

## CONSTRUCTAL DESIGN OF A X-SHAPED CAVITY COOLED BY CONVECTION

**Fernanda Bichet Link, feulink@gmail.com**

**Luiz Alberto de Oliveira Rocha, luizrocha@mecanica.ufrgs.br**

Department of Mechanical Engineering, Universidade Federal do Rio Grande do Sul, Rua Sarmento Leite, 425,  
90050-170 – Porto Alegre, RS, Brazil

**Elizaldo Domingues dos Santos, elizaldosantos@furg.br**

**Liércio André Isoldi, liercioisoldi@furg.br**

School of Engineering, Universidade Federal do Rio Grande,  
Cx.P. 474, Rio Grande, RS, 96201-900, Brazil

**Abstract.** *This paper applies Constructal design to study the geometry of a X-shaped cavity that penetrates into a solid conductive wall. The objective is minimizing the dimensionless maximal excess of temperature between the solid body and the cavity. There is uniform heat generation on the solid body. The cavity surfaces are cooled by convection heat transfer while the solid body is subjected to adiabatic conditions on its outer surfaces. The total volume and the cavity volume are fixed, but the lengths and thickness of the X-shaped cavity can vary. The emerged optimal configurations and performance are reported. The effect of the area fraction  $\phi$  which denotes the ratio between the cavity area and the total area of the geometry, and the ratio between the length and thickness of the branch cavity,  $H_1/L_1$ , on the dimensionless maximal excess of temperature is numerically investigated. The results show that the dimensionless maximal excess of temperature  $(\theta_{\max})_m$  decreases approximately 60% when the cavity fraction increases from  $\phi = 0.05$  to 0.25. The results also show that the X-shaped cavity performs approximately 45% better when compared to a C-shaped cavity under the same thermal conditions. The optimal X-shaped cavity is also in accordance with the optimal distribution of imperfections principle.*

**Keywords:** *constructal design, cavities, heat generation*

### 1. INTRODUCTION

Constructal theory has been used to explain deterministically the generation of shape in flow structures of nature (river basins, lungs, atmospheric circulation, animal shapes, vascularized tissues, etc) based on an evolutionary principle of flow access in time (Bejan, 2000; Bejan and Zane, 2012). That principle is the Constructal law: “for a flow system to persist in time (to survive), it must evolve in such way that it provides easier and easier access to the currents that flow through it” (Bejan and Lorente, 2008). This same principle is used to yield new designs for electronics, fuel cells, and tree networks for transport of people, goods and information (Beyene and Peffley, 2009). The applicability of this method/law to the physics of engineered flow systems has been widely discussed in recent literature (Kim et al., 2010; Azad and Amidpour, 2011).

One important subject in engineering is the study of fins arrays. This fact is concerned with its employability in many applications, such as, heat exchangers, microelectronics, cooling of internal combustion engines and electric motors (Kraus, 1997; Bejan and Almogbel, 2000). The study of fins has also been the subject of optimization by means of Constructal Design. Bejan and Almogbel optimized a T-shaped assembly of fins. The objective was to maximize the global thermal conductance subject to total volume and fin material constraints. After that, several configurations of assembly of fins were extensively studied (Xie et al., 2010; Lorenzini and Rocha, 2009; Lorenzini et. Al., 2011 and Biserni et al., 2004).

As cavities can be considered as spacing between adjacent fins, they also play an important role in the heat transfer field. Constructal Design has also been successfully applied to the study of cooling cavities intruded into conducting solids with uniform heat generation. Several shapes, from the elemental C-shaped cavity to the complex T-Y cavity, have been investigated (Rocha et al., 2007; Biserni et al., 2007; Lorenzini and Rocha, 2009 and Rocha et al., 2010).

This paper considers the constructal design of a X-shaped cavity cooled by convection heat transfer. We expect that if the system is free to morph, i. e. its lengths and thicknesses can vary freely the best shapes will emerge according to the Constructal Law.

### 2. MECHANICAL MODELING

Consider the conducting body shown in Fig. 1. The configuration is two-dimensional, with the third dimension ( $W$ ) sufficiently long in comparison with the height  $H$  and the length  $L$  of the volume occupied by the body. There is a X-shaped cavity intruded in the body. The solid is isotropic with the constant thermal conductivity  $k$ . It generates heat uniformly at the volumetric rate  $q'''$  [ $W/m^3$ ]. The outer surfaces of the solid are perfectly insulated. The generated heat current ( $q'''A$ ) is removed by convective heat transfer through the cavities walls. The heat transfer coefficient  $h$  is uniform over all the exposed surfaces.

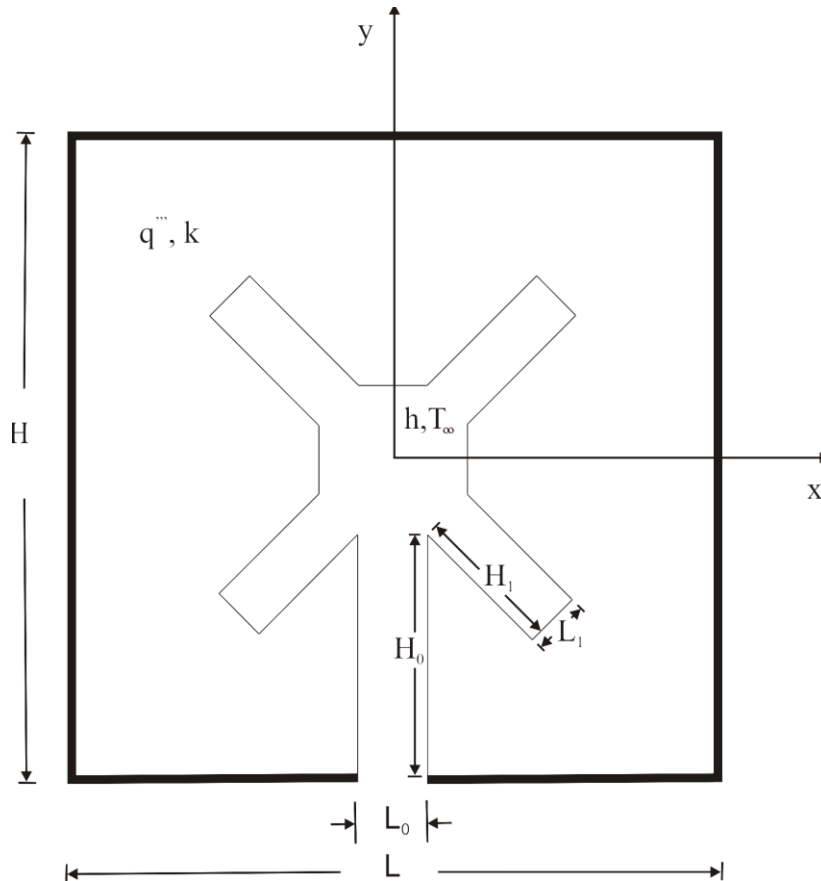


Figure 1. X-shaped Cavity.

The objective of the analysis is to determine the optimal geometry ( $L_0/L_1, H_0/L_0, H_1/L_1$ ) that is characterized by minimizing the maximal excess of temperature  $(T_{\max} - T_{\infty})/(q''' A)$ .

According to constructal design, this optimization can be subjected to two constraints, namely, the total area constraint,

$$A = H L \tag{1}$$

and the cavity area constraint,

$$A_c = 4 L_1 H_1 - L_0 H_0 + L_0 H - 2 L_1^2 \left[ \sin\left(\frac{\pi}{4}\right) \right]^2 + 2 H L_1 \sin\left(\frac{\pi}{4}\right) - 4 L_1 H_0 \sin\left(\frac{\pi}{4}\right) \tag{2}$$

Equations (1) - (2) can be expressed as the cavity fraction

$$\phi_c = \frac{A_c}{A} \tag{3}$$

The analysis that delivers the global thermal resistance as a function of the geometry consists of solving numerically the heat conduction equation along the solid region,

$$\frac{\partial^2 \theta}{\partial \tilde{x}^2} + \frac{\partial^2 \theta}{\partial \tilde{y}^2} + 1 = 0 \tag{4}$$

where the dimensionless variables are

$$\theta = \frac{T - T_{\infty}}{q'' A / k} \quad (5)$$

and

$$\tilde{x}, \tilde{y}, \tilde{H}_0, \tilde{L}_0, \tilde{H}_1, \tilde{L}_1, \tilde{H}, \tilde{L} = \frac{x, y, H_0, L_0, H_1, L_1, H, L}{A^{1/2}} \quad (6)$$

The outer surfaces are insulated and their boundary conditions are

$$\frac{\partial \theta}{\partial \tilde{n}} = 0 \quad (7)$$

The boundary conditions on the cavity surfaces come from balancing the conduction and convection heat transfer, and their dimensionless resulting values are given by

$$-\frac{\partial \theta}{\partial \tilde{n}} = \frac{a^2}{2} \theta \quad (8)$$

The parameter (a) that emerged in eq. (10 - 11) was already used by Bejan et al. in Ref. [13] and it is defined as

$$a = \left( \frac{2hA^{1/2}}{k} \right)^{1/2} \quad (9)$$

The dimensionless form of equations (1) and (3) are

$$1 = \tilde{H}\tilde{L} \quad (10)$$

$$\phi = 4\tilde{L}_1\tilde{H}_1 - \tilde{L}_0\tilde{H}_0 + \tilde{L}_0\tilde{H} - 2\tilde{L}_1^2 \left[ \sin\left(\frac{\pi}{4}\right) \right]^2 + 2\tilde{H}\tilde{L}_1 \sin\left(\frac{\pi}{4}\right) - 4\tilde{L}_1\tilde{H}_0 \sin\left(\frac{\pi}{4}\right) \quad (11)$$

The dimensionless maximal excess of temperature is our objective function and is defined as

$$\theta_{\max} = \frac{T_{\max} - T_{\infty}}{q'' A / k} \quad (12)$$

### 3. OPTIMAL GEOMETRY

The function defined by Eq. (12) can be determined numerically by solving Eq. (4) for the temperature field in each assumed configuration ( $L_0/L_1, H_0/L_0, H_1/L_1$ ) shown in Fig. 1, and calculating to see whether the maximal excess of temperature can be minimized by varying the configuration. In this sense, Eq. (4) was solved using a finite element code, based on triangular elements, developed in MATLAB environment, precisely the PDE (partial differential equations) toolbox (MATLAB, 2000). The appropriate mesh size was determined by successive refinements, increasing four times the number of elements from the current mesh size to the next mesh size until the stopping criterion has been satisfied. The following results were achieved using a range between 20.000 and 50.000 triangular elements. The accuracy of the numerical code has already been demonstrated in several studies (e.g. Lorenzini and Rocha, 2009) and will not be shown here.

Figure 2 shows that there is an optimum ratio  $(H_1/L_1)_o \approx 5.6$  which minimizes the dimensionless maximum excess of temperature when the degrees of freedom ( $H/L, L_1/L_0, H_0/L_0$ ) and the area fraction  $\phi$  are fixed. This optimal value is reached when the X-shaped cavity presents the highest value of  $\tilde{H}$ , i.e.  $\tilde{H} = 1$ . These results were expected because it confirms former observation that the cavity performs better when it penetrates almost completely into the

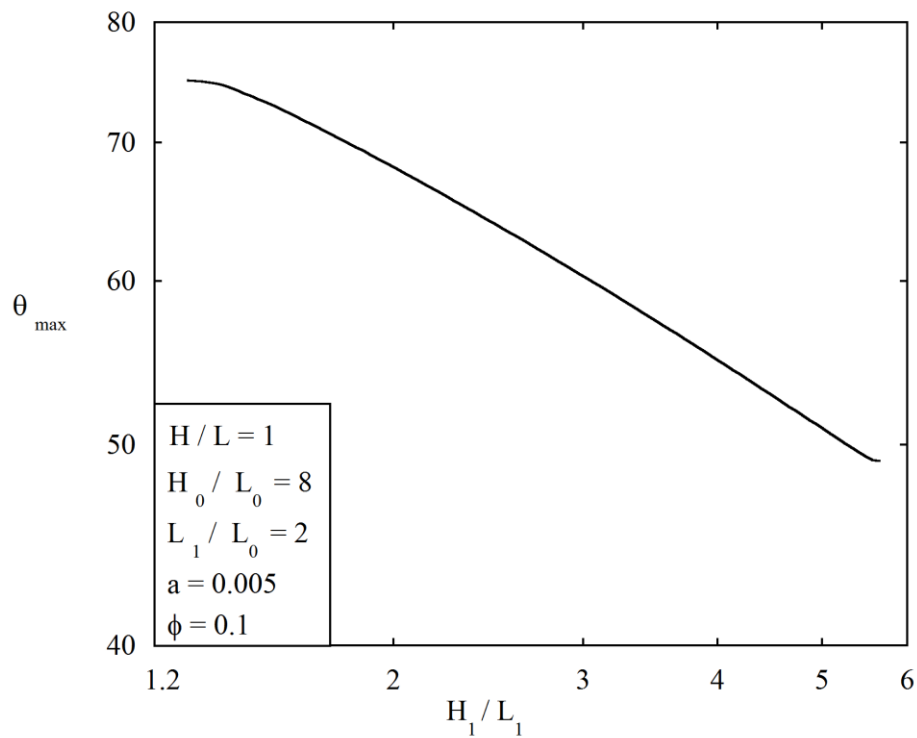


Figure 2. Behavior of the dimensionless maximal excess of temperature as a function of the ratio  $H_1/L_1$ .

solid body. Some of shapes simulated in Fig. 2 are shown in Fig. 3. Figure 3 (c) presents the best shape, i.e. the shape which allows the better distribution of imperfections (the hot spots). Note that the performance of the system improves when the hot spots move from the two superior corners of the solid body in Figs. 3 (a) e 3 (b) to both sides of the solid body in Fig. 3 (c).

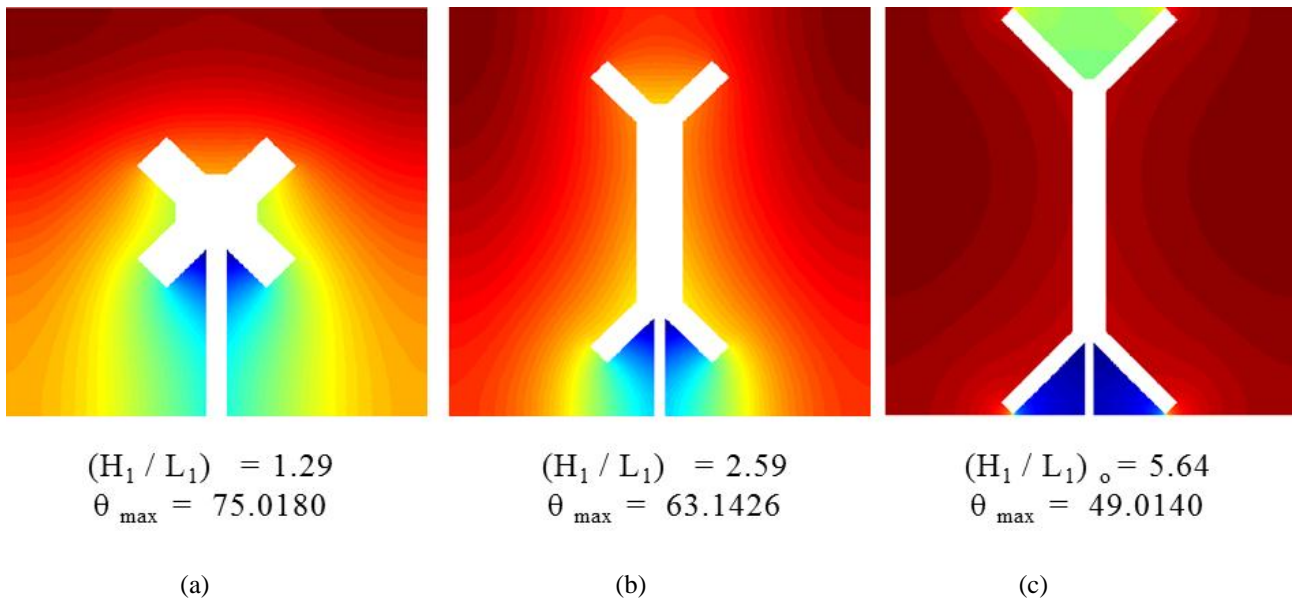


Figure 3. Some of the shapes calculated in Fig. 2 for fixed values of  $H/L = 1$ ,  $H_0/L_0 = 8$ ,  $L_1/L_0 = 2$  and  $\phi = 0.1$ , including the best one.

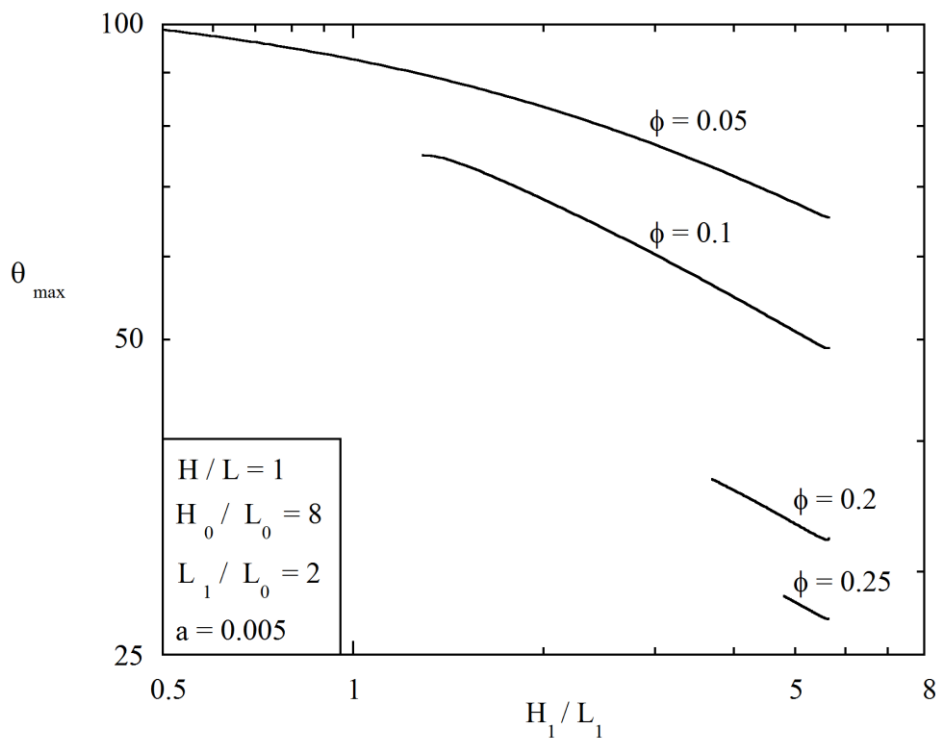


Figure 4. The effect of the cavity fraction,  $\phi$  in the maximal dimensionless excess of temperature as function of and the ratio between the lengths  $H_1/L_1$ .

Simulations performed in Figs. 2 are repeated using several values of the cavity fraction  $\phi$ . Figure 4 shows that there is an optimal ratio  $H_1/L_1$  that minimizes the dimensionless maximal excess of temperature for each studied cavity fraction,  $\phi$ . The minimal dimensionless maximal excess of temperature,  $(\theta_{max})_m$ , and the once optimized ratio  $(H_1/L_1)_o$  calculated in Fig. 4 are summarized in Fig. 5. This figure shows that  $(\theta_{max})_m$  decreases approximately 60% as the cavity fraction increases from  $\phi = 0.05$  to 0.25. In the other side, the optimal ratio  $(H_1/L_1)_o$  is almost insensitivity to changes in the cavity fraction and it can be approximate as  $(H_1/L_1)_o \approx 5.6$ . Some of the best shapes calculated in Fig. 4 are shown in Fig. 6. Again the optimal configuration shown in Fig. 6 (c) is the one where the hot spots are better distributed. It is interesting to notice that the X-shaped cavity performs approximately 45% better when compared to a C-shaped cavity studied by Rocha et al., 2010, when  $\phi = 0.1$  and  $\lambda = 0.005$ .

#### 4. CONCLUSIONS

This paper presents the constructal design of a X-shaped cavity cooled by convection. The results show that there is an optimal shape  $H_1/L_1 = 5.6$  that minimizes dimensionless maximal excess of temperature when the other degrees of freedom are kept constant for several values of the cavity fraction. Important observation is that the best shapes of the cavities are the one that penetrate almost completely into the solid body and distribute better its imperfections. It is also to worth that the dimensionless maximal excess of temperature  $(\theta_{max})_m$  decreases approximately 60% as the cavity fraction increases from  $\phi = 0.05$  to 0.25. The results also show that the X-shaped cavity performs approximately 45% better when compared to a C-shaped cavity under the same thermal conditions. This performance can still be improved by varying the other degrees of freedom:  $L_0/L_1$ ,  $H_0/L_0$ . These issues will be addressed in future works.

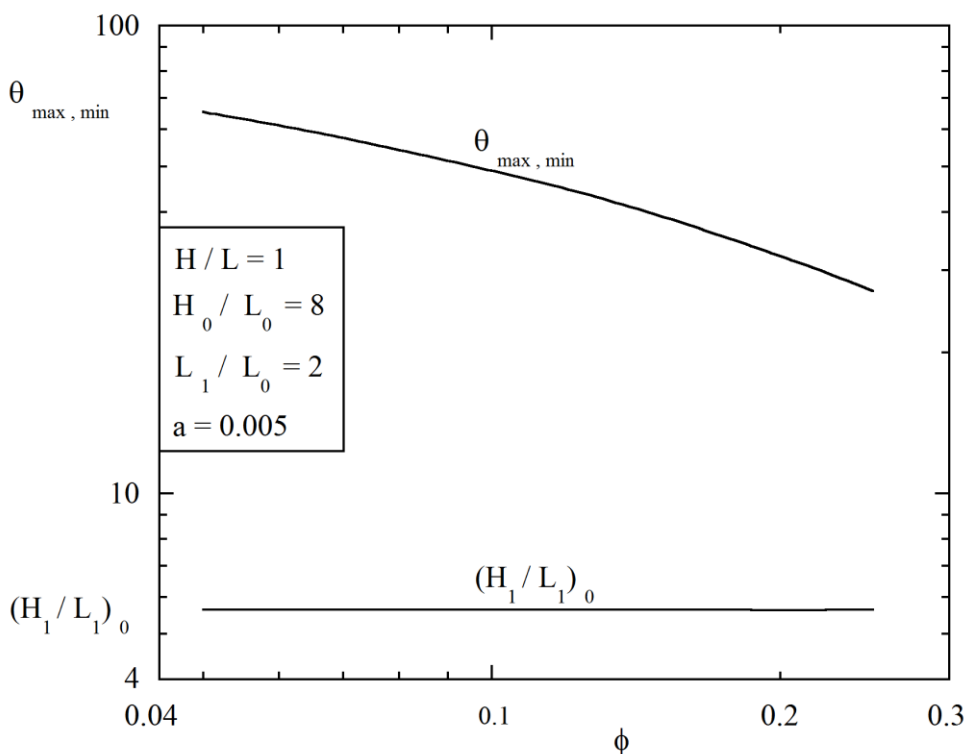


Figure 5. The behavior of the minimal dimensionless excess of temperature  $\theta_{\max,\min}$  and the once optimized ratio between the lengths  $(H_1/L_1)_o$  as function of the ratio between the thicknesses  $\phi$ .

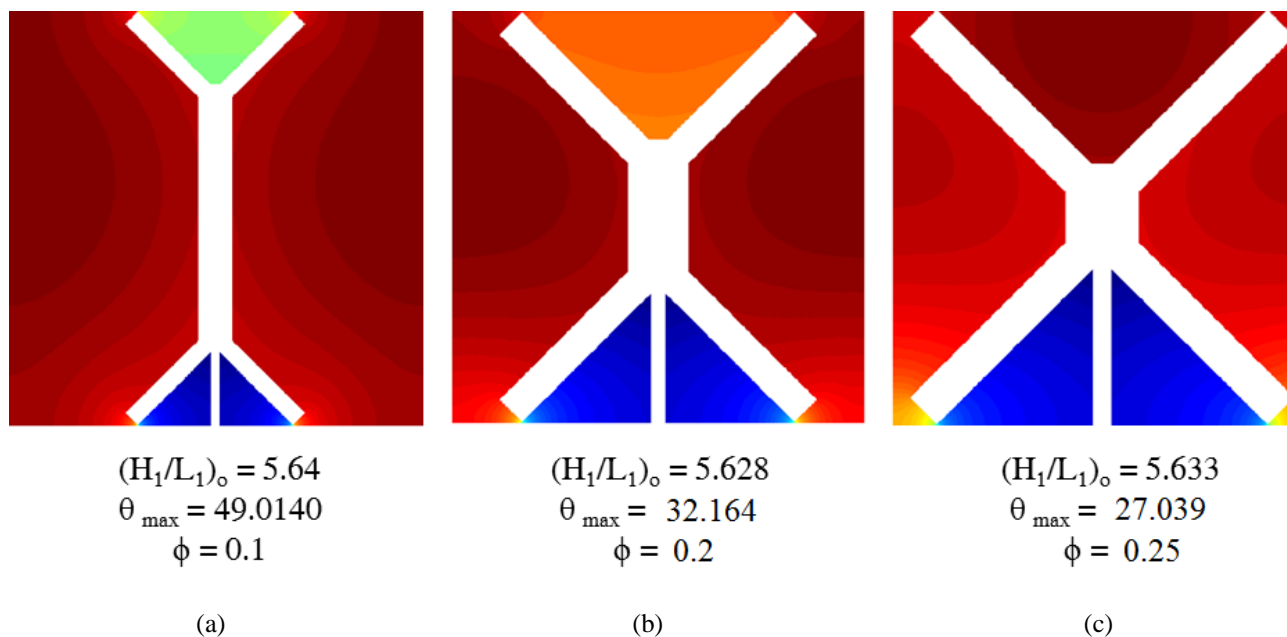


Figure 6. The best shapes calculated in Fig. 5 for fixed values of  $H/L = 1$ ,  $H_0/L_0 = 8$ ,  $L_1/L_0 = 2$ .

## 6. ACKNOWLEDGEMENTS

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