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Strain rate dependency of post-consumer recycled high-density polyethylene

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Abstract: This paper describes a study of the strain rates dependency of post-consumer recycled high-density polyethylene (HDPE) tested in tension. The main goals of this study are to experimentally characterize recycled HDPE and to propose a one-dimensional viscoelastic phenomenological model able to yield a physically realistic description of strain rate sensitivity and damage observed in tensile tests that can be used in engineering problems. The material parameters that appear in the model can be easily identified from just three tests performed at different constant strain rates. The results of tensile tests conducted at different prescribed strain rates are presented and compared to model estimations of damage progression, and show good agreement.

1. Introduction

The environmental problem nowadays is a factor of extreme importance in the industrial world, particularly in the case of plastic processing companies, as efforts are mainly focused on the reduction and recycling of wastes generated during transformation processes and also after product end use. Plastics have become one of the materials with the greatest growth in terms of consumption and in the amount of generated wastes [1].

Although the recycling capacity for plastics has been progressively increased, the fraction of plastics that end up in a landfill is still very significant. EU produced 82.8 millions/ton of packing waste and 48.1 mil/ton were recycled [2]. In the US, the total of plastic waste produced in 2008 were 30 millions/ton and only 7.1% were recycled [3]. In Brazil, 150 thousand/ton of waste HDPE are produced per year and only 72 thousand/ton were recycled [4]. Due to that fact, researchers over the last years have focused their studies in finding ways to reprocess that waste plastics as new products [5-10].

High-density polyethylene (HDPE) is one of the most widely used polymers, having a broad amount of applications such as bottles, containers and consumer goods. Post-consumer HDPE from bottles is an interesting source of recycled material because, on one hand, it cannot be used again and, on the other hand, its high melting viscosity makes direct transformation via injection molding very difficult. Recycled HDPE can be used in a growing number of potential applications, such as boxes or pallets, whenever the thermal, mechanical and impact properties of the recycled polymer are close to the ones of the virgin material.

The use of recycled HDPE has become commonplace; therefore, a better understanding of the behavior of this material is necessary. The purpose of this study is to propose a one-dimensional phenomenological damage model for describing the viscoelastic behavior of recycled HDPE in tensile tests for different strain rates at room temperature. The model equations combine enough mathematical simplicity to allow their application to engineering problems with the capability of describing complex nonlinear mechanical behavior (irreversible deformations, strain rate sensitivity and damage observed in tensile tests performed at different strain rates). The material constants that appear in the model can be easily identified from just three stress-strain curves obtained at different prescribed strain rates. The model equations can be obtained within the thermodynamic context described in previous work by da Costa Mattos et al. [11-13].

2. Material and Methods

2.1 Materials

Recycled HDPE from post-consumer plastic motor oil containers were obtained from SEPAN Services (Niterói, RJ, BR). The containers were drained to eliminate any oil residue and then washed with biodegradable soap. The containers were dried at 90ºC for 3 hours and then shredded into pellets.

The shredded recycled HPDE was compression molded in a steel frame according to ASTM D 638-08 [14]. Differential scanning calorimetry measurements (DSC F3-Maia Netzsch®) characterized the HDPE as having a melting temperature of 137ºC and a specific density of 0.95 g/cm^3. Figure 1 displays the recycled HDPE pellets in the steel mold for conformation and the finished specimen.

Figure 1 – Recycled HDPE tensile specimen mold and geometry.

2.2 Methods

Mechanical tensile tests were performed using a Shimadzu® AG-X universal testing machine with electro-mechanical sensors for the control of longitudinal strain in the active zone of the test specimens. Tensile tests at five different prescribed engineering strain rates were performed to quantify the strain rate dependency: $\dot{\varepsilon}_1 = 7.25 \times 10^{-5}\ s^{-1}$, $\dot{\varepsilon}_2 = 1.45 \times 10^{-4} s^{-1}$, $\dot{\varepsilon}_3 = 7.25 \times 10^{-4} s^{-1}$, $\dot{\varepsilon}_4 = 1.45 \times 10^{-3} s^{-1}$ and $\dot{\varepsilon}_5 = 7.25 \times 10^{-3} s^{-1}$.

3. Results and Discussion

3.1. Experiments

Figure 2 presents the true stress vs. strain curves for recycled HDPE obtained from the controlled-strain tensile tests at different constant engineering strain rates: $\dot{\varepsilon}_1 = 7.25 \times 10^{-5}\ s^{-1}$, $\dot{\varepsilon}_2 = 1.45 \times 10^{-4} s^{-1}$, $\dot{\varepsilon}_3 = 7.25 \times 10^{-4} s^{-1}$, $\dot{\varepsilon}_4 = 1.45 \times 10^{-3} s^{-}1$ and $\dot{\varepsilon}_5 = 7.25 \times 10^{-3} s^{-1}$.

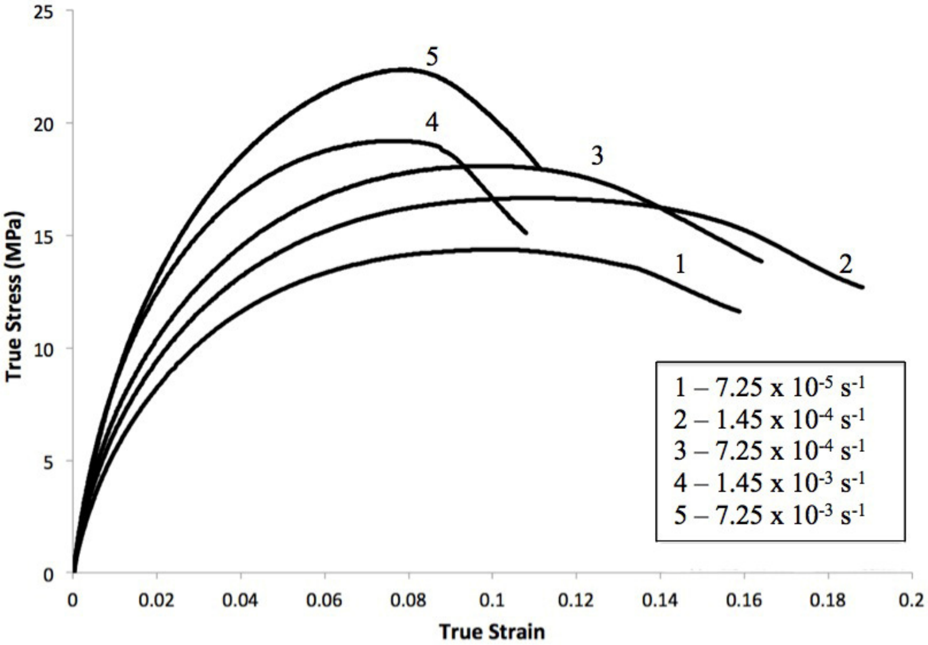

Figure 2 – True stress vs. true strain experimental curves for different strain rates.

The curves display a significant strain rate dependency in which the maximum strength and modulus of elasticity increase as the strain rate increases. Ductility decreases as the strain rate increases, and higher deformation levels are observed for lower strain rates.

3.2. Basic definitions and notations

Let us consider a simple tensile test in which the specimen has a gauge length L and a cross section A_o and is subjected to a prescribed elongation $\Delta L(t)$. The force necessary to impose such an elongation at a given instant t is denoted $F(t)$. The so-called engineering strain ε and the engineering stress σ are classically defined as follows:

$$\varepsilon(t) = \frac{\Delta L(t)}{L} \quad ; \quad \sigma(t) = \frac{F(t)}{A_o} \tag{1}$$

The so-called true strain ε_t and true stress σ_t are defined as follows:

$$\varepsilon_t = \ln(1+\varepsilon) \quad ; \quad \sigma_t = \sigma(1+\varepsilon) \tag{2}$$

From definitions (1) and (2), it is possible to obtain the following relations:

$$\varepsilon_t = \ln(1+\varepsilon) \;\Rightarrow\; \exp(\varepsilon_t) = \exp(\ln(1+\varepsilon)) \;\Rightarrow \varepsilon = \exp(\varepsilon_t) - 1 \Rightarrow \qquad \dot{\varepsilon} = \exp(\varepsilon_t)\,\dot{\varepsilon}_t$$

$$\dot{\varepsilon}_t = \frac{\dot{\varepsilon}}{(1+\varepsilon)} \tag{3}$$

In the phenomenological approach adopted in this study, an auxiliary variable D, called the damage variable, is introduced. This auxiliary variable can have values within the range $0 \leq D \leq 1$ and is presumed to be related to the loss of mechanical strength of the system due to damage (i.e., geometrical discontinuities such as microvoids and microcracks induced by mechanical deformation). If $D = 0$, the bar is considered “virgin,” and if $D = 1$, it is considered “broken” (meaning that it can no longer support mechanical loading). By definition, the damage variable can be directly related to the softening phenomenon in a tensile test and hence can be obtained experimentally. For a given stress, the damage variable is defined as follows (see figure 3):

$$\left(1 - D\right)\sigma_{\max} = \sigma \qquad \Rightarrow D = 1 - \left(\frac{\sigma}{\sigma_{\max}}\right) \tag{4}$$

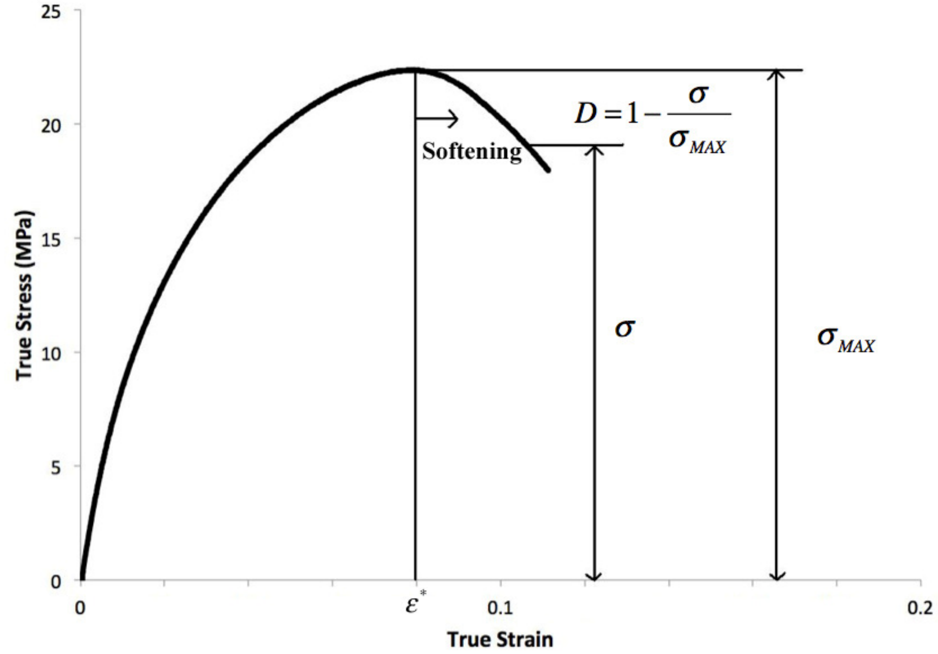

Figure 3 – Experimental identification of the auxiliary variable D in a tensile test.

$\sigma_{\max}$ is the ultimate or maximum stress, which is dependent on the strain rate. From definition (4), it can be verified that during a tensile test, $D = 0$ if the specimen exhibits strain hardening, and $0 < D \leq 1$ if the specimen exhibits strain softening.

3.3. Modeling

Assuming a fixed temperature, the following equation is proposed to model tensile tests of recycled HDPE at a variable strain rate:

$$\sigma_t = [1 - D(\varepsilon_t, \dot{\varepsilon})] \; [\; a(\dot{\varepsilon}) \; [1 - \exp(-b \; \varepsilon_t)] \;] \tag{5}$$

where the functions $D(\varepsilon_t, \dot{\varepsilon})$ and $a(\dot{\varepsilon})$ are defined as follows:

$$a(\dot{\varepsilon}) = a_1 \; \ln(\dot{\varepsilon}) + a_2 \tag{6}$$

$$D(\varepsilon_t, \dot{\varepsilon}) = c(\dot{\varepsilon}) \left\langle \varepsilon_t - \varepsilon^* \right\rangle^2 + d(\dot{\varepsilon}) \left\langle \varepsilon_t - \varepsilon^* \right\rangle \tag{7}$$

with

$$c(\dot{\varepsilon}) = c_3 \; \dot{\varepsilon}^3 - c_2 \; \dot{\varepsilon}^2 + c_1 \; \dot{\varepsilon} + b \tag{8}$$

$$d(\dot{\varepsilon}) = d_1 \; \dot{\varepsilon} - d_o \tag{9}$$

$$\varepsilon^*(\dot{\varepsilon}) = -\frac{1}{b(\dot{\varepsilon})} \ln(0.02) \tag{10}$$

$\left\langle \varepsilon - \varepsilon^* \right\rangle = \max\{(\varepsilon - \varepsilon^*), 0\}$, a_1, a_2, b, c_1, c_2, c_3, d_1 and d_o are positive material constants. It is important to note that $\dot{\varepsilon} = \dot{\varepsilon}_t \exp(\varepsilon_t)$. Equation 5 and auxiliary equations 6 to 10 form a complete set of constitutive equations.

This model is conceived for a given range of strain rates $\dot{\varepsilon}_{\min} \leq \dot{\varepsilon} \leq \dot{\varepsilon}_{\max}$. It is difficult to present a precise definition of the limiting strain rates $\dot{\varepsilon}_{\min}$ and $\dot{\varepsilon}_{\max}$. In the absence of a precise physical definition, it is suggested that a range from 7.25x10^{-5} s^{-1} to 7.25x10^{-3} s^{-1} be considered for the strain rate.

3.3. Parameter identification

The values of all 9 of the material constants (a_1, a_2, b, c_o, c_1, c_2, c_3, d_o and d_1) that appear in the theoretical model can be determined from just three tensile tests at constant engineering strain rates. These parameters are obtained in 2 steps. In the first step, the initial parts of the stress-strain curves, where the specimen exhibits strain hardening and, by definition, the damage variable is zero, are used. In the second step, the portions of the stress-strain curves where the specimen exhibits strain softening are used.

Table 1 - Material parameters

a_1	a_2 (MPa)	b	c_1	c_2	c_3	d_o	d_1
1.8028	32.24	42.3	98853	$-3\text{x}10^7$	$2\text{x}10^9$	1.71	443.94

In the initial part of the test at a constant engineering strain rate, the specimen exhibits strain hardening; therefore, by definition, the damage variable is zero. The true stress σ_t can be expressed as follows:

$$\sigma_t = [\ a(\dot{\varepsilon})\ [1 - \exp(-b\ \varepsilon_t)]\ ] \tag{6}$$

It is possible to verify the following from equation (6):

$$\lim_{\varepsilon_t = 0}(\sigma_t) = a(\dot{\varepsilon}) \tag{7}$$

Hence, $a(\dot{\varepsilon})$ is the maximum value of the stress σ_t for a given constant strain rate, as shown in figure 4.

From equation (6), the following can be verified:

$$\left.\frac{d\sigma_t}{d\varepsilon_t}\right|_{\varepsilon_t = 0} = a(\dot{\varepsilon})b(\dot{\varepsilon}) \tag{8}$$

Hence, if $a(\dot{\varepsilon})$is known, $b(\dot{\varepsilon})$ can be identified from the initial slope of the true stress vs. true strain curve, as shown in figure 4. The parameters $a(\dot{\varepsilon})$ and $b(\dot{\varepsilon})$ can also be identified using the least squares technique.

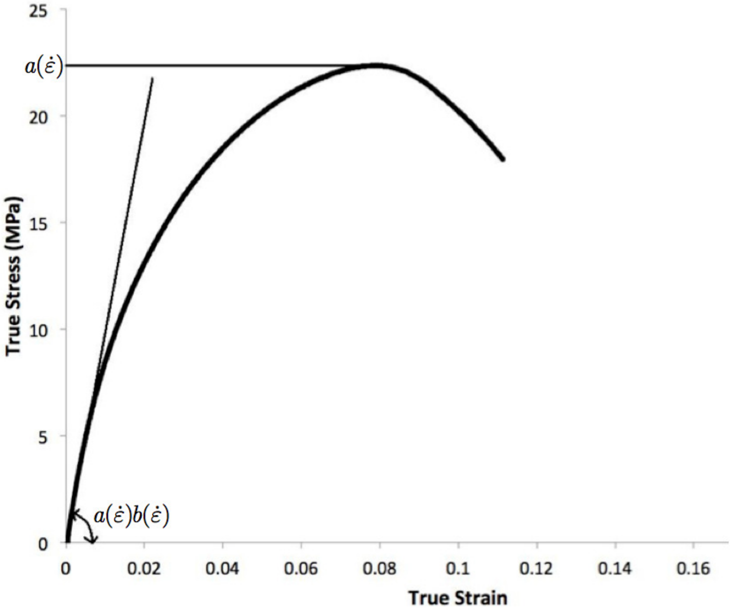

Figure 4 – Identification of $a(\dot{\varepsilon})$ and $b(\dot{\varepsilon})$parameters from the true stress vs. true strain curve.

The values of the proposed model parameters $a(\dot{\varepsilon})$ and $b(\dot{\varepsilon})$ can be determined using at least two tensile tests with different engineering strain rates; thus, the first part of the model can be calculated. When the specimen reaches the maximum stress, strain softening begins, and, by definition, the value of the damage variable is zero.

The damage vs. true strain curve can be obtained from the true stress vs. true strain curve. Figure 5 presents the damage curves associated with the tensile tests shown in figure 2.

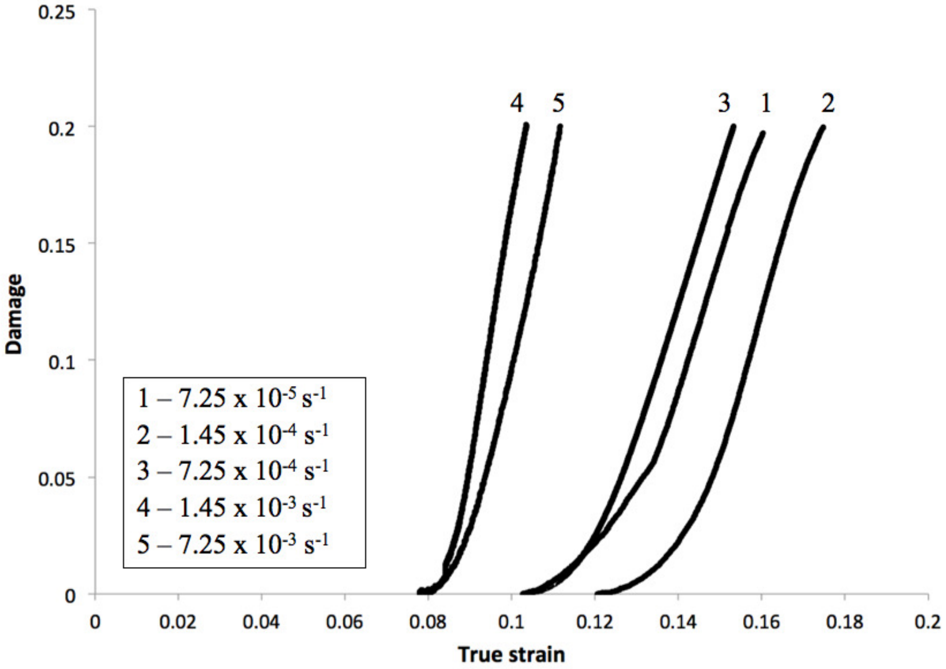

Figure 5 – Damage curves for tensile tests of recycled HDPE at different strain rates.

Figure 5 shows that during a typical tensile test, the value of the damage variable increases slowly, and for a design specification, it was decided to quantify the maximum value for the variable $D = 0.2$. No such material will be designed to work beyond its maximum strength; therefore, for this case, it was decided to model the material behavior until it reaches 20% damage. Such a damage variable value can be adopted as a quality limit for any design part. Figure 6 shows that the values of the damage parameters $c(\dot{\varepsilon})$ and $d(\dot{\varepsilon})$ presented in equation 9 can be determined from three tensile tests with different engineering strain rates.

To determine the accuracy of the model, samples of experimental results were cross-checked with the mathematical model.

Figure 6 presents the experimental and theoretical true stress vs. true strain curves for different strain rates.

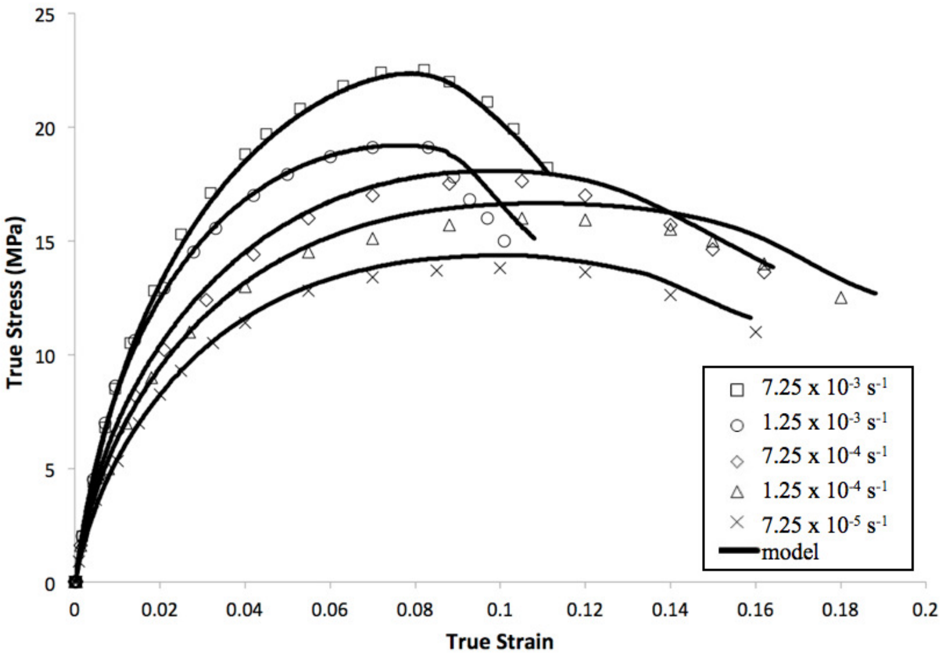

Figure 6 – Recycled HDPE true stress vs. true strain curves for different strain rates. Comparison with experimental results.

Clearly, figure 6 shows that the experimental results agree well with the model predictions obtained using estimated parameters. Due to the complexity of the material, small discrepancies are observed when the experimental results are compared to the model predictions. It is important to emphasize that although test results for 5 different strain rates are presented, just 3 tests are sufficient to predict the mechanical behavior of recycled HDPE.

4. Conclusions

In this study, the strain rate dependency of recycled high-density polyethylene (HDPE) is analyzed. In addition, a simple continuum damage model is proposed to describe the tensile behavior of HDPE. The strain rate significantly influences the modulus of elasticity, tensile strength and ductility of HDPE. The proposed model equations combine simple mathematical simplicity that facilitates their application to engineering problems with a physically realistic description of the mechanical behavior of HDPE. The intent of this study is to use this model formulation to obtain the maximum amount of information possible about the macroscopic properties of recycled HDPE from a minimum number of laboratory tests, saving time and experimental costs. Just three tensile tests at room temperature are needed to determine the values of the parameters of the model. The proposed model results agree well with experimental results for different strain rates.

5. Acknowledgements

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6. References

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