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CONSTRUCTAL OPTIMISATION OF CONJUGATE Y-SHAPED COOLING CHANNELS WITH INTERNAL HEAT GENERATION

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Abstract*. This paper presents the development of the three-dimensional flow architecture of conjugate cooling channels in forced convection with internal heat generation within the solid for a circular stem intrusion that braches into two elemental intrusions to form a Y-shaped channel configuration. The main objective is to optimize configuration in such a way that the peak temperature was minimized subject to the constraint of fixed global volume of solid material. The cooling fluid is driven through the channels by the pressure difference across the channel. The structure has hydraulic diameter and angle between the two tributary branches of the y-shaped configuration as degrees of freedom as design variables. The Y- shape of the channel is allowed to morph to determine the best configuration that gives the lowest thermal resistance. A gradient-based optimization algorithm is applied in order to search for the best optimal geometric configurations that improve thermal performance by minimizing thermal resistance for a wide range of dimensionless pressure difference. The effect of angle between the two tributary branches, applied pressure difference and heat generation rate on the optimal hydraulic diameter is reported. There are unique optimal design variables for a given pressure difference.Results obtained show that the effects of dimensionless pressure drop on minimum thermal resistance are consistent with those obtained in the open literature.*

Keywords: *Optimisation, Peak temperature, Constructal theory, Dynamic-Q, Y-shaped*

1. NOMENCLATURE

A_c	Cross sectional area of the channel, m^2	v_{el}	Elemental volume, m^3
A_s	Cross sectional area of the structure, m^2	W	Structure width, m
Be	Bejan number	w	Elemental width, m
C_P	Specific heat at constant pressure, J/kg K	x, y, z	Cartesian coordinates, m
d_h	Hydraulic diameter, m	**Greek symbols**	
H	Structure height, m	α	Thermal diffusivity, m^2/s
h	Elemental height , m	μ	Viscosity, kg/m.s
i	Mesh iteration index	ν	Kinematics viscosity, m^2/s , α is the angle between two tributary branch channels
k	Thermal conductivity, W/mK	ρ	Density, kg/m^3
L	Axial length, m	∂	Differential
N	Number of channels	∞	Far extreme end, free stream
n	Normal	ϕ	Porosity
P	Pressure, Pa	Δ	Difference
q_s'''	Internal heat generation density, W/m^3	∇	Differential operator
R	Thermal resistance	γ	Convergence criterion
T	Temperature, ^{0}C	**Subscripts**	
$\vec{u}$	Velocity vector, m/s	*opt*	Optimum
V	Global structure volume, m^3	*in*	Inlet

v_c	Channel volume, m^3	*max*	Maximum, peak
min	Minimum	*w*	Wall
s	Solid		

2. INTRODUCTION

Constructal theory and design (Bejan 1997; 2000), have emerged as an evolutionary design philosophy for developing flow architectures that offer greater flow access and system performance. This law is stated by Bejan (Bejan 1997; 2000), as: *For a finite-size system to persist in time (to live), it must evolve in such a way that it provides easier access to the imposed (global) currents that flow through it.*

The application of this theory started with Bejan and Sciubba (1992), who obtained a dimensionless pressure difference number for optimal spacing of board to board of an array of parallel plate to channel length ratio and a maximum heat transfer density that can be fitted in a fixed volume in an electronic cooling application using the method of intersection asymptotes. This philosophy has been applied to all the facets of flow system design, from biology and physics, to engineering and social organisation (Bejan, 2003, 2006, 2009, Bejan et al 2000; Weinerth 2010, Miguel, 2006, Nakayama et al, 2009, Reis and Gama, 2010, Reis et 2004, Wang et al. 2007 , Bello-Ochende and Bejan, 2006; Charles, and Bejan 2009).

In nature, water always takes the path of least flow resistance in the course of navigation in the river basin (Bejan and Lorente, 2008) . Thermodynamically, every system exhibits a level of imperfection due to entropy generation and leads to the degradation of performance of the system (Bello-Ochende and Bejan, 2003),. However, a system must adjust itself to operate maximally by optimizing the process and geometric configuration of the system to reduce the entropy.

In medicine (Wang et al. 2007) , this physical law can also be applied to the treatment of cancer; The spreading of cancer can be controlled by maintaining the temperature field of the unaffected tissues in the neighbourhood of the turmoil below the temperature that the cancer virus can survive. In the business world (Bejan et al., 2000) constructal theory shows that the transportation cost can also be minimized by optimizing the transportation routes of goods and products from one area to another in a dendritic form in order to get shortest and easier distance.

In heat transfer (Bejan, 2003), the peak temperature must be minimized at every hot spot of a system for better thermal performance and to avoid thermal stress by optimizing shape and geometry. In academia (Bejan, 2009) , the constructal law was used to optimize the hierarchal rankings of universities in the global flow of knowledge. Also in military defence, the constructal law was used to provide insight information on the optimization of warfare tactics and strategy (Weinerth, 2010)

Numerous works on the application of constructal theory to the design of shape and structure for heat transfer and fluid flow has been outlined by (Bello-Ochende et al., 2009; Muzychka 2005; Rocha, et al. 2005; Kim et al. 2007; Salimpour et al., 2010; Olakoyejo et al., 2012a.). The advantage of constructal law in the engineering field is that the flow architecture is not assumed in advance, but is the consequence of allowing the structure to morph. The applications of this theory have been reviewed most recently by Bejan and Lorente (2008), in which under certain global constraints, the best architecture of a flow system can be achieved as the one that gives less global flow resistances, or allows high global flow access. In other words, the shapes of the channels and unit structure that is subject to global constraint are allowed to morph. The development of heat exchangers and multiscale architectural by constructal theory was also, reviewed by Fan and Luo (2008).

Yilmaz *et al.* (2000) studied the optimum shape and dimensions for convective heat transfer of laminar flow at constant wall temperatures for ducts with parallel plate, circular, square and equilateral triangle geometries. Approximate equations were derived in the form of maximum dimensionless heat flux and optimum dimensionless hydraulic diameter in terms of the duct shape factors and the Prandlt number (*Pr*).

Da Silva et al. (2004), optimized the space allocation on a wall occupied by discrete heat sources with a given heat generation rate by forced convection using the method of constructal theory in order to minimize the temperature of the hot spot on the wall.

Also, Bello-Ochende *et al*. (2007) conducted a three-dimensional optimization of heat sinks and cooling channels with heat flux using scale analysis and the intersection of asymptotes method based on constructal theory to investigate and predict the design and optimization of the geometric configurations of the cooling channels. The theory was also applied to optimise the geometry of C- and H-shaped cavities respectively that intrude into a solid conducting wall in order to minimise the thermal resistance between the solid and the cavities (Rocha *et al*., 2010; Biserni et al., 2007). Muzychka (2007) studied and analyzed the optimization of micro tube heat sinks and heat exchangers for maximum thermal heat transfer by using a multiscale design approach. In his analysis, he was able to show that through the use of interstitial micro tubes, the maximum heat transfer rate density for an array of circular tubes increased.

This paper focuses on the study of three-dimensional, laminar forced convection cooling of solid structures. It examines the optimization of a fixed and finite global volume of solid materials with Y-shaped cooling channels, which experience a uniform internal heat generation. The objective is the building of a smaller construct to form a larger construct body that will lead to the minimization of the global thermal resistance or, inversely, the maximization of the heat transfer rate density (the total heat transfer rate per unit volume). The optimization process is carried out numerically under total fixed volume and manufacturing constraints.

3. MODEL

Figure 1a is the physical configuration of a solid body. The system consists of Y-shaped cooling channel in a solid structure of fixed global volume *V*. The solid body is generating an internal heat q_s'''. The body is cooled by forcing cooling fluid (water) from the left side into the Y-shaped cooling channels by a specified pressure difference ΔP. The fluid is assumed to be in single phase, steady and Newtonian with constant properties. An elemental volume shown in Fig. 1b consisting of a Y-shaped cooling channel and the surrounding solid in which the heat is generated was used for the analysis. The heat transfer in the elemental volume is a conjugate problem that combines heat conduction in the solid and the convection in the channels.

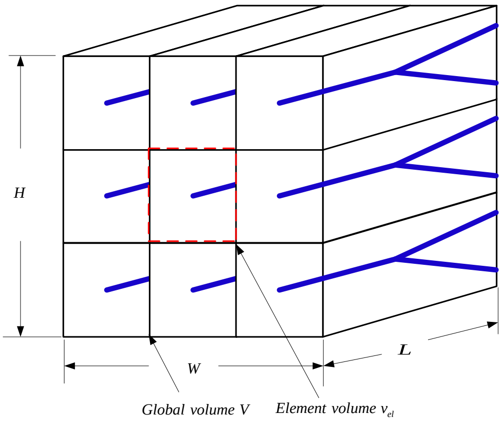


Figure 1a. Three-dimensional Y-shaped channels across a solid body with internal heat generation

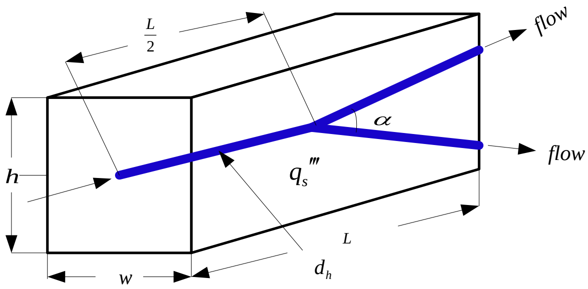


Figure 1b. The three dimensional computational domain of elemental volume with Y-shaped cooling channel

In Fig. 1b, an elemental volume constraint is considered and it is composed of an elemental Y-shaped cooling channel of hydraulic diameter d_h with the surrounding solid defined as:

$$v_{el} = whL, \qquad w = h \tag{1}$$

The volume of the elemental Y-shape channel is:

$$v_c = \frac{\pi}{4} d_h^2 \left(\frac{L}{2} + \frac{L}{\cos(\alpha/2)} \right) \tag{2}$$

where, α is the angle between two tributary branch channels. For a fixed length of the channel, we have, global volume as:

$$A_s = HW \tag{3}$$

However, porosity or void fraction of the unit structure is defined as:

$$\phi = \frac{v_c}{v_{el}} \tag{4}$$

The objective is to find the channel hydraulic diameter, d_h, which offer minimum resistance to heat and flow. The analysis also focuses on the extreme limits of $0 \le d_h \le \infty$ and $0 \le w \le \infty$.

The temperature distribution in the model was determined by solving the equation for the conservation of mass, momentum and energy. The fluid is assumed to be a single phase, steady state and Newtonian with constant thermo physical properties. The governing equations used for fluid flow and heat transfer analysis in the elemental volume of the structure are:

$$\nabla \cdot \vec{u} = 0 \tag{5}$$

$$\rho(\vec{u} \cdot \nabla \vec{u}) = -\nabla p + \mu \nabla^2 \vec{u} \tag{6}$$

$$\rho_f C_{Pf} (\vec{u} \cdot \nabla T) = k_f \nabla^2 T \tag{7}$$

The energy equation for a solid with internal heat generation is given as:

$$k_s \nabla^2 T + q_s''' = 0 \tag{8}$$

The continuity of the heat flux at the interface between the solid and the liquid is given as:

$$k_s \frac{\partial T}{\partial n}\bigg| = k_f \frac{\partial T}{\partial n}\bigg| \tag{9}$$

A no slip boundary condition is specified at the wall of the channel, $\vec{u}=0$, at the inlet ($x = 0$), $u_x = u_y = 0$, $T = T_{in}$ and

$$P = \frac{Be\alpha u}{L^2} + P_{out} \tag{10}$$

where *Be,* is the dimensionless pressure difference defined in Refs. (Bhattacharjee and Grosshandler 1998; Petrescu, 1994).
at the outlet ($x = L$), zero normal stress, $P_{out} = 1\ atm$
at the solid boundaries,

$$\nabla T = 0 \tag{11}$$

The minimum global thermal resistance could be expressed in a dimensionless form as:

$$R_{\min} = \frac{k_f \left(T_{\max} - T_{in}\right)_{\min}}{q_s''' L^2} \tag{12}$$

And it is a function of the optimized design variables and the peak temperature.

$$R_{\min} = f\left(d_h, \phi, \alpha, \left(T_{\max}\right)_{\min}\right) \tag{13}$$

The inverse of R is the global thermal conductance.

4. NUMERICAL PROCEDURE AND GRID ANALYSIS

The continuity, momentum and energy Eqs. (5) - (8) along with the boundary conditions (9) - (11) were solved by using a three-dimensional commercial package FLUENT™ (2001), which employs a finite volume method. The details of the method were explained by Patankar (1980). The FLUENT™(2001) was coupled with geometry and mesh generation package GAMBIT (2001) using MATLAB (2008) to allow the automation and running of the simulation process. After the simulation had converged, an output file was obtained containing all the necessary simulation data and results for the post-processing and analysis. The computational domain was discretized using hexahedral/wedge elements. A second-order upwind scheme was used to discretize the combined convection and diffusion terms in the momentum and energy equations. The SIMPLE algorithm was then employed to solve the coupled pressure-velocity fields of the transport equations. The solution is assumed to have converged when the normalized residuals of the mass and momentum equations fall below 10^{-6} and while the residual convergence of energy equation was set to less than 10^{-10}. The number of grid cells used for the simulations varied for different elemental volume and porosities. However, grid independence tests for several mesh refinements were carried out to ensure the accuracy of the numerical results. The convergence criterion for the overall thermal resistance as the quantity monitored is:

$$\gamma = \frac{\left|\left(T_{\max}\right)_i - \left(T_{\max}\right)_{i-1}\right|}{\left|\left(T_{\max}\right)_i\right|} \le 0.01 \tag{14}$$

where *i* is the mesh iteration index. The mesh is more refined as i increases. The $i-1$ mesh is selected as a converged mesh when the criterion (14) is satisfied.

5. NUMERICAL RESULTS

The simulation work began by fixing the length of the channel, applied pressure difference, porosity, angle α between two tributary branches, internal heat generation and material properties and we kept varying the values of the elemental volume and hydraulic diameter of the channel in order to identify the best (optimal) internal configuration that minimized the peak temperature. . The results was presented for the case where $0.01 \le w/L \le 0.095$ $Ø = 0.05$, $\alpha = 12^{\circ}$ and $\alpha = 15^{\circ}$. for a fixed structural length *L* and fixed applied pressure differences of ΔP=50 *kPa*. The thermal conductivity of the solid structure (silicon) is 148 W/m.K, and the internal heat generation within the solid was taken to be fixed at 100 W/cm^3. The thermo physical properties of water used in this study were based on water at 300 *K* and the inlet water temperature was fixed at this temperature.

Fig. 2 shows the existence of an optimum hydraulic diameter of the cooling channel where the peak temperature is minimized at any point in the channel. The channel hydraulic diameter has a significant effect on the peak temperature and the overall thermal resistance. It shows that there exists an optimal channel hydraulic diameter which lies in the range $0.005 \le d_h/L \le 0.015$ that minimized the peak temperature. These indicate that peak temperature decreases as d_h /L decreases and that a minimum value is reached beyond which the peak temperature begins to increase. Therefore, the global thermal resistance decreases as the hydraulic diameter increases. Any values of the hydraulic diameter outside the optimal point that is above or below the optimal ranges will cause the working fluid not to be properly engaged with the cooling process and hence increases in the values of the peak temperature.

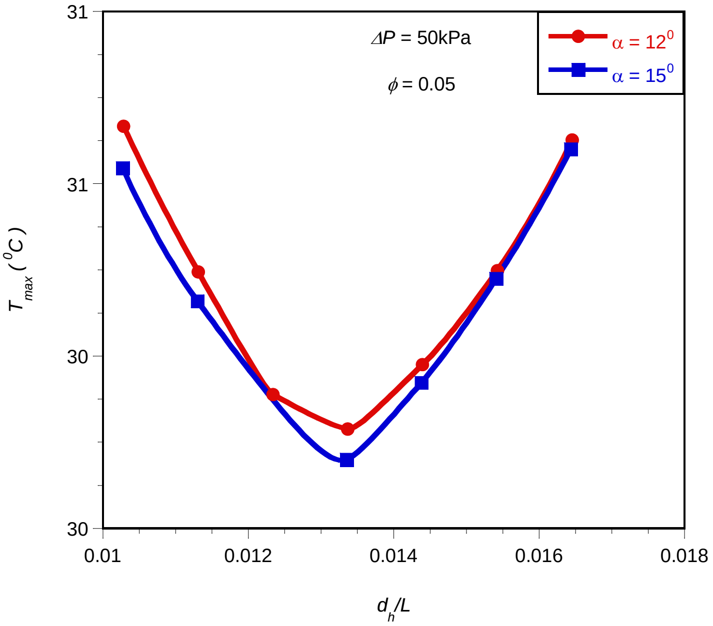


Figure 2. Effect of optimised hydraulic diameter d_h , on the peak temperature

6. MATHEMATICAL OPTIMISATION

An optimization algorithm that will search and identify the optimal design variables at which the system will perform at an optimum was coupled to computational fluid dynamic code. A numerical algorithm, Dynamic-Q **(**Snyman, and Hay 2002) was employed and incorporated into the finite volume solver and grid (geometry and mesh) generation package by using MATLAB. for more efficient and better accuracy in determining the optimal performance.

The Dynamic-Q is a multidimensional and robust gradient-based optimisation algorithm, which does not require an explicit line search. The technique involves the application of a dynamic trajectory LFOPC optimization algorithm to successive quadratic approximations of the actual problem **(**Snyman, 2005). The algorithm is also specifically designed to handle constrained problems where the objective and constraint functions are expensive to evaluate. The details of the Dynamic-Q and applications can be found in Refs **(**Snyman, and Hay 2002, Bello-Ochende et al, 2010; Olakoyejo et al., 2012a).

7. OPTIMISATION PROBLEM

The design variables constraint ranges for the optimisation are:

$$0.05 \le \phi \le 0.075, \qquad 12^0 \le \alpha \le 17^0, \qquad 0 \le w \le L, \quad 0 \le d_h \le w \tag{15}$$

The design and optimization technique involves the search for and identification of the best channel layout that minimizes the peak temperature, T_{max} such that the minimum thermal resistance between the fixed volume and the cooling fluid is obtained with the desired objectives function. Hydraulic diameter d_h and angle α between the two tributary branches are considered as design variables. The length of the channel was fixed. However, the elemental volume and the internal architecture of the cooling channel are allowed to morph (change). A number of numerical optimization and calculations were carried out within the design constraint ranges given in Eq. (15). The optimisation process was repeated for applied dimensionless pressure differences (*Be*) that correspond to ΔP = 10 *kPa* to ΔP = 50 *kPa* in order to show the optimal behaviour of the entire system

Figure 3a shows the minimized dimensionless global thermal resistance as a function of dimensionless pressure difference at optimized design variables for the configuration. The minimized dimensionless global thermal resistance decreases as the dimensionless pressure difference increases.

Figure 3b shows that the optimized hydraulic diameter decreases as the applied dimensionless pressure difference, porosity and angle between the tributary increase. We can say that there exists a unique optimal geometry for each of the applied pressure differences. The trends of these results are also in agreement with previous work **(**Bello-Ochende et al, 2010; Olakoyejo et al, 2012b).

Figures 4a and 4b show the temperature contours of the elemental volume and of the inner wall of the cooling channel with cooling fluid, respectively. The blue region indicates the region of low temperature and the red region indicates that of high temperature. The arrow indicates the direction of flow.

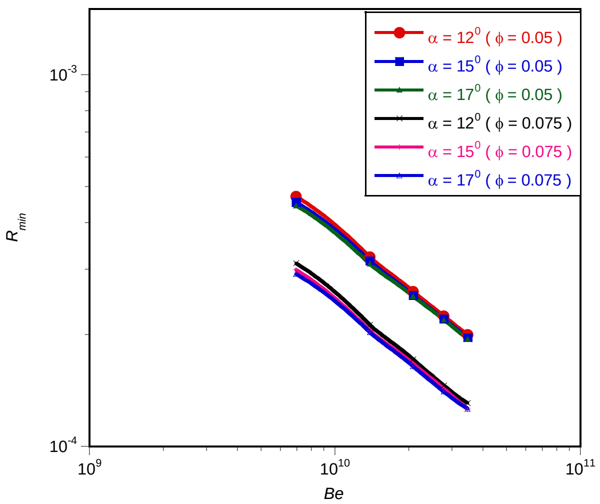


Figure 3a. Effect of dimensionless pressure difference on the minimised dimensionless global thermal resistance

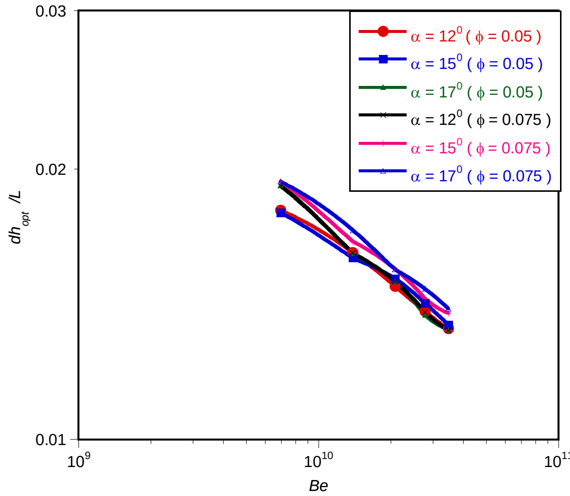
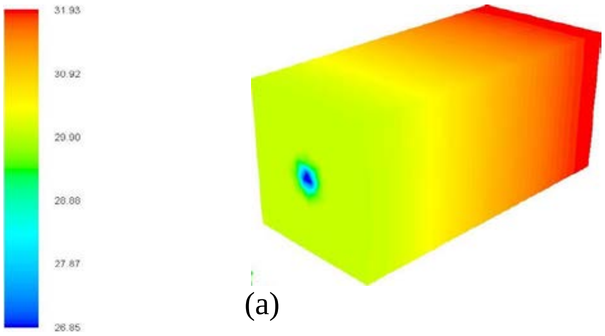
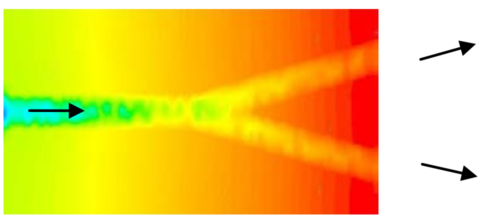


Figure 3b. Effect of dimensionless pressure difference on the optimised hydraulic diameter



(a)



(b)

Figure 4. Temperature distributions (a) on the unit structure and (b) on the cooling fluid and inner wall.

8. CONCLUSION

This paper studied the numerical optimization three-dimensional flow architecture of conjugate cooling channels in forced convection with internal heat generation within the solid for a circular stem intrusion that braches into two elemental intrusions to form a Y-shaped channel configuration based on constructal theory. The effects of different geometrical parameters such as the hydraulic diameters, porosity and and angle between the two tributary branches of the Y-shaped configuration were comprehensively studied. The numerical results obtained showed that there is an optimal geometry for the Y-shape channel configuration considered which minimizes the peak temperature and hence thermal resistance.

Also, the numerical analysis showed that the optimised geometry and minimised thermal resistance are function of the dimensionless pressure difference for different porosites. This shows the existence of unique optimal design variables (hydraulic diameters) for a given applied dimensionless pressure number for different porosity and angle between the two tributary branches of the Y-shaped configuration. The results also show that the minimized peak temperature decreases as the porosity and angle between the two tributary branches increase.

9. ACKNOWLEDGEMENTS

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