplied by a decreasing exponential term, according to Eq. 3. At the end of the test, the values of $\tilde{y}(t)$ and $\tilde{u}(t)$ are closed to zero. At the final time, the system must be under stationary oscillation.

$$\tilde{y}(t) = y(t)e^{-\alpha t}$$
$$\tilde{u}(t) = u(t)e^{-\alpha t} \tag{3}$$

where $\alpha > 0$.

As the signals modulated by the exponential tend to zero at the end of the test, can be applied the Fast Fourier Transform (FFT) as shown to follow:

$$\tilde{Y}(jw_i) \approx T\sum_{k=0}^{\infty} \tilde{y}(kT)e^{-jw_ikT} \approx T\sum_{k=0}^{N-1} \tilde{y}(kT)e^{-jw_ikT} = Y(jw_i + \alpha)$$
$$\tilde{U}(jw_i) \approx T\sum_{k=0}^{\infty} \tilde{u}(kT)e^{-jw_ikT} \approx T\sum_{k=0}^{N-1} \tilde{u}(kT)e^{-jw_ikT} = U(jw_i + \alpha) \tag{4}$$

where: $w_i = \dfrac{2\pi i}{NT}$; $i = 1,2,...,m$; $m = \dfrac{N}{2}$; N is the number of the samples of each signal and T is the sample time.

The modified Frequency Response Function (FRF) of the system is:

$$G(jw_i + \alpha) = \frac{Y(jw_i + \alpha)}{U(jw_i + \alpha)} = \frac{\tilde{Y}(jw_i)}{\tilde{U}(jw_i)} = \tilde{G}(jw_i) \tag{5}$$

where: $i = 1,2,...,m$

The modified frequency response of the system, Eq. (5), is enough for PID controller's project, not needing larger computational efforts for the estimate the frequency response $G(jw_i)$. However, if necessary, is possible to find it (Wang et al, 1997 e 1999, e Hang et al, 2002).

Although a good approach for the frequency response of the system is obtained with a single test, to improve the performance and to reduce the random errors provoked by external disturbances, the procedure is accomplished *Na* times, and the modified final transfer function, $\tilde{G}_p(jw_i)$, in the frequency domain is given by the average of all values of the system's transfer function, $\tilde{G}(jw_i)$, in each frequency:

$$\tilde{G}_p(jw_i) = \frac{1}{Na}\sum_{\beta=1}^{Na} \tilde{G}_\beta(jw_i) \tag{6}$$

where: β indicates the test, *Na* is the number of accomplished tests and $i = 1,2,...,m$.

Once several tests are accomplished for the determination of the frequency response function of the system, is possible to find the value of the coherence function in each frequency of interest, by applying the expression (Bendat and Piersol, 1986; Cardoso, 2002):

$$\gamma_{uy}^2(jw_i + \alpha) = \frac{\left|\sum_{\beta=1}^{Na} U_\beta^*(jw_i + \alpha)Y_\beta(jw_i + \alpha)\right|^2}{\left[\sum_{\beta=1}^{Na} U_\beta^*(jw_i + \alpha)U_\beta(jw_i + \alpha)\right]\cdot\left[\sum_{\beta=1}^{Na} Y_\beta^*(jw_i + \alpha)Y_\beta(jw_i + \alpha)\right]} \tag{7}$$

where: β indicates the sample; *Na* the total number of accomplished samples; $U(jw_i+\alpha)$ and $Y(jw_i+\alpha)$ are the FFTs of the signals $u(t)$ and $y(t)$ modulated by the exponential decline (α); $i = 1,2,...,m$ being m half of the points number of each sample and, the symbol * denotes the complex conjugated.

The Equation (7) allows determining in which frequencies the modified system FRF, calculated by the Eq. (6), is reliable, in other words, in which frequencies the identification was well succeeded. This information is used in the choice of the areas of PID controller's tuning.

3.2. The Tuning of the PID Controller

With the frequency response function of the system, or the modified FRF, the next step is to accomplish the controller's tuning. Basically this tuning method consists of adjusting the gains (*Kp*, *Ki*, *Kd*) of PID controller, minimizing the error among FRF of the system+controller in open-loop and desired FRF in open-loop, that reaches the requirements put by the operator.

Like this, initially is specified a desired frequency response of the system+controller in closed-loop that, for interpretation easiness and knowledge, can be the classic second order system:

$$H_r(s) = \frac{w_n^2}{s^2 + 2\xi w_n s + w_n^2} \tag{8}$$

where: w_n is the desired natural frequency and ξ is the desired damping factor and, must be specified by the operator.

Of the Equation (8) is derived the open-loop modified frequency response function, desired for the controlled system:

$$\tilde{G}_r\left(jw_i^f\right) = G_r\left(jw_i^f + \alpha\right) = \frac{H_r\left(jw_i^f + \alpha\right)}{1 - H_r\left(jw_i^f + \alpha\right)} = \frac{w_n^2}{(jw_i^f + \alpha)^2 + 2\xi w_n(jw_i^f + \alpha)} \tag{9}$$

where: w_i^f denotes the frequencies where the controller will be tuned and α is the coefficient of the exponential decline.

Considering PID controller given by the following structure:

$$G_c(s) = Kp + \frac{Ki}{s} + Kd \cdot s \tag{10}$$

or:

$$\tilde{G}_c\left(jw_i^f\right) = Kp + \frac{Ki}{\left(jw_i^f + \alpha\right)} + Kd \cdot \left(jw_i^f + \alpha\right) \tag{11}$$

The gains (*Kp*, *Ki*, *Kd*) of PID controller are found by fitting the open-loop FRF of the system+controller, $\tilde{G}_p\left(jw_i^f\right) \cdot \tilde{G}_c\left(jw_i^f\right)$, to the desired open-loop frequency response function, $\tilde{G}_r\left(jw_i^f\right)$. The proceeding is accomplished by minimizing the mean square error defined in the following equation:

$$min\,\vartheta\Big|_{Kp,Ki,Kd} = \left[\sum_{i=1}^{N_f}\left|\tilde{G}_p\left(jw_i^f\right)\tilde{G}_c\left(jw_i^f\right) - \tilde{G}_r\left(jw_i^f\right)\right|^2\right] \tag{12}$$

where: N_f is the number of frequencies of fitting.

The Equation (12) provides a system with N_f equations, where each equation corresponds at the one frequency in that the controller will be tuning. This system of N_f equations and only three variables (the PID gains) can be solved by the least square method.

The frequencies w_i^f are found taking the frequencies in that the coherence function is greater than *0.95*, or be where the identification of the system was accomplished with success. These areas are, in general, the static area (low frequencies) and the area close to the first natural frequency.

4. Numerical Evaluation

The proposed methodology was numerically evaluated, being used as references other works of PID tuning found in the literature (see Cardoso ,2002). An oscillatory system with low damping and several natural frequencies was also evaluated.

In all numerical tests were considered: *(i)* – noise in the input and output of the system of the order of 10% of the amplitude of the signals; *(ii)* – quantization errors in the input and output, looking for to simulate the signals acquisition; *(iii)* – unitary amplitudes for the feedback relay; *(iv)* – desired response specified with a damping factor $\xi=0.707$ (that assure a phase margin greater than 60^o) and a natural frequency (w_n) equal half the frequency of relay oscillation, or half the first natural frequency.

Were identified with success the DC gain of the system and the point of smaller frequency where the phase of the system is -90^o, in all tests. To follow some numerical simulations are presented.

Low order non-oscillatory system:

$$G(s)=\frac{0.57}{(8.60s+1)^2}e^{-18.70s}$$ - Hang et al (2002)

Main characteristics:
- monotonic
- non-oscillatory
- long dead time
- low order

PID controller found:
- $Kp = 0.1585$
- $Ki = 0.0271$
- $Kd = 5.8011$

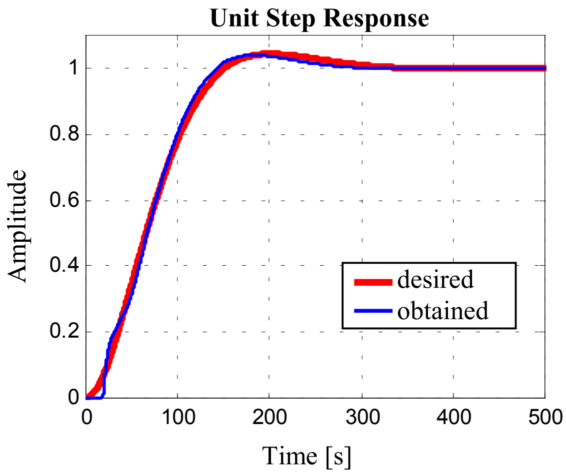

Figure 4. Low order non-oscillatory system.

High order non-oscillatory system:

$$G(s)=\frac{1.08}{(s+1)^2(2s+1)^3}e^{-10s}$$ - Wang et al (2001)

Main characteristics:
- monotonic
- non-oscillatory
- long dead time
- high order

PID controller found:
- $Kp = 0.0880$
- $Ki = 0.0285$
- $Kd = 1.8512$

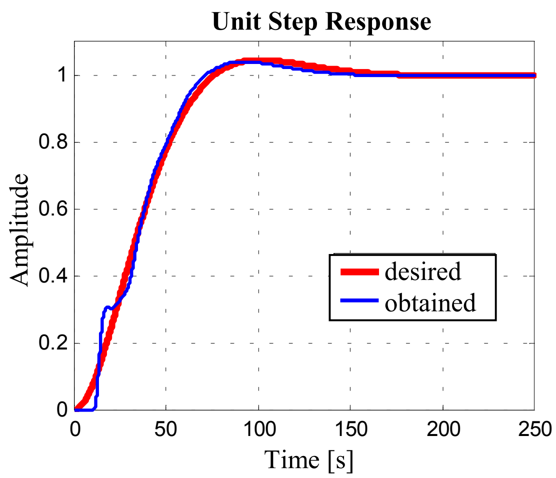

Figure 5. High order non-oscillatory system.

High order oscillatory system:

$$G(s)=\frac{1}{\left(s^2+2s+3\right)\left(s+3\right)}e^{-0.3s}$$ - Hang et al (2002)

Main characteristics:
- monotonic
- oscillatory
- high damping
- high order

PID controller found:
- *Kp = 0.8102*
- *Ki = 3.3221*
- *Kd =1.2723*

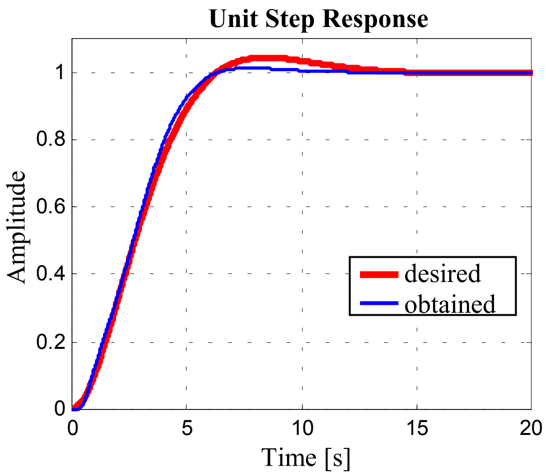

Figure 6. High order oscillatory system.

Non-minimal phase system:

$$G(s)=\frac{(1-5s)}{(5s+1)\left(4s^2+2s+1\right)}e^{-5s}$$ - Wang et al (1999)

Main characteristics:
- non-minimal phase
- oscillatory
- high damping

PID controller found:
- *Kp = 0.0796*
- *Ki = 0.0328*
- *Kd =1.8293*

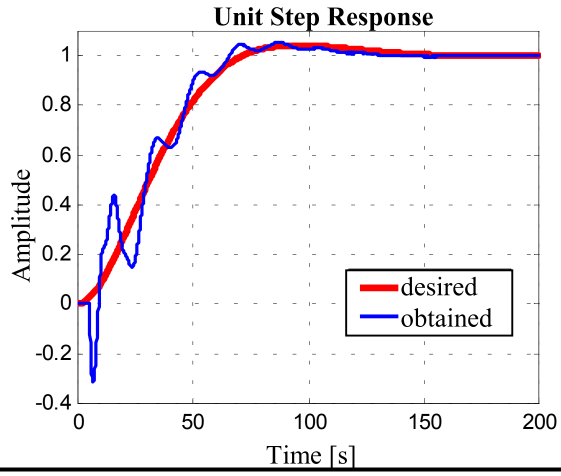

Figure 7. Non-minimal phase system.

Oscillatory system with low damping and several natural frequencies:

$$G(s)=\left[\frac{799.4}{s^2+0.2827s+799.4}+\frac{846.3}{s^2+2.909s+846.3}+\frac{13511.5}{s^2+2.325s+13511.5}\right]0.1489e^{-0.002s}$$

Natural frequencies and its damping:
- $f_1 = 4.50$ Hz - $\xi = 0.005$
- $f_2 = 4.63$ Hz - $\xi = 0.05$
- $f_3 = 18.50$ Hz - $\xi = 0.01$

Main characteristics:
- monotonic
- oscillatory
- low damping
- high order
- several natural frequencies
- very close two natural frequencies

PID controller found:
- *Kp = 0.0073*
- *Ki = 22.2670*
- *Kd = 0.0269*

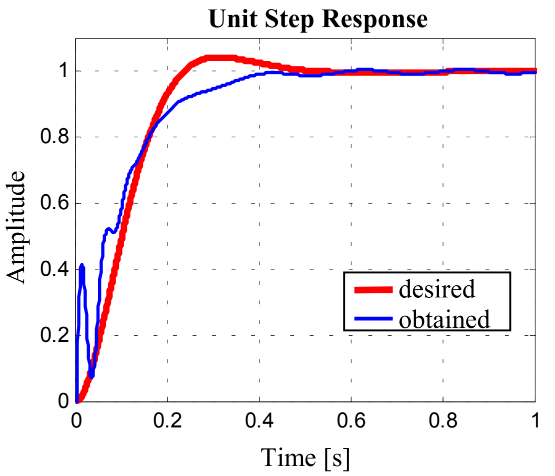

Figure 8. Oscillatory system with low damping and several natural frequencies.

5. Experimental Evaluation

The proposed methodology was applied to a continuous system, with infinites degrees of freedom. This system is constituted of a cantilever beam with piezoelectric actuators incorporated (Fig. 9). An electromagnetic proximity sensor is used to detect the beam traverse displacements in a certain point. The signals of this sensor go by a signal conditioner and are sent to the acquisition system. With the signal of the proximity sensor, the computer accomplishes the necessary calculations and, through the acquisition system, sends a voltage signal for the power amplifier of the piezoelectric actuator (PZT), sending the conditioned signal to PZT (one in each face of the beam) that is deformed. Once PZT is fixed to beam, it forces the beam provoking its movement.

Data of the cantilever beam with piezoelectric actuators incorporated:

- Dimensions:
  - Length x Width x Thickness of the beam: 400 x 34.5 x 1.2 mm
  - Position of the PZT from base: 115 mm
  - Position of the sensor from base: 240 mm
- Piezoelectric actuator:
  - Actuator ACX QP10N

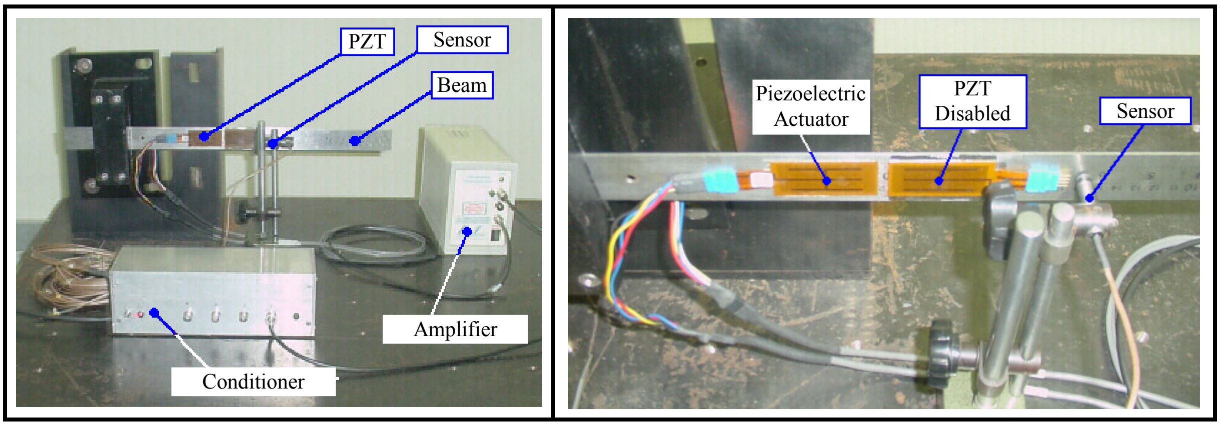

Figure 9. Cantilever beam system with piezoelectric actuators incorporated.

In the Figure 10 is showed the frequency response function (FRF) of the beam system obtained with a signal analyzer (in red/continuous) and the identified FRF with the proposed test (in blue/dashed). In this figure, the inverted triangle indicates the frequency of relay oscillation, which is very close to the first natural frequency of the system. For the proposed method, the coherence function informs a good confidence of estimation at low frequencies and at the area close to the first natural frequency of the system. In the identification test be used a sampling time of 3 ms and, was accomplished 10 tests with 20 seconds of time each. The relay amplitude was 0.12 V, the reference level (*nref*) equal to 0.9 and the compensator of the integrator type.

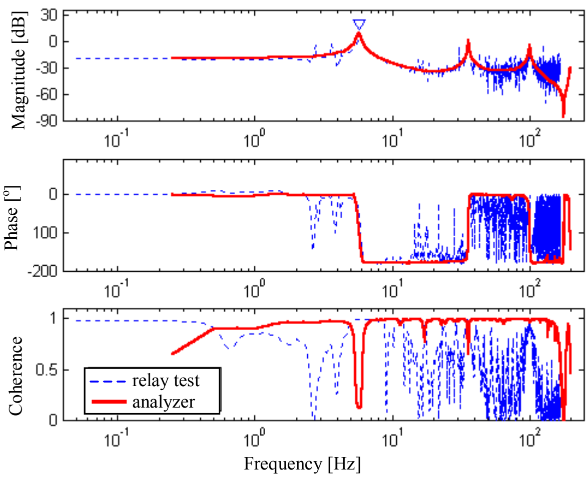

Figure 10. Frequency response function of the cantilever beam system: reference FRF and identified FRF.

Once the two areas of interest (low frequencies – DC gain – and area close to the first natural frequency of the system) were identified by proposed methodology, the desired response was specified with a damping factor (ξ) of 0.707 and a natural frequency (w_n) of 1.0 Hz. Fitting the open-loop response of the system+controller to the desired response in open-loop, was obtained the following PID controller's gains: $Kp = -0.0383$, $Ki = 37.9575$ e $Kd = 0.0276$.

In the Figure 11 the response of the beam system is showed when is specified a square type reference signal with amplitude of 0.2 V and frequency of 0.1 Hz.

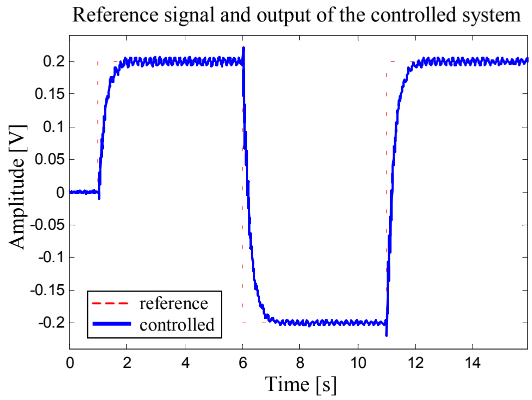

Figure 11. Controlled beam system when is specified a square reference signal.

6. Conclusions

The techniques of auto-tuning of PIDs controllers have showed more and more attractive, once the tuned controllers present a good performance with a simple and fast project. Several works exist regarding the auto-tuning of PID controllers. However, the present methods in the current literature are just applied to an or other system type and, it are not applicable to oscillatory systems with low damping and several natural frequencies.

The complete method proposed can be applied with success to a large range of processes, even to oscillatory systems with low damping and several natural frequencies. Although demands certain computational effort, mainly in the FRFs' calculation, the complete method can be implemented in digital signal processors (DSPs), as for example the TMS320C2407, or still in microprocessors of low processing capacity, since the acquired data are stored and processed after the identification test.

The method highlights by the high automation level, once few user interventions are make on the tuning process of PID controller.

Although the objective of this work was to evaluate in which systems the proposed method could be applied, the tuned PIDs controllers presented good performance. Specifying other values for the desired damping factor (ξ) and for the desired natural frequency (w_n), the controllers' performance can be improved.

In the non-minimal phase system, although the PID controller's structure is not capable to eliminate the non-minimal phase characteristics, it kept the response of the controlled system very close to the desired response. The proposed method showed very attractive, with good performance and, fast and simple project for the tested systems, even to oscillatory systems with low damping and several natural frequencies.

The proposed method can be modified to project a PI controller or a PD controller, as also is possible to specify several types of desired responses.

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8. References

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