A MESHLESS METHOD APPLIED TO THE HEAT TRANSFER ANALYSIS OF A DIESEL ENGINE PISTON

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Abstract. *The recent advances in computer technology have provided powerful tools for the numerical simulation of heat transfer phenomena within engine pistons. Numerical schemes such as finite volume method, finite difference method and the finite element method are commonly used. The success of discretization schemes mainly relies on the mesh of good quality. Thus, mesh generation often poses a challenge for the numerical simulations associated with industrial and environmental applications, especially for those problems with complex geometry. Recent development in the automatic mesh generation techniques for mesh-based methods relieves the associated difficulties. However, maintain detailed structural information about the computational mesh is still expensive, making mesh generation, modification, and re-meshing a challenging task for programmers, mathematicians and engineers. In this paper we analyzed the steady-state heat transfer within a piston of a diesel engine, using a meshless method, based on the Method of Fundamental Solutions. Preliminary results are presented for different engine operating conditions. Results are also compared with data obtained through other numerical techniques, showing a good agreement between the solutions.*

Keywords*: internal combustion engines, meshless methods.*

1. INTRODUCTION

The in-cylinder heat transfer has been generally recognized as a major factor influencing internal combustion engines efficiency and exhaust emissions. Therefore, the analysis of in-cylinder heat transfer is of importance for fuel economy, as well as for environmental preservation.

The solution of heat transfer problems in internal combustion engines is a very complex task due to different reasons, including, the cyclic temperature variation of gases inside the engine; the irregular geometry of the parts involved in the heat transfer process, and the variance of heat transfer coefficients between different part faces, which may vary during the cycle. Therefore, the estimation of heat transfer coefficients constitute, in itself, a problem. A review of available theoretical and experimental works on the subject was presented by Borman and Nishiwaki (1987).

Different expressions for the time-varying heat transfer coefficient between gases and piston have been suggested in the literature (Borman and Nishiwaki, 1987, Heywood, 1988, Kornhouser and Smith, 1994). However, stress computations do not require the knowledge of the cyclic variation of temperature in the piston and are based on time-averaged heat transfer coefficients (Borman and Nishiwaki, 1987, Singh et al, 1986, Prasad and Samria, 1990). Such is the case because the temperature only varies in a very small depth below the surface during each cycle.

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Recent development in the automatic mesh generation techniques for mesh-based methods relieves the associated difficulties. However, maintain detailed structural information about the computational mesh is still expensive, making mesh generation, modification, and re-meshing a challenging task for programmers, mathematicians and engineers.

To overcome the above mentioned difficulties, mesh-free and meshless methods are been developed. Mesh-free methods are being applied to several direct and inverse problems in heat transfer (Alves et al, 2008; Valle and Colaço, 2008; Colaço et al, 2009; Magalhães et al, 2008; Colaço et al, 2006a,b). From the viewpoint of kernel interpolation/approximation techniques, many mesh-free methods are based on the moving least square technique. This group of mesh-free methods has been successfully applied to many practical but difficult problems in engineering that are to be solved by the traditional mesh-based methods.

In this paper we used a meshless technique, based on the Method of Fundamental Solutions (Kupradze and Aleksidze, 1964) to study a steady-state temperature distribution within a piston of a Diesel internal combustion engine.

2. PHYSICAL PROBLEM

The physical problem considered here is the transient heat conduction in a diesel engine piston. The piston is assumed to be axi-symmetric, so that asymmetries due to the piston pin and oil cooling channels are neglected. The geometry and coordinates (in millimeters) relevant for this study are presented in Figure 1 (Colaço et al, 2010):

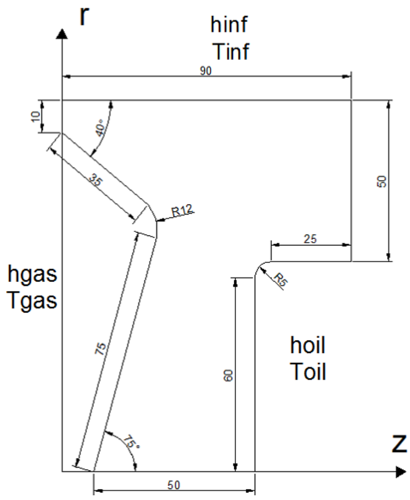


Figure 1 - Geometry and coordinates (dimensions in mm)

The piston is heated through its top surface by the gases inside the combustion chamber. The gas temperature (T_{gas}) and the heat transfer coefficient between gases and piston (h_{gas}) are assumed to be constant. The piston is cooled by oil on its bottom surfaces and by a coolant fluid flowing through passages in the cylinder wall. The oil temperature (T_{oil}), as well as the heat transfer coefficient between oil and piston (h_{oil}) are supposed to be constant. The heat transfer to the coolant fluid is taken care by using a constant overall heat transfer coefficient (h_∞), which takes into account the heat transfer from the piston to the cylinder wall, the conduction through the wall, and the convection from the wall to the coolant fluid. The fluid temperature (T_∞) is assumed to be constant.

The mathematical formulation of such physical problem is given by:

$$\nabla^2 T(r,z) \qquad \text{for } (r,z) \in \Omega_1 \tag{1.a}$$

$$-k\frac{\partial T(r,z)}{\partial \mathbf{n}_i} = h_i(T - T_i) \qquad \text{on the boundary surface } \Gamma_i \tag{1.b}$$

$$\frac{\partial T}{\partial r} = 0 \qquad \text{on the symmetry axis } (r=0) \tag{1.c}$$

where h_i, T_i and $\frac{\partial T}{\partial \mathbf{n}_i}$ are, respectively, the heat transfer coefficient, the fluid temperature and the normal derivative of temperature at boundary surface Γ_i. The variable k is the thermal conductivity.

For the solution of the problem given by Eqs. (1) we have used the Method of Fundamental Solutions as described next.

3. SOLUTION METHODOLOGY

The Method of Fundamental Solutions (MFS) is a meshless technique, which is integration free, for the numerical solution of certain elliptic boundary value problems. It was first proposed by Kupradze and Aleksidze (1964), and it has solution if the fundamental solution of the governing equation is known. The MFS has been used to solve different kinds of problems such as biharmonic problems, radiation-type boundary conditions, diffusive-convective problems, free boundary problems, potential problems and elastostatic and acoustic problems (Fairweather and Karageorghis, 1998). The MFS is also known as the desingularized method, the charge simulation method or the superposition method in the mathematical and engineering literature (Alves and Chen, 2005).

The main idea of the MFS consists of approximating the solution of the problem by means of a linear combination of fundamental solutions with respect to some source points that are placed on a fictitious boundary outside or inside the domain. Good results are usually obtained with equally distributed collocation points on the boundary and a similar distribution of source points on the artificial boundary.

For this method, the unknown quantity (the temperature in the context of this paper) is approximated by an expansion series, as represented in the following equation:

$$T(r,z) = \sum_{i=1}^{N} \beta_i \phi_i(r,z) \qquad \text{for } (r,z) \in \Omega_1 \tag{2}$$

where β_i are the unknown coefficients to be determined, $\phi_j(r,z)$ is the fundamental solution of the elliptic partial differential equation considered and N is the number of source points.

If $\mathbf{P}=(r,z)$ and $\mathbf{P}_i=(r_i,z_i)$ are two points in Ω_1, we can define:

$$R = \sqrt{(r_i + r)^2 + (z_i - z)^2} \tag{3}$$

For the axysimmetric problem, as the one considered in this paper, the fundamental solution of the Laplace equation is given as (Karageorghis and Fairweather, 1999):

$$\phi_i(\mathbf{P}-\mathbf{P}_i) = \frac{4K(k)}{R} \qquad \text{for 2D axysimmetric} \tag{4}$$

where $K(k)$ is the complete elliptic integral of the first kind (Karageorghis and Fairweather, 1999), defined as:

$$K(k) = \int_0^{\pi/2} \frac{1}{\sqrt{1-k^2 \sin^2 \theta(Q)}} d\theta(Q) \tag{5}$$

and

$$k = \sqrt{\frac{4\, r\, r_i}{R^2}} \tag{6}$$

For the normal derivative of temperature, Karageorghis and Fairweather (1999) developed an expression, given by Eq. (7):

$$\frac{\partial \phi_i(\mathbf{P}-\mathbf{P}_i)}{\partial \mathbf{n}_i} = \frac{2\left\{R^2\left[E(k)-K(k)\right](1-k^2) - 2r_i(r_i+r)E(k)\right\}}{r_i R^3 (1-k^2)} \hat{\mathbf{n}}_r - \frac{4(z_i - z)}{R^3(1-k^2)} \hat{\mathbf{n}}_z \tag{7}$$

where $E(k)$ is the complete elliptic integral of the second kind (Karageorghis and Fairweather, 1999), defined as:

$$E(k) = \int_0^{\pi/2} \sqrt{1-k^2 \sin^2 \theta(Q)}\, d\theta(Q) \tag{8}$$

Equations (2)-(8) can be then used to solve the physical problem given by Eq. (1) for the domain Ω_1 with a proper choice of source points located outside of the domain (Alves et al, 2008; Valle and Colaço, 2008; Colaço et al, 2009; Magalhães et al, 2008; Colaço et al, 2006a,b).

4. RESULTS AND DISCUSSION

For the results presented below, the values of various parameters were chosen as follows (Colaço et al, 1996):

(i) Oil temperature: T_{oil}=50°C;
(ii) Heat transfer coefficient to the oil: h_{oil}=175W/m^{2o}C;
(iii) Piston thermal conductivity: k=54W/m^{2o}C;
(iv) Heat transfer coefficient to the gases in the combustion chamber: h_{gas}=290 W/m^{2o}C.

Other parameters of interest for the analysis were varied for the study of several test-cases. The values of such parameters appear below when required.

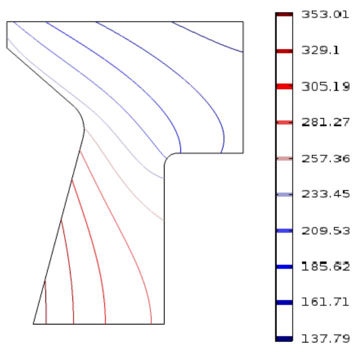
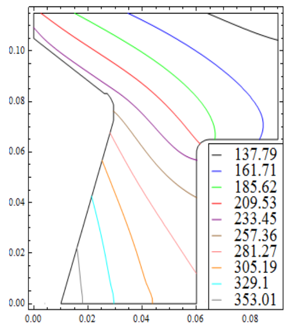
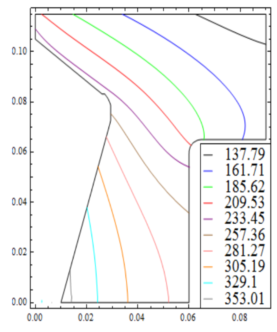
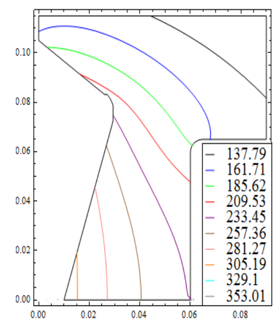
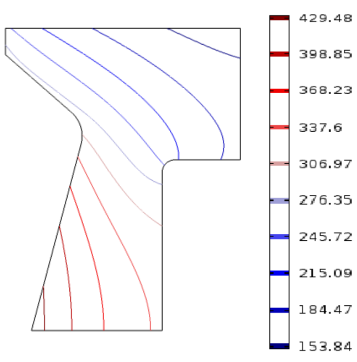
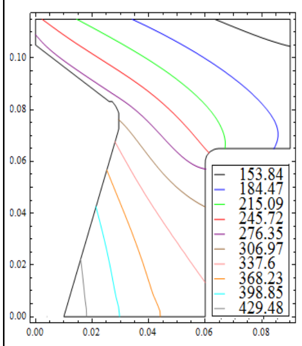
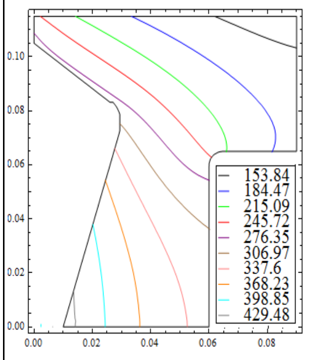
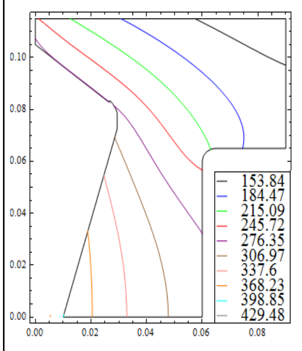
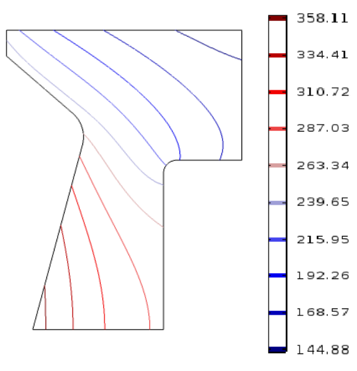
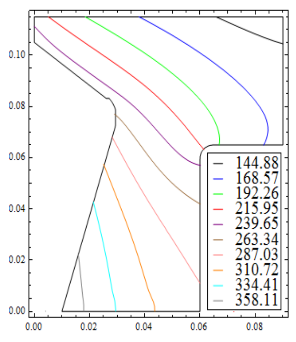
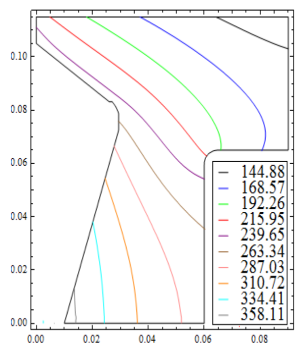
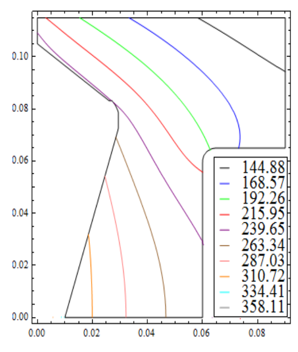
The test-cases studied in this paper are summarized in table 1.

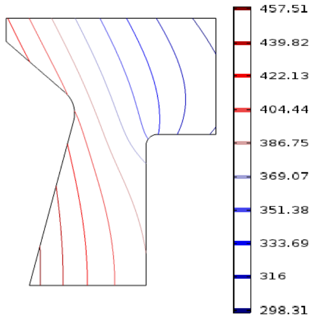
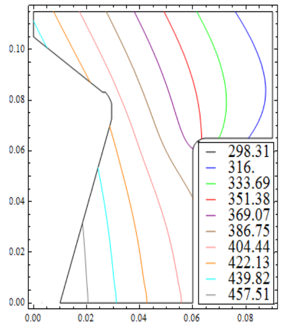
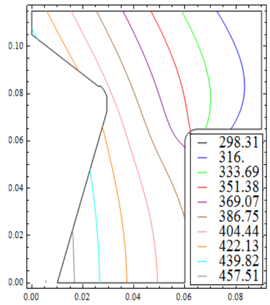
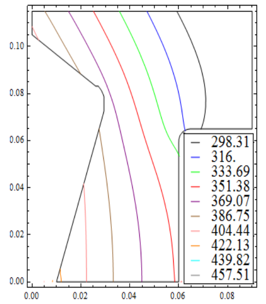
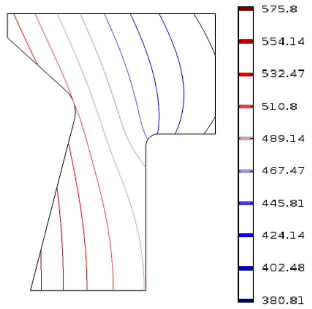
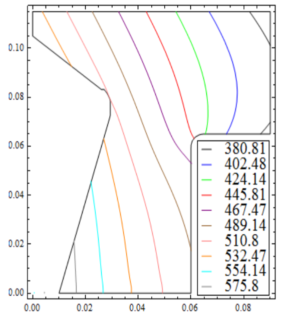
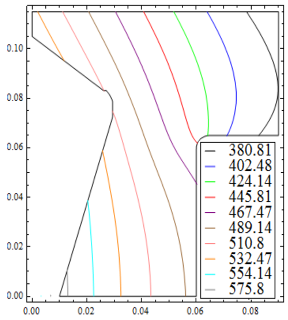
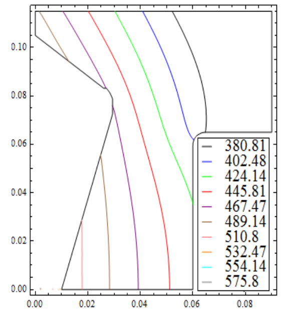
Table 1 - Test-cases

Case	Coolant	T_∞(°C)	h_∞(W/m²°C)	T_{gas}(°C)
1	water	85	1400	800
2	water	85	1400	1000
3	water	85	1200	800
4	air	25	123	800
5	air	25	123	1000

Table 2 shows the results obtained using the software COMSOL and the ones obtained using the Method of Fundamental Solutions (MFS) with three different distances d between the boundary of the domain and the source points, located outside the domain of interest.

Table 2 - Results (All values presented are in °C)

Case	**COMSOL**	**Method of Fundamental Solutions (Current work)**		
		(*d*=3.0)	**(*d*=2.0)**	**(*d*=1.0)**
1				
2				
3				

4				
5				

By analyzing the results presented in Table 2, it is possible to conclude that the behavior of the isothermal curves obtained by the MFS for the three distances is similar to the ones obtained by using finite differences. This can be concluded from all the five test cases.

The results of the Method of Fundamental Solutions shown in Table 2 were obtained by using the boundary points and source points that are illustrated in Figure 2 and the COMSOL solutions used the mesh presented in Figure 3. For the COMSOL solution we used a very refined mesh, in order to use this result as benchmark solution for the present paper. The mesh used by the COMSOL had 12962 triangular elements and each one had a maximum size of 1.15 mm, a minimum size of 0.0023 mm and an average area of 0.4938 mm².

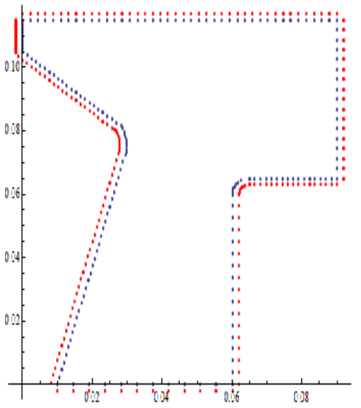


Figure 2 – Boundary points (blue) and source points (red)

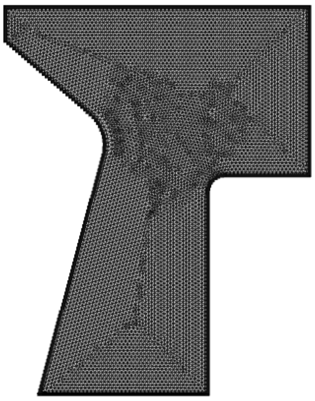


Figure 3 – Mesh used in the COMSOL

In order to obtain the solutions found in Table 2, COMSOL required a CPU time of about 3 seconds. For the Method of Fundamental Solutions, the CPU time necessary to build the linear system and to solve it is presented in Table 3. All test cases were run on a Intel Core 2 Duo 2.27Ghz with 4 Gb of RAM.

Table 3 – CPU time for the MFS

Case	CPU time for the Method of Fundamental Solutions (s)		
	d=3.0	*d*=2.0	*d*=1.0
1	9.53	9.44	9.41
2	9.64	9.58	9.59
3	9.92	9.99	9.75
4	9.84	9.85	9.80
5	9.75	9.95	9.86

It is possible to conclude from Table 3 that the Method of Fundamental Solutions required a greater average CPU time than COMSOL. However, the Method of Fundamental Solutions used in this work was implemented in the Mathematica software. The requested time could be much lower if it was implemented in C++ or Fortran.

Five points of the domain were chosen, as shown in Figure 4, and the values of their temperatures were calculated and compared against the COMSOL solution as presented in Table 4. The relative errors between these values were also calculated.

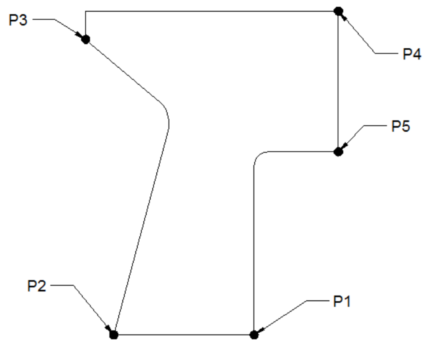


Figure 4 – Points used to calculate the errors

A detailed convergence analysis can be provided from Table 4. The results obtained from the distance *d*=2.0 presented the best agreement with the COMSOL solution, mainly for the locations P1, P2 and P3. The distance *d*=3.0 also obtained good results for P1, P2 and P3, but with greater errors than *d*=2.0. However, both these distances had greater errors in P4 and P5 in most test-cases. The distance *d*=1.0 presented, with few exceptions, like the results of P4 and P5 in some test cases, bad convergence for the chosen points. This all indicates that there's an optimum distance for this distribution of points and source points shown in Figure 2 between *d*=2.0 and *d*=3.0.

Table 4 – Results for P1, P2, P3, P4 and P5

	Point	COMSOL	Method of Fundamental Solutions					
			***d*=3.0**		***d*=2.0**		***d*=1.0**	
		***T*(°C)**	***T*(°C)**	**Error**	***T*(°C)**	**Error**	***T*(°C)**	**Error**
Case 1	P1	277.03	286.61	3.46%	274.09	-1.06%	242.78	-12.36%
	P2	364.96	371.20	1.71%	361.91	-0.84%	328.74	-9.92%
	P3	242.18	242.53	0.14%	242.02	-0.07%	237.92	-1.76%
	P4	125.84	137.07	8.92%	135.79	7.91%	130.17	3.44%
	P5	156.70	166.63	6.34%	164.96	5.27%	155.27	-0.91%
Case 2	**Point**	**COMSOL**	**Method of Fundamental Solutions**					
			***d*=3.0**		***d*=2.0**		***d*=1.0**	
		***T*(°C)**	***T*(°C)**	**Error**	***T*(°C)**	**Error**	***T*(°C)**	**Error**
	P1	333.02	345.35	3.70%	329.33	-1.11%	290.04	-12.91%
	P2	444.79	452.81	1.80%	440.96	-0.86%	399.32	-10.22%
	P3	286.51	286.96	0.16%	286.34	-0.06%	281.78	-1.65%

	P4	138.53	153.18	10.58%	151.64	9.46%	145.56	5.07%
	P5	179.20	192.35	7.34%	190.37	6.23%	178.94	-0.15%
Case 3	**Point**	**COMSOL**	**Method of Fundamental Solutions**					
			***d*=3.0**		***d*=2.0**		***d*=1.0**	
		***T*(°C)**	***T*(°C)**	**Error**	***T*(°C)**	**Error**	***T*(°C)**	**Error**
	P1	282.47	291.81	3.31%	279.53	-1.04%	248.21	-12.13%
	P2	369.95	376.01	1.64%	366.90	-0.82%	333.76	-9.78%
	P3	252.92	253.40	0.19%	252.79	-0.05%	247.87	-2.00%
	P4	133.03	144.34	8.50%	142.95	7.46%	136.67	2.74%
	P5	163.29	173.06	5.98%	171.34	4.93%	161.20	-1.28%
Case 4	**Point**	**COMSOL**	**Method of Fundamental Solutions**					
			***d*=3.0**		***d*=2.0**		***d*=1.0**	
		***T*(°C)**	***T*(°C)**	**Error**	***T*(°C)**	**Error**	***T*(°C)**	**Error**
	P1	387.83	399.32	2.96%	391.28	0.89%	349.81	-9.80%
	P2	466.35	475.88	2.04%	469.17	0.60%	426.87	-8.47%
	P3	433.04	445.44	2.86%	442.36	2.15%	407.76	-5.84%
	P4	289.46	308.09	6.44%	303.97	5.01%	269.01	-7.06%
	P5	296.04	310.43	4.86%	307.26	3.79%	257.87	-12.89%
Case 5	**Point**	**COMSOL**	**Method of Fundamental Solutions**					
			***d*=3.0**		***d*=2.0**		***d*=1.0**	
		***T*(°C)**	***T*(°C)**	**Error**	***T*(°C)**	**Error**	***T*(°C)**	**Error**
	P1	488.03	493.59	1.14%	483.59	-0.91%	432.42	-11.39%
	P2	586.63	590.48	0.66%	582.14	-0.77%	529.81	-9.69%
	P3	550.32	552.64	0.42%	548.84	-0.27%	506.19	-8.02%
	P4	369.98	378.73	2.36%	373.69	1.00%	331.04	-10.52%
	P5	374.88	381.70	1.82%	377.90	0.81%	339.62	-9.41%

5. CONCLUSIONS

In this paper we presented some preliminary results of a meshless method, based on the Method of Fundamental Solutions, to analyze the heat transfer in a diesel engine piston. The results were compared with the ones obtained in the COMSOL software for a very refined grid.

It is safe to say that the MFS presented good results for all the test-cases due to the low discrepancy between them and the ones obtained from the COMSOL solution. However, to obtain accurate results it is still necessary to first discover the distance *d* that has the best convergence for the distribution of boundary points and source points chosen. This distance will be optimized for its best value in an extension of this work.

6. ACKNOWLEDGEMENTS

The authors would like to thank the Brazilian agencies for the fostering of science, *Conselho Nacional de Desenvolvimento Científico e Tecnológico* (CNPq), *Coordenação de Aperfeiçoamento de Pessoal de Nível Superior* (CAPES) and *Fundação Carlos Chagas Filho de Amparo à Pesquisa do Estado do Rio de Janeiro* (FAPERJ), for the financial support for this work. Mr. Pasqualette is also greateful to the PIBIC/UFRJ/CNPq scholarship.

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