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Robust Disturbance Rejection with Time Domain Specifications in Control Systems Design

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Abstract. *The robust design of compensators aiming disturbance rejection with time domain specifications is discussed in this paper from a perspective of loop shaping. The constraints to provide robust disturbance rejection are derived as functions of the disturbance reference model, which contains the associated time domain specifications. It is shown that the larger is the distance between the nominal plant model and the reference model to be followed the more restrictive the design constraints are. The problem can be posed as a model tracking compensator and the proposed procedure may reduce the conservativeness normally associated with its design. The plant model is assumed subject to unstructured uncertainties and the design specifications are written in the usual form of loop shape constraints. Hence techniques like $H_{¥}$ or LQG/LTR can be applied as design tools. In order to illustrate the application of the proposed methodology we consider a multivariable mixture tank as an example.*

Keywords. *Disturbance Rejection, Model Matching, 2-D Control, Robust Control, Loop Shaping, Time Specifications*

1. Introduction

Nowadays most multivariable linear control design techniques are carried out in the frequency domain. Many textbooks contain an exhaustive presentation of the theme (Green, 1995; Helton, 1998; Skogestad, 1996; Skogestad, 1996). Nevertheless in many practical situations part of the specifications is given in the time domain. The use of H_2 and H_∞ theories requires that specifications in the time domain be expressed in the frequency domain before they can be applied. For servo problems, time domain constraints are intuitive. They are also quite natural in many regulatory problems. For SISO systems these specifications can often be translated into the frequency domain, but this is not the case for MIMO systems in general

Model matching and 2-D control are two approaches that can be used to indirectly handle time domain specifications to solve servo problems and also may be adapted to treat the disturbance rejection with time domain constraints. However, even when applied to servo problems, the loop gain is normally made too high in order to make the input/output transfer matrix close enough to the identity matrix so that the overall system behaves approximately as the reference model containing the given time specifications.

The objective of the design approach presented in this paper is to adapt the Model matching and 2-D control schemes deriving conditions to guarantee robust disturbance rejection with a prescribed time domain behavior. Attention is restricted to the case where the disturbances can be measured. In particular, we propose a design technique and argue that it may be less conservative than usually.

Conditions to guarantee setpoint tracking over given frequency ranges within prescribed accuracy are well known and will be omitted here. Although the conditions for non measured disturbance rejection and measurement error rejection are also well known they are included in this paper just for the sake of completeness since together with the measured disturbance rejection they form the so called regulatory problem.

The plant model is assumed subject to unstructured uncertainties and the design specifications are written in the usual form of loop shape constraints. Hence techniques like H_∞ or LQG/LTR (Athans, 1986; Doyle, 1981) can be applied as design tools.

In order to illustrate the application of the proposed methodology we consider a multivariable mixture tank as an example.

The paper is structured as follows.

Section 2 contains a preliminary discussion on how the disturbance rejection problem with time domain specifications can be posed as a model tracking problem.

In section 3 the design conditions required for robust disturbance rejection with time domain specifications are written in both the form of loop sensitivity constraints and in terms of constraints on the loop gain shape.

An analysis of the control magnitude associated to model following is performed in section 4.

A numerical example is presented in section 5 to illustrate the proposed methodology.

Section 6 contains the conclusions of the paper.

2. Preliminary Discussion

Consider the system represented in Fig. (1). $P(s)$ and $K(s)$ are respectively the plant and compensator transfer matrices. The transfer matrix $\Sigma(s)$ is a filter and will be called *disturbance reference model*. $d(s)$ is the non measured disturbance reflected at the plant output, $f(s)$ is the measured disturbance reflected at the plant input, $n(s)$ is the measurement error, $r(s)$ is the system input and $y(s)$ is the system output. All signal and transf er matrices are assumed to have compatible dimensions.

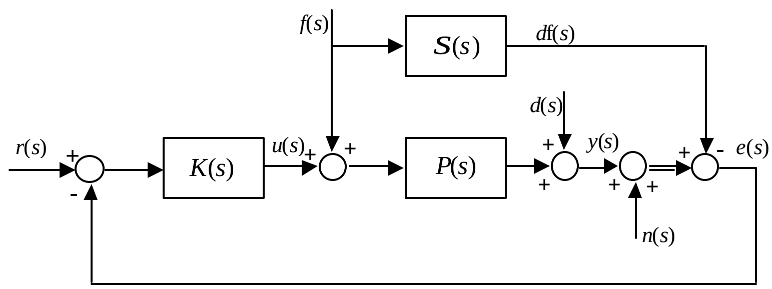

Figure 1. Control problem.

When there is no measured disturbance ($f(s)=0$), the design problem turns into the classical one and the conditions to guarantee setpoint tracking, non measured disturbance rejection and measurement error rejection over given frequency ranges within prescribed accuracy are well known. On the other hand, taking only the measured disturbance, i.e., for $r(s)=d(s)=0$, the block diagram in Fig. (1) becomes as shown in Fig. (2). This figure is equivalent to the one considered in reference (Jonckheere, 1999) in order to solve an aircraft propulsion control problem.

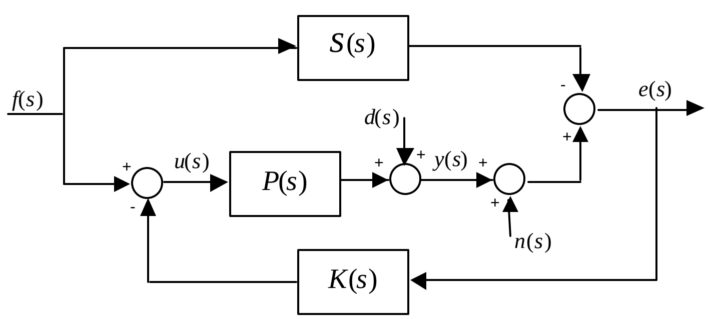

Figure 2. Model tracking structure.

Notice that the control problem depicted in Fig. (2) is a two-degree of freedom problem. By simple block manipulation it becomes the diagram in Fig. (3) – the 2-D control with a pre-filter (Leonardi, 2002a).

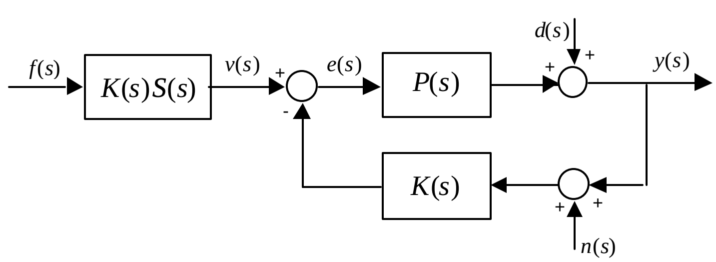

Figure 3. The 2-D control structure.

Design aiming measured disturbance rejection with time domain specifications can thus be viewed as a model matching problem and in this case it can be reduced to making the closed-loop transfer matrix from $v(s)$ to $y(s)$ close to $K^{-1}(s)$ in the frequency range where the matching between $\Sigma(s)$ and the transfer matrix from $r(s)$ to $y(s)$ is sought (Kwakernaak, 1996).

In reference (Maciejowski, 1989) (see page 14) he writes about the choice of the pre-filter: "we can consider first the problem of designing $K(s)$ to obtain desired $S(s)$ and $T(s)$, and subsequently design $\Sigma(s)$ to give a suitable" transfer matrix from $f(s)$ to $y(s)$. A high loop gain is required in order to get a closed-loop transfer matrix close to $K^{-1}(s)$ and the choice of how high it is made may give rise to a conservative design if it is taken higher than necessary.

As proposed in (Leonardi, 2002b), the *model tracking problem* is understood in this paper in the following sense. We look for a compensator $K(s)$ such that the norm of the transfer matrix from $f(jw)$ to $e(jw)$ (see fig. 2) be below some prescribed value in a given frequency range – this requirement will be called *model following*. Additionally we wish that the contributions to the output $y(jw)$ of both the non measured disturbance $d(jw)$ and the measurement error $n(jw)$ be below given values in given frequency sets.

In what follows we adopt the perspective of a set membership for transfer matrices of the plant model considering, in particular, the multiplicative representation of the modeling error. We assume that an upper bound is given for the spectral norm of the multiplicative error matrix in the form of a scalar function $e_M(w)$ (Doyle, 1981).

3. Loop Shaping

In what follows the symbol $\|\cdot\|$ represents the Euclidean norm of complex vectors. $s_i[\cdot]$, $s_{\min}[\cdot]$ and $s_{\max}[\cdot]$ denote the *i*-th, the minimum and the maximum singular values of $[\cdot]$, respectively.

3.1. Nominal Plant Model

Assume that the plant dynamics are given by their nominal model. Then, for the systems represented in Fig. (3), the following set of equations hold:

$$y(s) = S(s)d(s) + S(s)P(s)\left(I + K(s)\Sigma(s)\right) f(s) - S(s)P(s)K(s)\, n(s), \tag{1}$$

$$e(s) = S(s)d(s) + f(s) + S(s)n(s), \tag{2}$$

$$u(s) = -S(s)K(s)\, d(s) + S(s)\left(I + K(s)\Sigma(s)\right) f(s) - S(s)K(s)\, n(s). \tag{3}$$

where $S(s) = \left(I + P(s)K(s)\right)^{-1}$ is the loop sensitivity matrix.

At this point, since we are obviously assuming that $\Sigma(s)$ is stable, it should be clear that system stability is determined solely by the closed-loop containing $P(s)$ and $K(s)$. Because this is a classical situation, stability is not addressed in the paper.

3.1. 1. Measured Disturbance Rejection

Assume that $a_f > 0$ (typically $a_f << 1$) is a given number which expresses the desired accuracy of the rejection of measured disturbance in a given set of frequencies Ω_f in the sense that $\|e(jw)\| / \|f(jw)\| \leq a_f \quad (w \in \Omega_f)$. Typically $\Omega_f = \{w \in R : w \leq w_f\}$ for a given w_f. Assuming that $d(s) = n(s) = 0$, to accomplish model following we get the following condition from Eq. (2):

$$s_{\max}[S(jw)] \leq \frac{a_f}{s_{\max}[P(jw) - \Sigma(jw)]} \quad (w \in \Omega_f) \tag{4}$$

Hence, as it should be expected, the sensitivity must decrease as the distance[1] between the plant and the disturbance reference model increases. The same occurs as a_f decreases. Nevertheless, depending on the specific problem at hand, this condition may be not so restrictive and the sensitivity may be not necessarily low.

[1] Distance is obviously understood here as measured by the spectral norm of the difference of the two transfer matrices.

When the right-hand side of (4) is much smaller than 1, this condition can be rewritten approximately as

$$s_{\min}\left[P(jw)K(jw)\right] \geq \frac{s_{\max}\left[P(jw)-\Sigma(jw)\right]}{a_f} \quad (w \in \Omega_f). \tag{5}$$

Similarly as above, this condition shows that the loop gain should increase with both the distance between P and Σ and the inverse of a_f.

3.1. 2. Non Measured Disturbance Rejection

Suppose that $\Omega_d = \{w \in R : w \leq w_d\}$ is a given frequency set where the non measured disturbance $d(s)$ predominantly has its energy. Assume also that $f(s)=0$ and $n(s)=0$. For a given $a_d > 0$ (typically $a_d << 1$) we express the non measured disturbance rejection condition as

$$\frac{\|e(jw)\|}{\|d(jw)\|} \leq a_d \quad (w \in \Omega_d). \tag{6}$$

From Eq. (2) we then get the following well-known sufficient condition:

$$s_{\max}\left[S(jw)\right] \leq a_d \quad (w \in \Omega_d), \tag{7}$$

which leads to the following approximate condition (Da Cruz, 1996):

$$s_{\min}\left[P(jw)K(jw)\right] \geq \frac{1}{a_d} \quad (w \in \Omega_d) \tag{8}$$

when $a_d << 1$.

3.1. 3. Measurement Error Rejection

Suppose that $\Omega_n = \{w \in R : w \geq w_n\}$ is a given frequency set where the measurement error predominantly has its energy. Assume also that $f(s)=0$ and $d(s)=0$. For a given $a_n > 0$ (typically $a_n << 1$) we express the measurement error rejection condition as

$$\frac{\|y(jw)\|}{\|n(jw)\|} \leq a_n \quad (w \in \Omega_n). \tag{9}$$

From Eq. (1) we then get the condition of measurement error rejection:

$$s_{\max}\left[T(jw)\right] \leq a_n \quad (w \in \Omega_n), \tag{10}$$

where

$$T(s) = S(s)P(s)K(s). \tag{11}$$

Alternatively, from Eq. (1) it follows the following approximate form (Da Cruz, 1996):

$$s_{\max}\left[P(jw)K(jw)\right] \leq a_n \quad (w \in \Omega_n). \tag{12}$$

when $a_n << 1$.

3.2 Model Uncertainties

First of all, recall that for the multiplicative model error adopted the stability robustness condition is given by (Doyle, 1981)

$$s_{\max}[T(jw)] < e_M(w) \quad (\forall w) \qquad (13)$$

Assuming that $e_M(w) < 1$ for $w \in \Omega_f \cup \Omega_d$, then conditions (4) and (7) above modify respectively to (Green, 1995):

$$s_{\max}[S(jw)] \le \frac{a_f[1 - e_M(w)]}{s_{\max}[P(jw) - N(jw)]} \quad (w \in \Omega_f) \qquad (14)$$

and

$$s_{\max}[S(jw)] \le a_d[1 - e_M(w)] \quad (w \in \Omega_d). \qquad (15)$$

Assuming that $a_n << 1$, condition (10) can be rewritten in the following approximate form (Da Cruz, 1996):

$$s_{\max}[T(jw)] \le \frac{a_n}{1 + e_M(w)} \quad (w \in \Omega_n). \qquad (16)$$

As expected the effect of model uncertainty is to make the constraints on S and T more restrictive.

From Eq. (1) we can see that $S(s)P(s)(I + K(s)\Sigma(s))$ has no arbitrary dynamics at high frequencies where, in general, $P(s)$, $\Sigma(s)$ and $K(s)$ exhibit low gains. Hence, the overall transfer matrix is approximately equal to $P(s)$. In view of this the value of w_f is not arbitrary but related to both $P(s)$ and $\Sigma(s)$. This means that the model following condition (14) does not necessarily implies model matching. This is the reason why we call the procedure *model tracking*. Simulations carried out by now indicate it is reasonable to expect nice matching up to one decade above the reference model bandwidth. In general this is enough to ensure good model following.

It should be emphasized that (14)-(16) together with (13) are the key conditions to design robust compensators in order to achieve model tracking.

To close this section recall that the conditions for robust design can also be expressed in terms of the loop gain in the usual way (Doyle, 1981).

4. Control Magnitude Analysis for Model Following

In this section we restrict the analysis to the nominal plant.

From Eq. (3) it is straightforward to see that

$$u(s) - f(s) = K(s)[I + P(s)K(s)]^{-1}[\Sigma(s) - P(s)]f(s) \qquad (17)$$

Assume that P, Σ and a_f are such that $s_{\max}[P(jw) - \Sigma(jw)]/a_f >> 1$ for $w \in \Omega_f$. Thus if condition (5) holds, it follows that $s_{\min}[P(jw)K(jw)] >> 1$. In this case Eq. (18) leads to

$$u(jw) - f(jw) \cong P(jw)^{-1}[\Sigma(jw) - P(jw)]\, f(jw) \qquad (18)$$

if we assume that both P and K are square and P^{-1} exists.

From Eq. (19) it then follows immediately that

$$\sup_{\|f \neq 0\|} \frac{\|u(jw) - f(jw)\|}{\|f(jw)\|} \cong s_{\max}\left[P^{-1}(jw)\left[\Sigma(jw) - P(jw)\right]\right] \quad (19)$$

This equation shows that the worst-case relative increment of the control magnitude is approximately the same as the relative distance between both the plant and the disturbance reference models. Hence disturbance reference models that are distant from the plant model require a large control magnitude to be followed. This is in accordance with condition (14) which shows that the larger is the distance between the plant and the disturbance reference models the more restrictive the condition of model following is.

5. Numerical Example

In order to illustrate the application of the proposed methodology we consider the stirred tank of reference (Kwakernaak, 1970). Its nominal linearized state model is given by

$$\begin{aligned}\dot{x}(t) &= \begin{bmatrix} -0,01 & 0 \\ 0 & -0,02 \end{bmatrix} x(t) + \begin{bmatrix} 1 & 1 \\ -0,25 & 0,75 \end{bmatrix} u(t) \\ y &= \begin{bmatrix} 0,01 & 0 \\ 0 & 1 \end{bmatrix} x(t)\end{aligned} \quad (20)$$

Since the main contribution of this work focus on the disturbance rejection with time domain specifications, other design specifications are omitted in this example. Besides, as model uncertainties turns the design constraints just more restrictive, they are omitted in this instance and may be considered included in the design specifications.

Consider as control system specifications that disturbance at both control channels shall have its influence on the output rejected as if filtered by a second order system

$$\Sigma_{ii}(s) = \frac{0.25\, s}{s^2 + 0.7s + 0.25} \quad (i = 1,2)\,, \quad (21)$$

within a tolerance of 10% (i.e., $a_f = 0.1$) in the frequency range that extends up to one decade above the reference model bandwidth.

From the time domain specifications we thus have

$$\Sigma(s) = \begin{bmatrix} \Sigma_{11}(s) & 0 \\ 0 & \Sigma_{22}(s) \end{bmatrix}, \quad (22)$$

The H_∞ mixed-sensitivity framework will be used to obtain $K(s)$ (Green, 1995; Helton, 1998; Skogestad, 1996; Skogestad, 1996).

To simplify we assume that model uncertainties have already been taken into account in the definition of the weighting matrix

$$W_1(s) = 10 \cdot \left[P(s) - \Sigma(s)\right]. \quad (23)$$

Once the plant $P(s)$ is in the right-hand side of (4) its input variables may be scaled in such a way as to get it close to the identity in low frequencies. In this case condition (4) can be rewritten as

$$s_{\max}\left[S(jw)\right] \leq \frac{a_f}{s_{\max}\left[P(jw)S_u(jw) - \Sigma(jw)\right]} \quad (w \in \Omega_f)\,, \quad (24)$$

where S_u is a square non-singular matrix with compatible dimensions[2].

[2] Obviously, S_u is a part of the compensator.

For simplicity S_u is considered here as a constant matrix and equal to the plant inverse at low frequencies, namely,

$$S_u = \lim_{s \to 0} P^{-1}(s)\,. \tag{24}$$

Notice that scaling the plant input variables does not change the multiplicative modeling error. Hence both performance and stability robustness conditions are not affected. Figure (4) shows the singular value Bode plots of matrices relevant for the model following design. $W_1(s)$ is the weighting matrix for the mixed-sensitivity procedure.

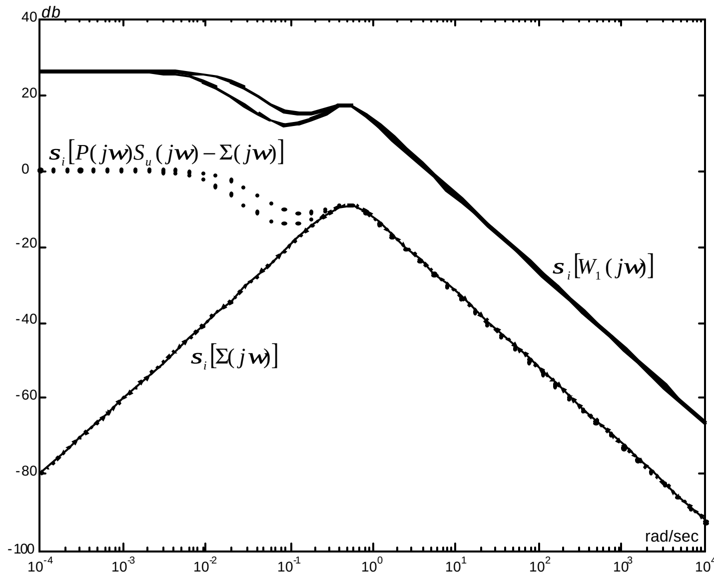

Figure 4. Bode plots for Model Following.

Notice that $s_i[W_1(jw)]$ is situated 20dB above $s_{\max}[P(jw)-\Sigma(jw)]$.

With the sake of illustration, the time response of the closed-loop system and the control variables have been evaluated for the model following design.

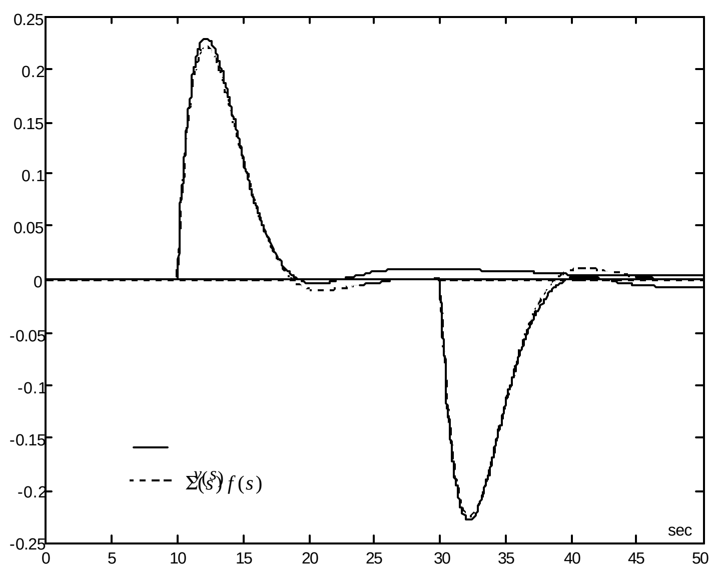

Figure 5. Closed-loop time response.

Two unit steps disturbances applied at instants 10 sec. and 30 sec. with positive and negative signs, respectively, have been considered. Simulation results have been plotted in Fig. (5). As it can be seen, the process outputs follow closely the corresponding ones of the disturbance reference model. The control time history is plotted in Fig. (6).

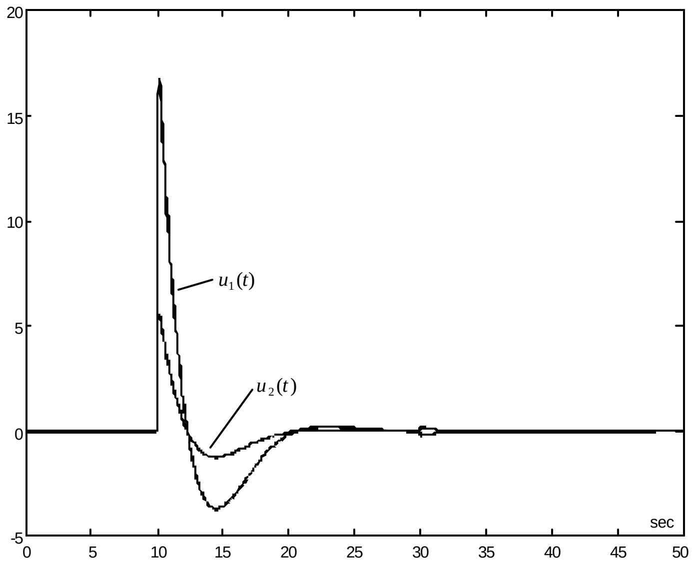

Figure 6. Control time history.

6. Conclusions

This work discussed the robust control design for the rejection of measured disturbance with time domain specifications. The model tracking problem has been posed as a natural way of dealing with time domain specifications in a frequency design context.

It has been shown that the model following condition depends directly on the distance between the reference model and the plant nominal model - the larger the distance between both models the more restrictive the condition is.

It has been shown that the relative increase in the control magnitude to attain model following is approximately the same as the relative distance between the plant and the disturbance reference models. Hence, as expected, the control magnitude increases with the distance between the nominal and disturbance reference models.

Since the disturbance reference model $\Sigma(s)$ is an explicit part of the compensator, small adjustments may generally be done after the design has been completed. This possibility can be useful in practical applications where fine tuning is required during control systems startup.

The mixed sensitivity formulation of the H_∞ control theory has been used in a numerical example to illustrate the application of the methodology. Nevertheless, since the design conditions are ultimately expressed as constraints on Bode Diagrams of the system, any loop-shaping technique could be equally used.

7. Acknowledgments

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