A NEW SET OF VORTEX IDENTIFICATION PARAMETERS BASED ON THE STRAIN ACCELERATION TENSOR

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Abstract. *A mathematical definition of a vortex became an important issue in Fluid Mechanics specially after the recognition of the importance of coherent structures on the turbulence dynamics. The birth, evolution, dissipation and death of a vortical coherent structure plays a crucial role on the understanding of turbulence as a phenomenon. In this context, the evaluation of the strain acceleration, as the strain itself is crucial to determine the flow tendency. When those two entities presents ortogonal or out-of-phase behavior, its correct to classify the region as a vortex. The performance of a new set of vortex identification parameters are evaluated for the so called ABC flow (u,v,w)=(A sin z + C cos y, B sin x + A cos z, C sin y + B cos x), which is an example of a laminar Trkalian Beltramian flow exhibiting chaotic behavior. Those vortex identification parameters differ from the parameters presented in previous articles from the same authors.*

Keywords: *Vortex, Coherent vortical structure, Frame invariance*

1. INTRODUCTION

A mathematical description of a vortex, with full acceptance, is one great lacuna in fluid mechanics. Several points distinguish the proposed criteria and its hypothesis limits from their application to any type of flow. Some criteria present alternative formulations for compressive or non-newtonian flows, but fail to identify vortices on regions of high vorticity, for example. Most of proposed criteria are based on kinematic assumptions. Some intuitive criteria, like vorticity magnitude or spiralling streamlines are fundamented on the velocity field and their behavior along space. Even some proposed criteria which are based on dynamic assumptions have their mathematical description as a function of kinematic entities. Since a vortex definition has not find yet a common description, there is not much to say about the association of dynamic entities em vortex identification in the literature.

The correct description of a vortex can lead to advances in many distinct areas in fluid mechanics: Mixing in non-newtonian fluids and in reactive flows where viscosity and density are variable fields along the flow. The turbulence is another important field where those criteria can be applied, enhancing the analysis and modeling of its related phenomena.

Although some intuitive ideas and observation can lead us to some basic conclusions, the lack of a mathematical description around the subject limits the application comprehensiveness to some simple flows. The description, for example, of a oil-water emulsion formation depends, among other reasons, on how the local mixing is responsible for breaking the oil droplets into small ones. If a vortex identification criterion could be mathematically inserted into the momentum equations for each phase, those phenomena could be associated.

The strain acceleration is being pointed out as a possible way to achieve a set of vortex parameters. The work of Haller (2005) presented a new concept to the vortex identification, proposing to identify vortices in regions where the acceleration of strain defies the behavior of the strain itself. The author was one of the first researchers in the field that explicit comment on the necessity of an objective vortex mathematical description.

The present work is intended to present a new concept of vortex identification parameters based on the strain acceleration and apply those to the so-called ABC flow. The new set of anisotropic parameters shows a connection between 2d and 3d flow, which was not possible in the previous parameters proposed by the authors.

2. CLASSICAL VORTEX IDENTIFICATION CRITERIA

2.1 Hunt et al. (1988) criterion

The criterion proposed by Hunt et al. (1988) is intrinsically related to a competition between vorticity and rate-of-strain where, in the case of a vortex, vorticity wins. The authors define a vortex as a connected region in space in which

$$Q = \frac{1}{2}[\|\mathbf{W}\|^2 - \|\mathbf{D}\|^2] > 0, \tag{1}$$

where $\mathbf{W}$ and $\mathbf{D}$ are, respectively the skew-symmetric and symmetric parts of the velocity gradient and the operator $\|\ \|$ indicates the Euclidean norm of a tensor.

2.2 Chong et al. (1990) criterion

A second criterion was formulated by Chong et al. (1990). This criterion is based on the fact that, when vorticity vanishes, the eigenvalues and eigenvectors of the velocity gradient are (the same as the rate-of-strain) real, since the velocity gradient, in this case is symmetric. The so-called Δ-criterion is given by a region where

$$\Delta = \frac{III_{\mathbf{L}}^2}{2} + \frac{Q^3}{27} > 0, \tag{2}$$

where $III_{\mathbf{L}}$ is the third invariant (determinant) of the velocity gradient. The Δ-vortex is a larger region than a Q-vortex, since $Q > 0$ is equivalent to $\Delta > 0$. This also shows that, to produce complex eigenvalues, the vorticity intensity measured by its norm, may not overcome the rate-of-strain intensity with the same measurer.

2.3 Jeong and Husssain (1995) criterion

Another very important criterion in the literature was proposed by Jeong and Hussain (1995). This criterion is based on a pressure minimum at the vorticity plane. The gradient of the Navier-Stokes equation can be separated into a symmetric and skew-symmetric parts, which is satisfied, for an incompressible Newtonian fluid, when

$$\lambda_2^{\mathbf{D^2}+\mathbf{W^2}} < 0, \tag{3}$$

where $\lambda_2^{\mathbf{D^2}+\mathbf{W^2}}$ is the intermediate eigenvalue of tensor $\mathbf{D^2} + \mathbf{W^2}$. It is interesting to notice that, although expressed by kinematic quantities, this criterion is based on dynamical arguments.

3. OTHER IMPORTANT CRITERIA

3.1 Tabor and Klapper (1994) criterion

Tabor and Klapper (1994) presented a systematic study on the stretching and alignment dynamics in general flows and came up with an interesting kinematic tensor very relevant to the present work. This tensor,Ω, measures the rate of rotation of the eigenvectors of $\mathbf{D}$, defined as

$$\Omega = \dot{e}_i^D e_i^D \tag{4}$$

where $e_i^{\mathbf{D}}$ is an eigenvectors of $\mathbf{D}$. They used, in that study, the Relative-rate-of-rotation tensor, $\bar{\mathbf{W}}$, the difference between the vorticity tensor and Ω. Therefore, a Q_s-criterion criterion can be constructed as

$$Q_s = \frac{1}{2}[\|\mathbf{W} - \Omega\|^2 - \|\mathbf{D}\|^2] > 0 \tag{5}$$

It is worth noticing that the relation between Q and the vorticity number, N_k is analog to the relation between Q_s and the stress-relieving parameter, R_D, proposed by Astarita (1979), defined as

$$R_D = -\frac{tr\bar{\mathbf{W}}^2}{tr\mathbf{D}} \tag{6}$$

$Q_s > 0$ is equivalent to $R_D > 1$.

3.2 Kida and Miura (1998) criterion

The criterion proposed by Kida and Miura (1998) follows the same principle considered by Jeong and Hussain (1995). The difference lies on the fact that the pressure minimum is calculated at the plane defined by the eigenvector correspondent to the smallest eigenvalue of the pressure Hessian. The vortex core is defined as a region where the skew-symmetric part of the velocity gradient projected on this plane overcomes its symmetric part.

3.3 Zhou et al. (1999) criterion

The so-called λ_{ci}-criterion was introduced by Zhou et al. (1999). It is based on the Δ-criterion of Chong et al. (1990). When $\Delta > 0$, the velocity gradient has two complex eigenvalues $\lambda_{cr} + i\lambda_{ci}$. The imaginary part λ_{ci} is identified as the swirling strength of the vortex. The criterion consists of a $\lambda_{ci}^2 > \delta$, where δ is a threshold generally chosen as percentage of its maximum value. When $\delta = 0$, $\Delta > 0$ and $\lambda_{ci} > \delta$ are equivalent.

3.4 Haller (2005) criterion

Another criterion based on a non-local vortex definition was presented by Haller (2005). He considered a vortex as a set of fluid trajectories that avoid the so-called hyperbolic domain, a domain defined as a region in space where the fluid defies, in a certain sense, the trend suggested by the rate-of-strain. To define the hyperbolic domain Haller (2005) uses (half of) the second Rivlin-Ericksen tensor, $\mathbf{A}_2$, the covariant convected time derivative of the rate of deformation tensor, $\mathbf{A}_1 = 2\mathbf{D}$, defined as

$$(\mathbf{A})_2 = (\mathbf{A}')_1 + (\mathbf{A})_1(\mathbf{L}) + (\mathbf{L})^T(\mathbf{A})_1 \tag{7}$$

where $\mathbf{L}$ is the transpose of the velocity gradient. For flows where the first invariant of $\mathbf{D}$ vanishes ($I_{\mathbf{D}} = 0$, isochoric flows) and the third invariant of $\mathbf{D}$ is a non-zero quantity ($III_D \neq 0$), he defines an elliptical cone on the basis of the eigenvectors of D, (e_1, e_2, and e_3) as

$$d\xi_3^2 = ad\xi_1^2 + (1+a)d\xi_2^2 \tag{8}$$

where $d\xi = d\xi_1 e_1 + d\xi_2 e_2 + d\xi_3 e_3$ is an infinitesimal vector and a is the ratio between the greatest and smallest eigenvalues of $\mathbf{D}$. The hyperbolic domain is a region in space where the second Rivlin-Ericksen tensor is positive definite in the elliptical cone defined by Eq.(8).

3.5 Chakraborty et al. (2005) criterion

Chakraborty et al. (2005) proposed a further step on the analysis of Zhou et al. (1999) by adding to the swirling strength criterion, an inverse spiraling compactness, measured by the ratio $\frac{\lambda_{cr}}{\lambda_{ci}}$. This ratio can be seen as local version of the non-local quantity introduced by Cucitore et al. (1999).

4. STRAIN ACCELERATION-BASED CRITERIA

In previous works, we have developed two methods for an anisotropic comparison between the diagonal and off-diagonal components of a matrix. Following Haller (2005) and Thompson (2008) we use the matrix associated with the second Rivlin-Ericksen tensor. The first method which will be called here line-method is to compare, in the diagonal components of the tensor $\mathbf{A}_2^{\mathbf{A}_1}$, acceleration tensor on the basis of the strain tensor, $\mathbf{L}$, the part of each component that comes from the diagonal and off-diagonal component of tensor $\mathbf{A}_2^{\mathbf{A}_1}$

$$AR_i^A = \frac{(A|_{ii})^2}{(A^2)|_{ii}} \tag{9}$$

The parameters are re-ordered after the calculation, in order to the AR_1^A contain the higher value in each point and AR_3^A the smaller values. This methodology is necessary since the sign in the relation between the strain main directions and the coordinate system can change without relation with its neighborhoods, resulting in discontinuous fields.

The new set of anisotropic parameters is concept to evaluate the characteristic polynomial of the current strain acceleration with the characteristic polynomials of the same tensor in two situations: one with just elements in the main diagonal and the second one with elements in off-diagonal position, resulting in three parameters

$$\begin{aligned}
AR_1^A &= \frac{(A|_{11} * A|_{22})^2}{(A|_{11} * A|_{22})^2 + (A|_{12})^2} \\
AR_2^A &= \frac{(A|_{11} * A|_{33})^2}{(A|_{11} * A|_{33})^2 + (A|_{13})^2} \\
AR_3^A &= \frac{(A|_{22} * A|_{33})^2}{(A|_{22} * A|_{33})^2 + (A|_{23})^2}
\end{aligned}$$

5. RESULTS

The ABC flow is a classical flow due to its chaotic behavior even for laminar flows (Dombre et al., 1986).Arnold (1965), seeking steady inviscid chaotic flow, proposed a Trkalian flow where $\zeta = 1$ or $\mathbf{w} = \mathbf{v}$. The ABC flow in cartesian coordinates is given by

$$\begin{aligned}
u &= A\sin z + C\cos y \\
v &= B\sin x + A\cos z \\
u &= C\sin y + B\cos x
\end{aligned}$$

Figure 1 shows the values of the isotropic normalized ratio that compares linear acceleration deformation, in the sense provided by the covariant convected time derivative, to angular acceleration gradient (in the same sense). Higher values correspond to hyperbolic-like behavior.All the criteria are normalized in order to obtain the same basis for comparison so if a certain region presents values below 0.5, this region remains in a vortical region, according to the criterion.

6. FINAL REMARKS

We have presented a new theoretical analysis to capture directional tendencies of stretching material elements. These directional quantities are able also to delineate coherent structures that are present in turbulent flows as shown in previous papers. The general results are complex in nature and the full interpretation are in order. The new set of vortex identification parameters can make possible to create a connection between 2d and 3d flows as a simplification of the second description. It is also possible to observe the similarities between both set, which is important to certify the robustness of the method.

7. ACKNOWLEDGEMENTS

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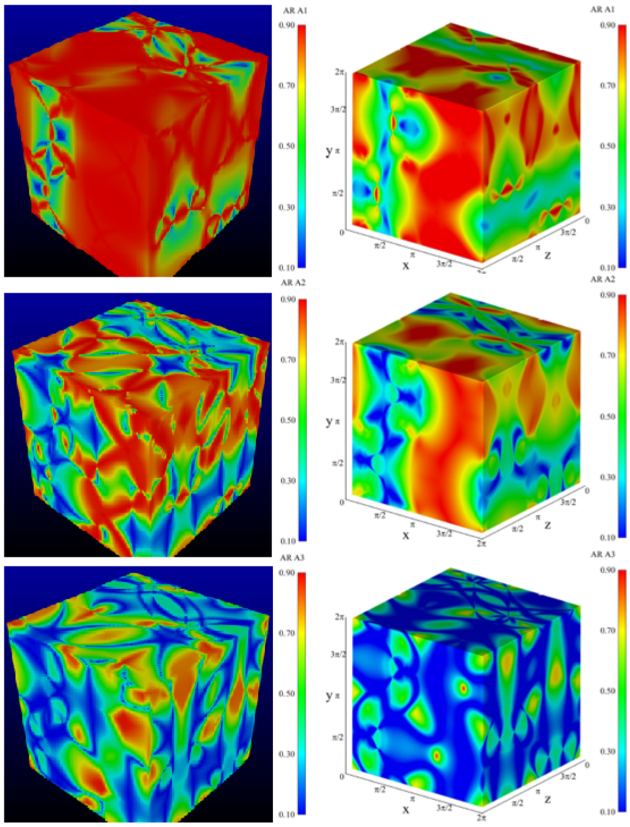

Figure 1. Comparison between the legacy anisotropic parameters (right) and the new set of anisotropic parameters (left).