

Computational Simulation Using PZT as Sensor Elements for Damage Detection on Impacted Composite Cylinders

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ABSTRACT

Composite material is very attractive for structural applications due to its inherent mechanical properties and low weight. The improvement in manufacture process allow composite materials be used even as primary structures in modern aircraft design such as Boeing 787 without loss of airworthiness. The uses of structural health monitoring (SHM) can improve even more the application of composite materials in aircraft structures. In this paper, it is performed an investigation about the damage effects on the structural response, considering filament winding composite tubes damaged by impact loading. It is important to highlight that piezoelectric transducers were used in order to obtain the dynamic behavior of the composite tubes. Therefore, firstly, undamaged circular cross section tubes (cylinders) made of composite material are investigated using modal analyses via Finite Element Methods (FEM). Second, Progressive Failure Analyses (PFA) via FEM were carried out for the cylinders under low energy impact load. During the impact simulations, the damage was evaluated by a new material model implemented as a FORTRAN subroutine (VUMAT – User Material for explicit integration analyses), which was linked to the finite element program ABAQUS. After the impact simulations, numerical modal analyses of damaged cylinder were carried out and, the results were compared to undamaged model. Finally, based on the comparison of the results, there is a discussion about the advantages and limitation of vibration method in order to be applied on SHM for composite cylinders.

Keywords: Structural health monitoring, filament winding, composite structures, vibration-based method.

1 INTRODUCTION

The application of composite materials in aeronautical industry have been increasing in the last decades, even in large civil transportation aircrafts, mostly due to composite high stiffness and low weight [1]. Its intrinsic anisotropy allows achieve an optimal material performance, regarding the structure geometry and applied loadings. Composite materials can provide lighter structures without loss of airworthiness, which is a very attractive characteristic for aeronautical industry. However, it is well know how the inspection affects the maintenance costs. Thus the maintenance program must be well planned, but predictions of the inspections are hard task, mostly for composite structures. In addition, unexpected loads could affect the structural integrity, changing the original maintenance plan and, the damage processes for composite materials are more complex than for metals. For example, intra-ply failures and delamination can occur [2], reducing the structural strength. These types of damages can be caused by low velocity impact. There are several sources of low velocity impact, such as dropping tool, impact of small debris, etc. [3], which causes damage in the structure. However, the damage is dependent on the geometry, laminate thickness, stacking sequence, energy level and boundary conditions [4]. Therefore, several studies of impacted composite plates were conducted by different researchers ([5], [6]), but only few studies were made regarding impact on curved geometry ([7], [8]). Despite the high strength in fiber direction, out-of-plane loads, for example impact loads due to bird strike or dropping tool in a composite structure, could lead to severe damage. In a metallic structure this kind of damage is easier to detect, on the other hand, for carbon fiber structures, it is more complicated [7].

Regarding the previous commented issues, the structural health monitoring (SHM) techniques arise as viable solution to reduce the inspections and to optimize the structural maintenance program, performing the repair only when it is necessary, reducing the operational costs. SHM has been approached in the literature as the ‘acquisition, validation and analysis of technical data to facilitate life-cycle management decisions’ [9]. For composite structures, it is possible to produce the structure inserting piezoelectric sensors in order to detect and measure the damage during the life time of the structure. Thus, there are several studies about SHM in the literature, for example, Rytter [10] defined four levels to evaluate damage in a structure. In fact, damage can be often related to a modification of physical parameters such as mass, stiffness or damping. Thus, some vibration-based methods have been developed to be applied on structural health monitoring. The application of modal analysis is one of the most popular approaches, using the Frequency Response Function for detecting damage in structures [11]. Previous studies showed that modal shapes and damping can be also used to detect damage. Other applications in this area involve modal energy and transfer functions. Vibration-based methods have been employed with some success to detect aircraft structural damage ([12–15]). There are several studies about beams and plates, where cracks are originated from the specimen surface due to the applied normal stress [16]. However, the major problem in these methods is related to damage sensitivity. Therefore, normally, modal and/or vibration based techniques are considered “global methods”. In spite of this consideration, other researchers ([17–19]) also investigated life in service and system reliability, using SHM systems, which are composed by vibration-based methods to detect, localize and quantify the structural damage. Farrar and Doebling [20] performed a review of some methods, which assess the vibration response in order to detect, localize and quantify the structural damage. In fact, those methods have used the differences in natural frequencies and modes between the undamaged and damaged structure due to modifications of the structural stiffness, mass and damping for monitoring the structure. Raghavan and Cesnik [21] presented the state of the art in the field of guided-wave structural health monitoring (SHM). Fan and Qiao [22] showed a review on modal parameter-based damage identification methods for beams and plates. The authors also discussed about the damage identification algorithms, emphasizing signal processing.

As shown by the scenario described above, there are few scientific contributions about SHM applied for composite cylindrical structures. Thus, it is possible to conclude that more investigations of vibration-based method to be applied on SHM are required mainly for composite cylinders. Hence, in this paper, it is performed an investigation about the damage effects on the structural response, considering filament winding composite tubes damaged by impact loading. It is important to highlight that piezoelectric transducers were used in order to obtain the dynamic behavior of the composite tubes. Therefore, firstly, undamaged circular cross section tubes, (cylinders) made of composite material (resin epoxy and carbon fiber), are investigated using modal analyses via Finite Element Methods (FEM). Second, Progressive Failure Analyses (PFA) via FEM were carried out for the cylinders under low energy impact load. During the impact simulations, the damage was evaluated by a new material model [23] implemented as a FORTRAN subroutine (UMAT – User Material for explicit integration analyses), which was linked to the finite element program ABAQUS™. After the impact simulations, numerical modal analyses of damaged cylinder were carried out and, the results were compared to undamaged model. Finally, based on the comparison of the results, there is a discussion about the advantages and limitation of vibration method in order to be applied on SHM for composite cylinders.

2 COMPOSITE CYLINDER, PIEZOELECTRIC TRANSDUCER AND DROP TEST

The cylinder in composite material has a piezoelectric transducer bonded on the outer surface (cf. Figure 1a). The composite cylinders are made of fourteen layers with stacking sequence equal $[90/60/-60/90/60/-60/90]_s$. The cylinder geometries consist on outer diameter of 163.7 mm, length of 150 mm and total thickness of 3.4 mm. The piezoelectric transducer (MIDÉ™ QP10n, cf. Figure 1b) geometries consist on length of 50.8 mm, width of 25.4 mm and thickness of 0.5 mm. In order to parameter of the development of finite element models, subroutines in Python were written. After that, these subroutines were used by the program ABAQUS™. The composite cylinder properties were obtained by Ribeiro *et al.* [23] and the piezoelectric transducer properties were obtained by Medeiros [24].

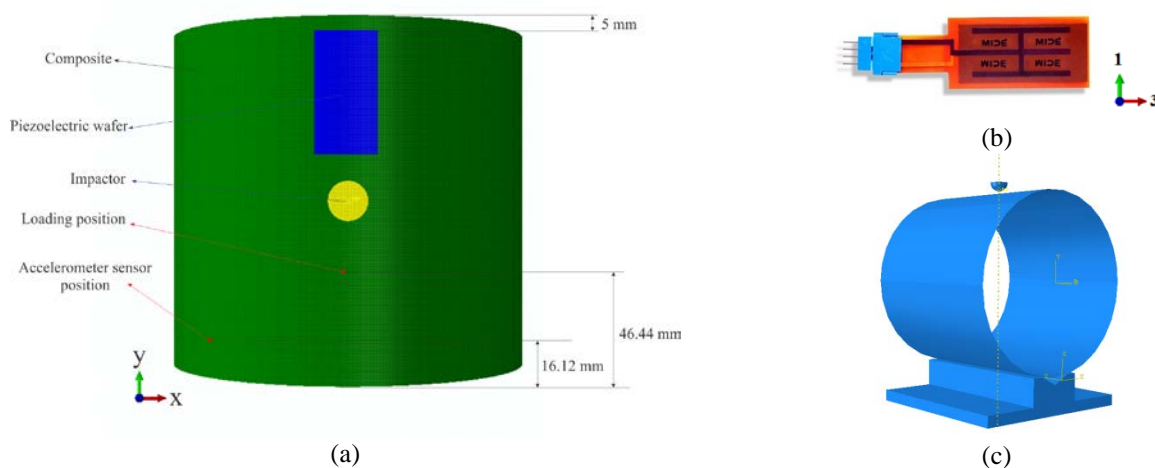


Figure 1: (a) Finite element model: composite cylinder with the piezoelectric transducer bonded on the outer surface; (b) piezoelectric transducer (MIDÉ™ QP10n); (c) drop test

It is noteworthy that for the composite material, the local coordinate system (1-2-3) is defined by the fiber reinforcement, *i.e.* 1-direction is aligned to the reinforcement, the 2-direction is normal to the reinforcement and 3-direction is normal to the plane of the layer. However, for the

piezoelectric transducer, 3-direction (local coordinate system) is related to the longitudinal direction of the insert, and 1- and 2-directions are related to the direction of the cross section of the insert. Therefore, the polarization direction is aligned to the 3-direction of the piezoelectric transducer.

Another important aspect of finite element model is to ensure the mechanical coupling between the cylinder and the piezoelectric transducer. Once assured that mechanical coupling, it is necessary to define appropriate conditions for reading the electric potential in the piezoelectric transducer. The dielectric properties of the MIDÉ QP10n provide homogeneous distribution of the induced electrical charges on the free surface of the transducer. Thus, two electrical boundary conditions can be considered: 1) Open-Circuit (OC); 2) Short-Circuit (SC). For case 1, all the nodes of the piezoelectric transducer surface bonded to the cylinder are considered electrically grounded and, the free surface should respect to equipotential condition. For case 2, the nodes of the base and top are grounded; so that the electrical potentials at the top and bottom of each insert are not variables and maintained constant. Hence, the transducer behaves as a short-circuit device. Physically, the generated surface charges are disposed outside the transducer by conductive wires in order to keep it discharged [25].

Figure 1c shows the impact test, which will be simulated via FEM. In fact, the principle of impact test via a drop tower apparatus is very simple. The drop test consists on a certain mass, which is dropped from a certain height, hitting the test coupon.

3 FINITE ELEMENT MODEL AND NUMERICAL ANALYSES

A four node reduced integration shell element with six degree of freedom (DOF) per node (defined as S4R - ABAQUSTM) were used to model the cylinder. Nevertheless, when performing a PFA, some elements could become excessively distorted, aborting the analysis. In order to overcome this issue, the mesh was refined until eliminating the severe element distortion. The piezoelectric transducer is modeled using eight-node coupled plane elements (defined as C3D8E - ABAQUSTM), with four DOF per node, which are the displacements in x, y and z directions and the electric voltage. Therefore, measurement of the electric potential in a specific node on the free surface will correspond to local information under an applied strain. In practice, the free surface of each transducer is covered with an electrode, which ensures a uniform level of induced electric potential (equipotential) in this position. In order to ensure an ideal perfect bonding between the piezoelectric transducer and the cylinder, the nodes on the bottom surface of the transducer is mechanically coupled to the ones on the top surface of the cylinder, using the “tie” tool at ABAQUSTM. The grounding nodes of the piezoelectric surfaces bonding with the cylinder are accomplished by requiring the potential equal zero at any stage of strain. The purpose of grounding is to define a reference value for the induced voltages on the nodes of the free surface, which can be measured.

The first numerical simulations consist on performing dynamic implicit analyses, following some procedures described below and applying the Fast Fourier Transform (FFT) to obtain the Frequency Response Function (FRF). The first numerical simulation consists on performing vibration analysis of the undamaged cylinder (*cf.* Figure 2a). Thus, a pulse loading was applied on the top of the cylinder close to the piezoelectric transducer, simulating the excitation of a hammer. The input is a transverse force with amplitude equal 1N concentrated in a node of the FE model (Figure 2a). The frequency of the force is applied in the range 0-1000 Hz, considering size step of 1.0 Hz. Also, the nodes of the FE model belonging to the lines of the cylinder, which are in contact with the base of the drop tower (Figure 1c), have been restricted all DOF ($U_x = U_y = U_z = UR_x = UR_y = UR_z = 0$) (Figure 2a). Based on the input and output signals (accelerometer and piezoelectric transducer), it is applied a Fast Fourier Transform (FFT) to obtain the frequency response function (FRF).

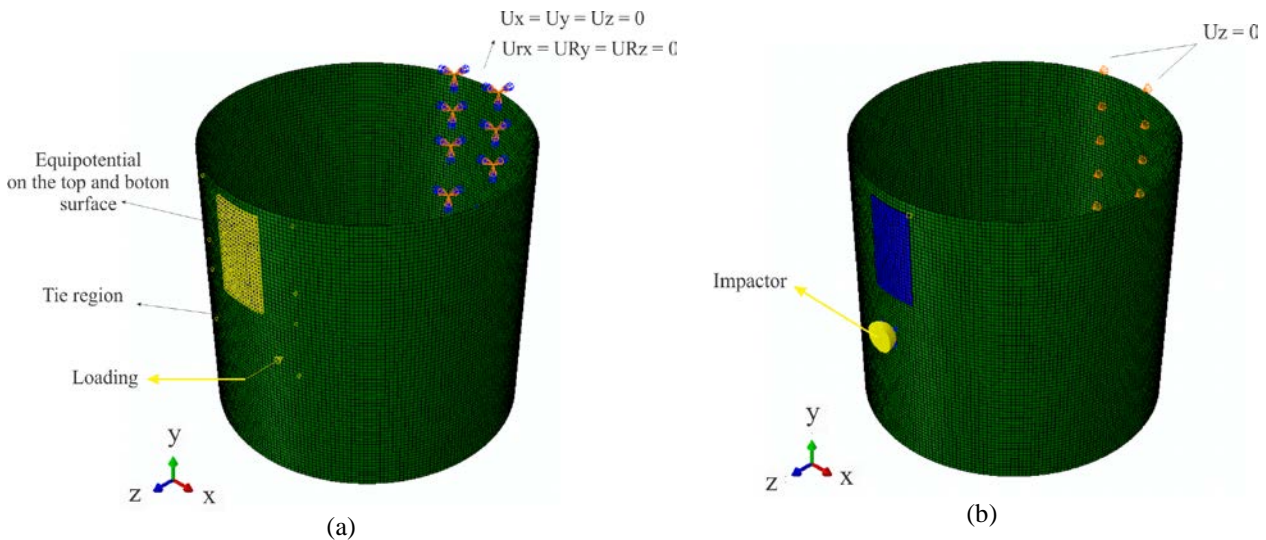


Figure 2: (a) Boundary conditions for frequency response of the structure before and after impact; (b) Boundary conditions for impact analysis

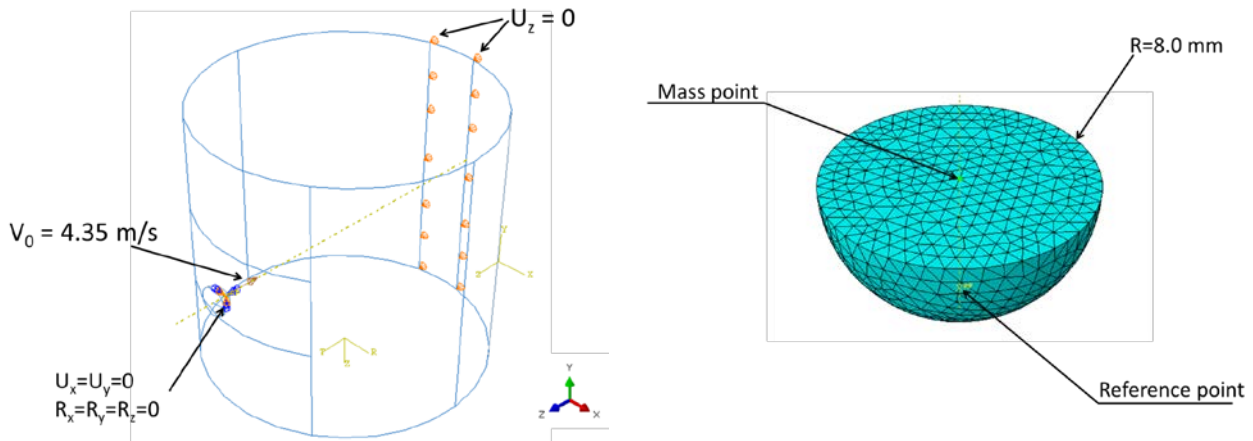


Figure 3: (a) Finite element model geometry and (b) impactor mesh

The second numerical simulations are related to the investigation of how the damage evolves in composite cylinders under impact loads. The present work uses a new damage model proposed by Ribeiro et al. [23] which is linked to the finite element dynamic explicit algorithms (ABAQUSTM/Standard). This model is simple to be implemented and has a low computational cost, which makes it suitable to simulate complex finite element models. Also, the model parameters are easy to be obtained (as shown by Ribeiro *et al.* [23]). The boundary conditions for impact simulations are used in order to simulate a V-base (Figure 1c and Figure 3a). The impactor is modeled using discrete rigid triangular elements (defined as R3D3 - ABAQUSTM) and the mass of 3.24 Kg is applied in the mass point as shown by Figure 3b. Also, all rotations (R_x , R_y , R_z) as well as the U_x and U_y are restricted. Initial velocity of 4.35 m/s is applied in the impactor (cf. Figure 3a). The contact between the impactor and the cylinder were modeled by Hard Contact algorithm for normal interactions and Penalty Method for tangential interactions. Other important issue in simulating of the impact in composite filament winding cylinders is the dissipation of the impact energy. Part of the impact energy is dissipated by irreversible process as damage (intra-ply failures and delamination) and other part is dissipated by damping effects. However, damping in composite materials are dependent on several factors as fiber volume fraction, composite lay-up,

environmental factors, force magnitude, etc. [25]. Also, the structure geometry has an important influence in the impact response. ABAQUSTM provides the Rayleigh's model for direct integration dynamic analysis in order to simulate damping effects [26]. Thus, the Rayleigh's model introduces damping in the structure as a linear combination of mass and stiffness system matrices [27]. It is important to mention that damping can be treated in two different ways, as a material property or as numerical artifices, which oppose to the excitation [27].

The third numerical simulations consist on performing modal analyses for damaged cylinder, obtained from the impact analyses, described earlier. Thus, a pulse loading was applied close to the piezoelectric transducer as shown by the first numerical simulations. The input is a transverse force with amplitude equal 1N concentrated in a node of the FE model (Figure 2a). The frequency of the force is applied in the range 0-1000 Hz with size steps of 1.0 Hz. Also, the nodes of the FE model belonging to the lines of the cylinder, which are in contact with the base of the drop tower (Figure 2), have been restricted all DOF ($U_x = U_y = U_z = UR_x = UR_y = UR_z = 0$). It should be noted that these boundary conditions and loading were applied to both intact and damaged cylinder. Based on the input and output signals (accelerometer and piezoelectric transducer), it is applied a Fast Fourier Transform (FFT) to obtain the frequency response function (FRF).

4 RESULTS AND DISCUSSIONS

Several types of metrics can be used for damage detection, localization and quantification. In this work, it was used the Frequency Response Function. Regarding the transfer function response methods, the differences between the damaged and undamaged structures were compared and, then, the differences can identify the damage. This method is easier to be implemented and also gives a good prediction of the structural integrity ([28], [29]). The open-circuit electrical boundary condition is the condition for the final structural modeling and it obtains the FRFs from the dynamic implicit analysis. Considering that the result is null i.e. the two tablets in SC. It is noteworthy that models of the cylinder restricted in U_z in two lines along the length with piezoelectric inserts bonded presents linear response to pulse inputs. For each excitation frequency, there is a typical configuration of elastic deformation in the structure. Hence, a uniform distribution of the voltage is induced in the equipotential surface of each piezoelectric wafer. The absolute value of the voltage induced in each sensor depends on the degree of elastic strain to which it is subjected. In general, the maximum voltage amplitude of each sensor is associated with strains caused by some modes of the structure vibration, which occurs for excitation frequencies sufficiently close to the respective natural frequencies.

Firstly, a frequency analyses, that consists on determine the natural mode shapes and frequencies of an object or structure during free vibration, is carried out in an undamaged cylinder. Thus, results from Eigen-frequency are presented in Figure 4. These results provide a first approach for comparing with the dynamics implicit analysis. The Figure 4 and Table 1 show the mode shapes, for undamaged model, between 0 to 1000 Hz.

Table 1: Mode shape for an undamaged cylinder with piezoelectric transducer

Mode	1	2	3	4	5	6
Frequency [Hz]	105.66	194.31	293.02	526.50	616.86	949.17
Figure 4	(a)	(b)	(c)	(d)	(e)	(f)

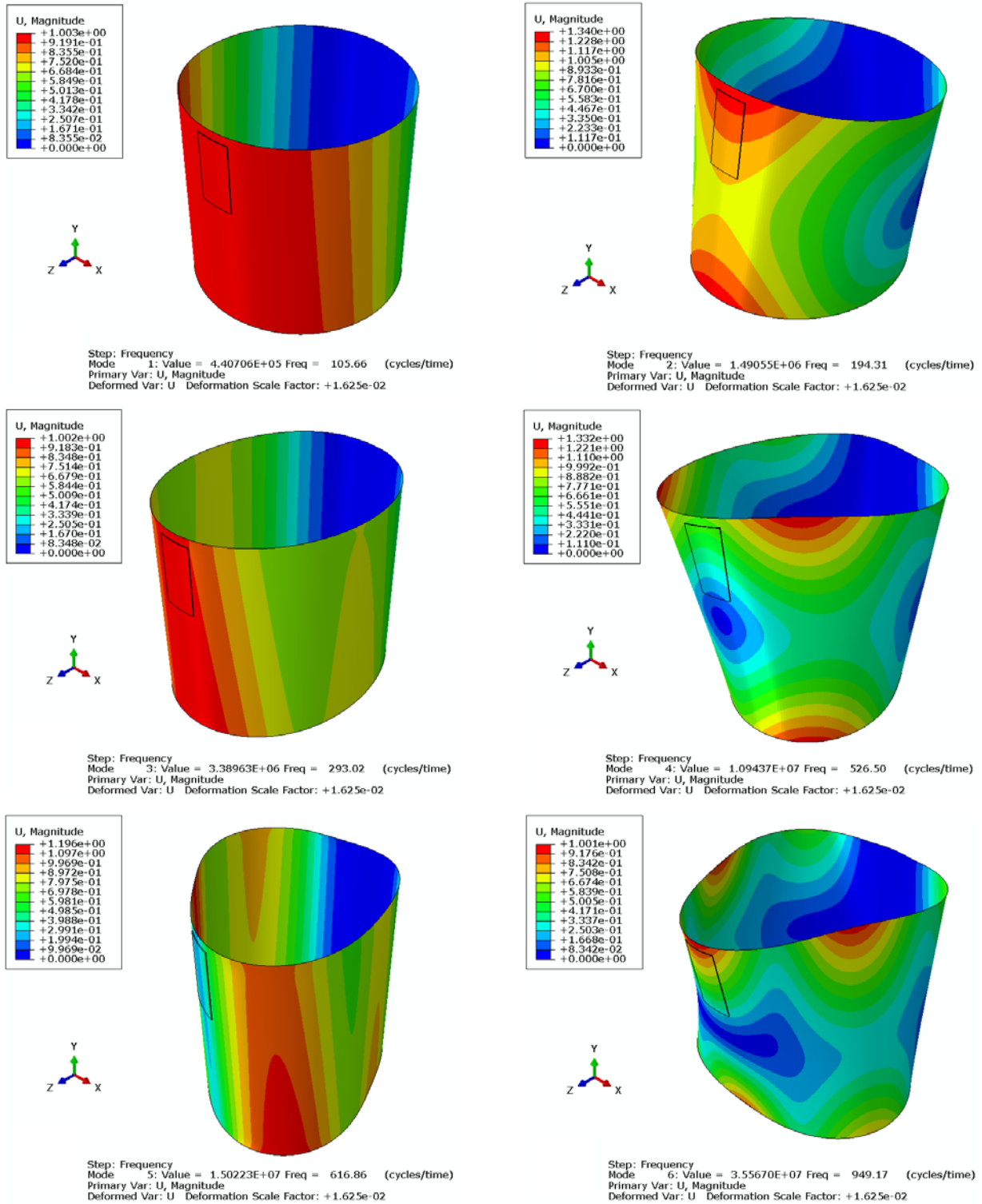


Figure 4: (a) Finite element model geometry and (b) impactor mesh

Figure 5 shows the results for the impacted cylinder and for damage cylinder (damaged area in red for layer 1). Moreover the Figure 6 and Figure 7 show the Frequency Function Response for damage and undamaged cylinder, considering the acquisition from the piezoelectric transducer and from a node of the FE model, respectively. It is important to mention that this node represents an accelerometer (Figure 1a). Initially, it can be observed that the introduction of damage in the structure changed the shape and the amplitudes of the FRF.

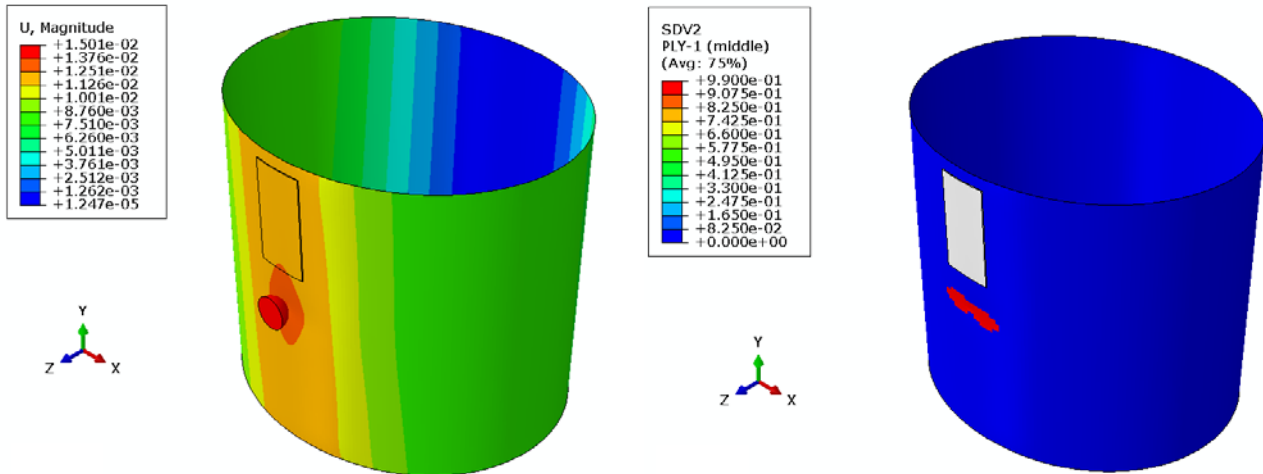


Figure 5: (a) Impact analyses – cylinder is hit by the impactor; (b) Cylinder after impact – damage zone in red layer 1.

The results for the piezoelectric transducer are discussed first. The Figure 6 shows the magnitude log-scale plot of the electric potential calculated with the FEM analysis in the 0 Hz – 1000 Hz frequency range. Six resonance peaks are well distinguished. These small frequency changes are typical in damage location and are one of the major difficulties in the identification of the location of damage.

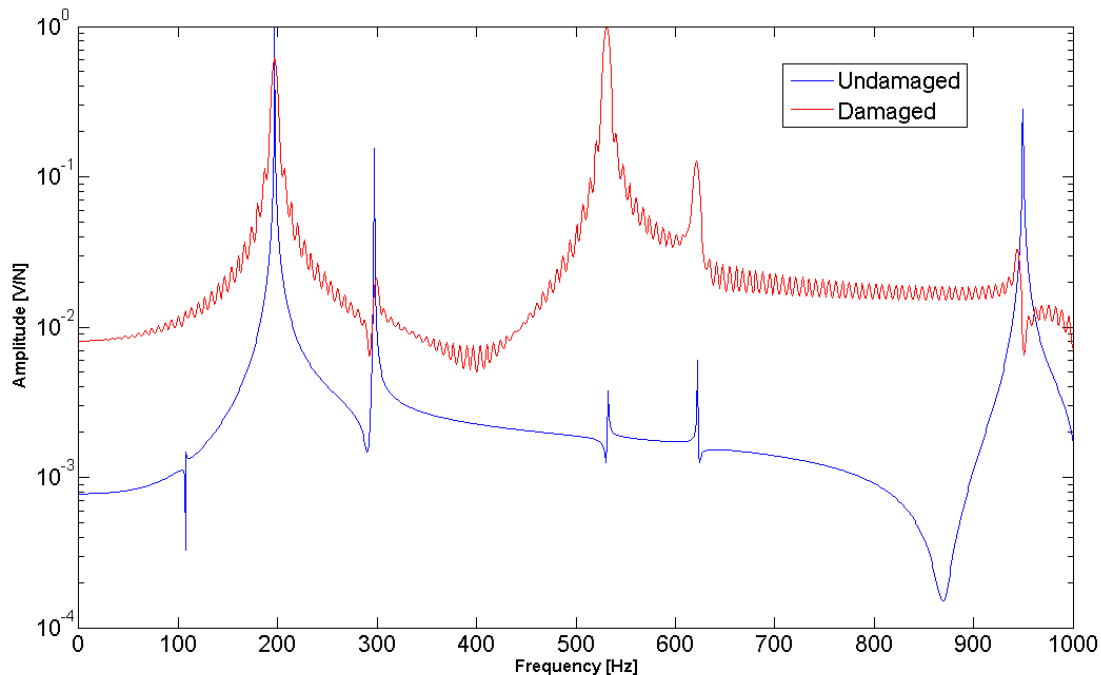


Figure 6: FRF using piezoelectric transducer: cylinder intact vs. damaged cylinder

Finally, it discusses the results for a node of the FE model. The Figure 7 shows the magnitude log-scale plot of the acceleration calculated with the FEM analysis in the 0 Hz – 1000 Hz frequency range. It can be observed that is small frequency differences between undamaged and damaged model. However, there is significant amplitude difference.

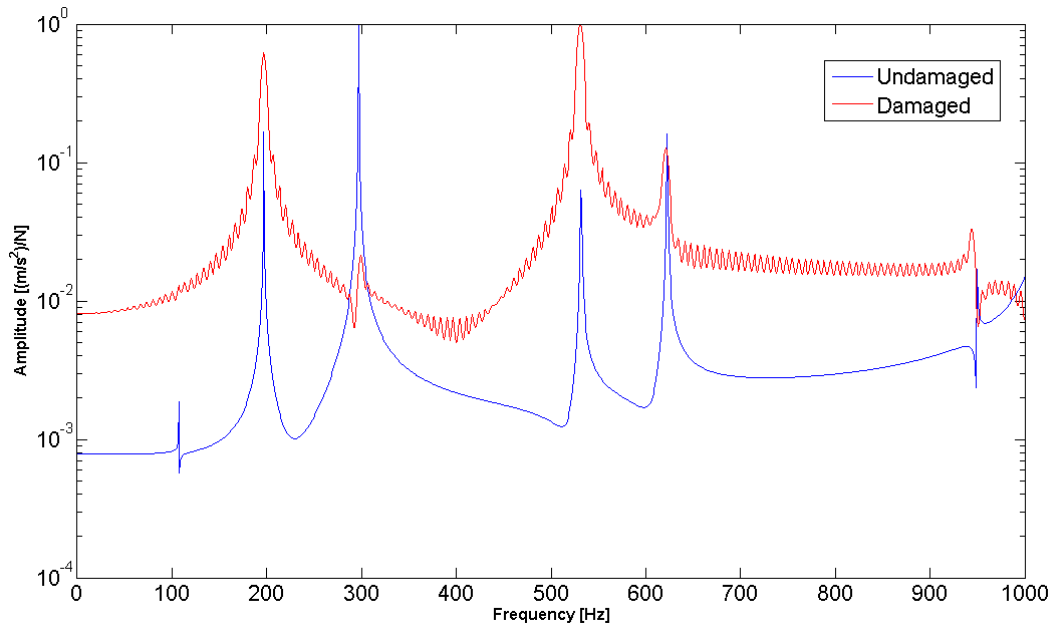


Figure 7: FRF using a node of the FE model: cylinder intact vs. damaged cylinder

5 CONCLUSION

Based on the results, it is concluded that the application of vibration-based methods for detecting damage on cylinders made of composite material is feasible. However, it is necessary that there are changes on the overall stiffness of the structure; so it is necessary to have large damage.

The numerical models used to simulate intact and damaged composite cylinder can be used as an alternative to design and evaluate SHM systems, which uses piezoelectric sensors. Furthermore, the results using only one PZT sensor indicate a severe limitation of the applied methods to provide information about the location or type of damages. In fact, to these tasks, it is recommended a net of PZT sensors. Therefore, the applied method with only one PZT sensor is suitable for a system SHM, which is simply used to identify the presence or not of the damage in the structure.

Although the numerical results have shown some limitations of the vibration-based methods, there are many advantages in employing the method based on FRFs in a SHM system for composite cylinders. For example, vibration-based methods combined with modal analysis provide global as well as local information on structural health condition and do not require direct human accessibility to the structure. In some cases, the PZT sensor can be embedded in the laminate structure. Also, the methods are cost effective and easy to operate, and has the potential for on-line damage detection with appropriate structural modeling. Therefore, it is possible to conclude that there is a great future perspective for the application of vibration-based methods using PZT sensors on SHM systems for composite structures.

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