Proceedings of ENCIT 2012

14th Brazilian Congress of Thermal Sciences and Engineering
November 18-22, 2012, Rio de Janeiro, RJ, Brazil

THE HILBERT-HUANG TRANSFORM: AN APPLICATION ON THE SHEDDING PROCESS

R. S. S. Horszczaruk, rodrigo.santiago@ufrgs.br

S. V. Möller, svmoller@ufrgs.br

Programa de Pós-Graduação em Engenharia Mecânica – PROMEC
Universidade Federal do Rio Grande do Sul – UFRGS
Rua Sarmento Leite, 425
90050-170 Porto Alegre, RS, Brasil

Abstract. *The use of mathematical tool for the treatment of data signal is very common, especially in turbulence flows. Because of the nature of the turbulence, the demand for tools to analyze non-stationary and non-linear signals becomes valid in the characterization of flow around bluff bodies. This kind of study leads to analysis of shedding process around these bodies, which reveal parameters such as vortex shedding frequency. This paper propose a new tool, associated to other mathematical tools currently used, called Hilbert-Huang Transform which will be applied to the analysis of turbulent flow around a single cylinder. The main features of the Hilbert-Huang transform and its advantages as a tool for analysis of turbulent flows are presented. A brief discussion and comparison with others methods used is made.*

Keywords*: Hilbert-Huang transform, Empirical Mode Decomposition, turbulent flow, shedding process, Fast Fourier transform.*

1. NOMENCLATURE

$c_n(t)$	n-component of IMF
D	diameter – m
f	frequency – Hz
$\bar{f}(t)$	mean frequency of the interval – Hz
$E(t)$	energy in time domain
$E(\omega)$	energy in frequency domain
F_S	sampling frequency – Hz
$H[c_i(t)]$	Hilbert transform of i-component of IMF
$H(\omega,t)$	Hilbert spectrum
$h(\omega)$	Marginal spectrum
$P_{xx}(f)$	Fourier spectrum
$P_{xx}(a,b)$	continuous wavelets spectrum
$P_{xx}(\omega,t)$	Hilbert power spectral density
Re	Reynolds number ($U\,D/\nu$)
RP	real part of complex number
$r_n(t)$	n-residue of IMF
S	Strouhal number ($f\,D/U$)
t	time – s
U	velocity – m/s
$x(t)$	generic function in time domain
$x(f)$	generic function in Fourier domain
$\tilde{X}(a,b)$	generic function in wavelet domain (continuous)
$\Phi_i(t)$	instantaneous phase – rad
$\omega_i(t)$	instantaneous frequency – rad/s

1. INTRODUCTION

The use of mathematical tools for signal analysis is very common in many fields of engineering, in particular with regard to turbulent flow around bluff bodies in order to investigate the shedding process around these bodies. One of the most important parameters in the analysis of flow in a circular cylinder is the dimensionless frequency associated with vortex shedding, which is the Strouhal number, which depends on the shedding frequency, the cylinder diameter and the velocity of the free stream (Blevins, 1990).

In experimental turbulence study, the RMS-values, auto spectral density functions, as well as auto and cross-correlation functions, are used, considering turbulence as a random phenomenon. Currently, this process is studied from the Fourier spectral analysis for stationary flows, and also by the wavelet analysis for non-stationary flows. Usually, random data are discrete time series.

This work suggests the use of a tool that enables the analysis of non-stationary and non-linear turbulent signals named Hilbert-Huang Transform (HHT) (Huang et al, 1998), which is composed by Empirical Mode Decomposition (EMD), which is a direct and intuitive method, for the decomposition of the signal, and the Hilbert Spectral Analysis (HSA).

The decomposition is based on the simple assumption that all the data consist of different simple intrinsic modes of oscillations (Huang and Shen, 2005), each of these modes of oscillation is called Intrinsic Mode Functions (IMF) and are obtained from the envelopment of the maxima and minima values and the average signal from this envelopment.

Repeating this procedure iteratively, the decomposition of the original signal is obtained. The Hilbert transform is used to obtain a baseband spectral distribution of signal energy in the time frequency domain. This spectral analysis is made possible by applying the Hilbert transform to each of the intrinsic mode functions of the original signal. This process is called Hilbert Spectral Analysis (HSA). In the literature, an interesting comparison between the wavelet and Hilbert-Huang transforms applied to signals from earthquakes is made (Shi and Luo, 2003). The authors show that the Hilbert-Huang transform performs a direct decomposition of the original signal and can show more clearly its intrinsic properties.

2. EXPERIMENTAL TECHNIQUE

The measurement of velocity and velocity fluctuations in the wake of were made of circular cylinders with two different diameters with hot wire anemometry technique in an aerodynamic channel is performed to investigate the shedding process.

The test section, shown on Fig. 1, with 146 mm height and width of 193 mm. Air, at room temperature, is the working fluid, driven by a centrifugal fan of 0,75 kW, passed by a diffuser and a set of honeycombs and screens, which reduce the turbulence intensity in the channel about 1%. A frequency inverter controls the fan speed, where the flow velocity in the aerodynamic channel is controlled from 0 to 15 m/s. To measure the velocity reference is used a Pitot tube fixed before to the test section. The cylinders in use have a diameter of 9.5 and 32 mm and are rigidly mounted in vertical position inside the channel. The cylinders blockage ratio is 4,92%, for cylinder diameter 9.5 and 16.5%, for cylinder diameter 32 mm and the incidence angle of the flow on the cylinder is 90°. The experiment was performed with a Reynolds number Re = 8.91 x 10^4 for cylinder diameter 9.5 mm and Re = 2.9941 x 10^4 cylinder of diameter 32 mm.

For the measures of velocity fluctuations, a constant temperature hot wire anemometry is used by means of DANTEC *StreamLine* and single straight probes. One probe was positioned at the wake, XXX mm downstream of the cylinder diameter 9.5 mm and 20 mm downstream of the cylinder diameter 32 mm.

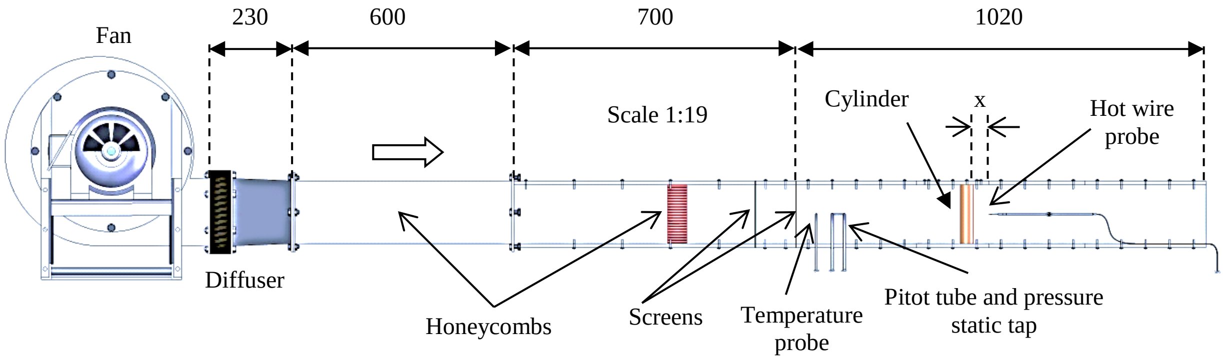


Figure 1. Schematic view of aerodynamic channel (measures in mm)

Data acquisition was performed with a 16-bit A/D board (NATIONAL INSTRUMENTS 9215-A) with USB interface, with a sampling frequency of 3000 Hz and a low pass filter at 1000 Hz.

With the intention to compare Fourier, Wavelet and HHT, the data set was acquired in steady state flows at same velocity value.

Computations of the Fourier transform, wavelet transform and HHT were performed using the Matlab © software.

The experimental data were analyzed by statistical, spectral, wavelets tools and HHT results in time-frequency domain, which allows the detection of non-permanent and non-linear flow structures.

3. MATHEMATICAL TOOLS

3.1 Fourier and Wavelet Transforms

The statistical (or time domain) analysis consists on determining the first four moments of the probability density function: mean (average), standard deviation, skewness and kurtosis. The spectral (or frequency domain) analysis can be done through the power spectral density function (PSD). The joint time-frequency domain analysis was made trough wavelet transform. The wavelet analysis can be applied to time varying signals, where the stationary hypothesis cannot be maintained, to allow the detection of non permanent flow structures.

The Fourier transform of a discrete time series gives the energy distribution of the signal in the frequency domain evaluated over the entire time interval. The Fourier spectrum is defined as

$$P_{xx}(f) = |\hat{x}(f)|^2 \tag{1}$$

The first attempt to deal with nonstationary processes was the Windowed Fourier Transform. However, due to the aliasing of high and low frequency components that do not fall within the frequency range of the window, the windowed Fourier transform is inaccurate for time-frequency location of transient features.

While the Fourier transform uses trigonometric functions as basis, the bases of wavelet transforms are functions named wavelets, with finite energy and zero average that generates a set of wavelet basis.

The continuous wavelet transform of a function x(t) is given by:

$$\tilde{X}(a,b) = \int_{-\infty}^{\infty} x(t)\psi_{a,b}(t)dt \tag{2}$$

where ψ is the wavelet function and the parameters a and b are respectively scale and position coefficients ($a,b \in \Re$) and $a > 0$.

The respective wavelet spectrum is defined as:

$$P_{xx}(a,b) = \left|\tilde{X}(a,b)\right|^2 \tag{3}$$

In the wavelet spectrum, Equation (2), the energy is related to each time and scale (or frequency) (Daubechies, 1992). This characteristic allows the representation of the distribution of the energy of the signal over time and frequency domains, called spectrogram.

The velocity signals were analyzed using wavelet transforms to obtain the energy distribution of the turbulent flow over time-frequency domain. The continuous wavelet spectrum was obtained through continuous wavelet transform. The discrete wavelet transform was used to decompose the measured signal in wavelet approximations divided in frequency bands (Indrusiak et al., 2005).

In this work, Daubechies “db20” functions were used as bases of discrete wavelet transforms.

3.2 Hilbert-Huang Transform

The Hilbert-Huang Transform (HHT) is applied in the treatment of non-linear and non-stationary signals and it is made up of Huang Transform and Hilbert spectral analysis. The HHT is ruled by the empirical mode decomposition (EMD), known as the Huang Transform. The EMD assumes that any data set consists of different, simple, intrinsic modes of oscillation that need not be sinusoidal. Based on this, each mode of oscillation from high frequency to low frequency is derived in an objective manner from the recorded complex data. We call each of these oscillatory modes an intrinsic mode function (IMF). As discussed by Huang et al., 1998, the EMD method is necessary to deal with data from non-stationary and non-linear processes. This new method is intuitive, direct, and adaptive, with an a *posteriori*-defined basis, from the decomposition method, based on and derived from the data. The decomposition is made from the identification of all the local maxima. Connect all the local maxima by a cubic spline to produce the upper envelop of data, i.e., $x(t)$, and repeat the procedures for the local minima to produce the lower envelop of $x(t)$. All the data should be encompassed by the upper and lower envelopes. Their mean of these envelopes is designated by $m_1(t)$, and the difference between the data $x(t)$ and $m_1(t)$ provide us the first component $h_1(t)$, i.e.,

$$h_1(t) = x(t) - m_1(t) \tag{4}$$

Ideally, $h_1(t)$ should be an IMF, but all the conditions of an IMF should be achieved; the conditions are [Huang and Shen, 2005]:

(1) In the whole dataset, the number of maxima and minima as well as the number of zero-crossings must either equal or differ at most by one, and
(2) At any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero.

Not satisfied the condition of an IMF, the process is repeated. In the subsequent sifting process, $h_1(t)$ is treated as the data, then

$$h_{11}(t) = h_1(t) - m_{11}(t) \tag{5}$$

where $m_{11}(t)$ is the mean of the upper and lower envelopes of $h_1(t)$. Repeating k times until $h_{1k}(t)$ to satisfy the conditions of an IMF, we have

$$h_{1k}(t) = h_{1(k-1)}(t) - m_{1(k-1)}(t). \quad (6)$$

It is named the first IMF component $c_1(t)$ from the data, because $c_1(t) = h_{1k}(t)$. The component $c_1(t)$ will contain the finest-scale of the highest frequency component of the signal. The residue $r_1(t)$, given by

$$r_1(t) = x(t) - c_1(t) \quad (7)$$

contains longer-period components, is treated as new data and subjected to the same sifting process as described above. This procedure can be repeated to obtain all the subsequent $r_j(t)$'s

$$r_j(t) = r_{j-1}(t) - c_j(t) \quad (8)$$

The sifting process can be ended on any of the following predetermined criteria: a) either the component $c_n(t)$ or the residue $r_n(t)$ becomes so small that it is less than a predetermined value of consequence, or b) the residue $r_n(t)$ becomes a monotonic function from which no more IMFs can be extracted [Huang e Shen, 2005]. The original data can be expressed by the sum of the IMF components plus the final residue, thus we obtain

$$x(t) = \sum_{j=1}^{n} c_j(t) + r_n(t) \quad (9)$$

The components of EMD are usually physical meaningful, for the characteristic scale are defined by the physical data.

The HHT is completed by the Hilbert spectral analysis (HSA) which consists in the application of the Hilbert transform on each IMF components obtained. For one IMF $c_i(t)$ in Eq. (14), we can express the Hilbert transform as

$$H[c_i(t)] = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{c_i(t')}{t - t'} dt' \quad (10)$$

From this definition, an analytic signal may be given by

$$z_i(t) = c_i(t) + jH[c_i(t)] = a_i(t)e^{i\Phi_i(t)} \quad (11)$$

where

$$a_i(t) = \sqrt{c_i^2(t) + H^2[c_i(t)]} \quad (12)$$

$$\Phi_i(t) = arctan\frac{H[c_i(t)]}{c_i(t)} \quad (13)$$

The instantaneous frequency is obtained from Eq. (18) as

$$\omega_i(t) = \frac{d\Phi_i(t)}{dt} \quad (14)$$

After applying the Hilbert transform to each IMF component, the original signal can be expressed as the real part (RP) in the following form [Cheng et al, 2008]:

$$x(t) = RP\sum_{i=1}^{n} a_i(t)e^{i\Phi_i(t)} = RP\sum_{i=1}^{n} a_i(t)e^{i\int \omega_i(t)dt} \quad (15)$$

At this moment, the residue $r_n(t)$ is left out on purpose, for it is either a monotonic function or a constant. Eq. (16) gives both amplitude and frequency of each component as functions of time. This frequency-time distribution of the amplitude is designated as the Hilbert spectrum $H(\omega, t)$:

$$H(\omega, t) = RP\sum_{i=1}^{n} a_i(t)e^{i\int \omega_i(t)dt} \quad (16)$$

Defined the Hilbert spectrum, we can also define the marginal spectrum $h(\omega)$ [Huang and Shen, 2005] as

$$h(\omega) = \int_0^T H(\omega,t)dt \tag{17}$$

where T is the total data length. The marginal spectrum offers a measure of the total amplitude (or energy) contribution from each frequency value.

We can use the Hilbert spectrum to obtain the local power spectral density, because $H(\omega,t)$ accurately describes the varying rule of signal amplitude according time and frequency, so when energy is taken into account, the local power spectral density is given by [Can-yang et al, 2008]

$$P_{xx}(\omega,t) = \frac{|H(\omega,t)|^2}{2} \tag{18}$$

Based on the temporal-frequency feature of the HHT, the energy of random process or the samples has been localized. The energy is expanded on the plane of time and frequency, as Eq. (23), then, the local power spectral density is got.

The Hilbert energy spectrum is defined by $H^2(\omega,t)$ that describes the energy-frequency-time distribution. Leaving out the residue $r_n(t)$, the HHT of $x(t)$ should be energy conservation, namely, the relation could be obtained:

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H^2(\omega,t)d\omega dt \tag{19}$$

where $|x(t)|^2$ is the energy density of the signal $x(t)$. Thus, the instantaneous energy $E(t)$ is defined by [Huang et al, 1998]

$$E(t) = \int_{-\infty}^{\infty} H^2(\omega,t)d\omega \tag{20}$$

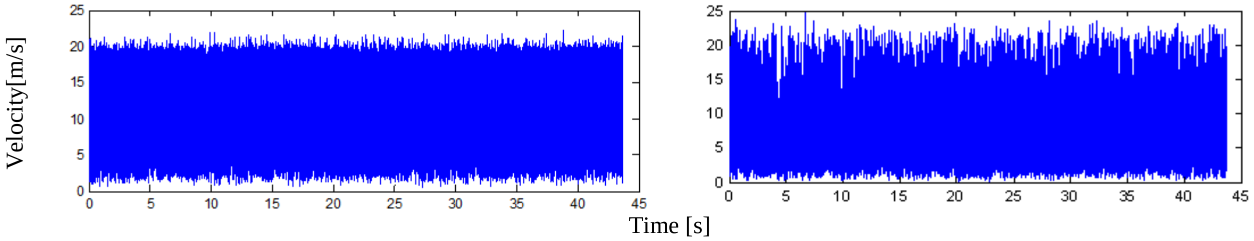
This $E(t)$ provides the energy distribution of the signal in the time domain. If we want the energy distribution in frequency domain [De-jie et al, 2008] we will have

$$E(\omega) = \int_{-\infty}^{\infty} H^2(\omega,t)dt \tag{21}$$

All mathematical analysis was made with MATLAB ® software and its specific toolboxes for statistical, spectral and wavelet analysis.

4. RESULTS

The circular cylinders of diameters 9.5 mm and 32 mm are studied. From the original data signal, Fig. 2, the autospectral density shows the shedding frequency (Fig. 2) and the presence of two harmonics. The frequency in Fig. 3a is 333.98 Hz, corresponding to a Strouhal number S = 0.213 and in Fig 3b the frequency is 105.5 Hz with a Strouhal number of 0.22. The value are higher than the classic value show of S = 0.21, due to the high blockage in the test section.



(a) (b)

Figure 2: Data signal: shedding process measured with a hot wire.
(a) Cylinder diameter 9.5 mm, (b) cylinder diameter 32 mm.

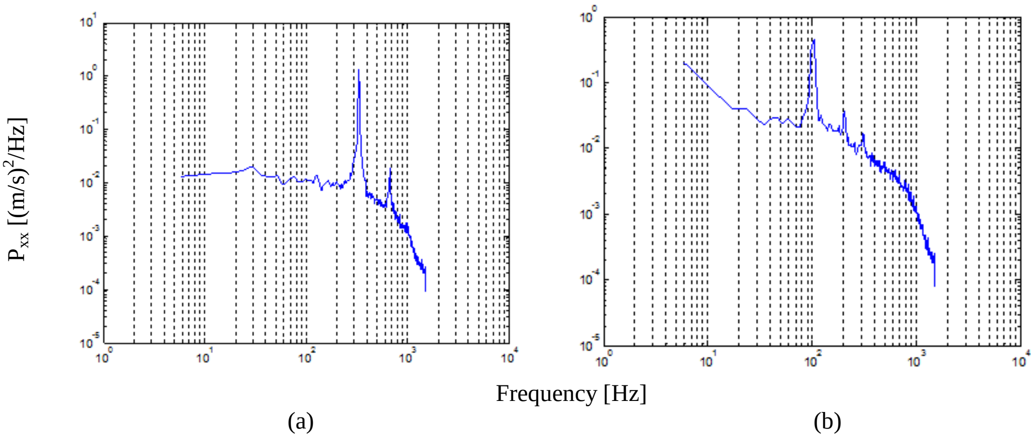


Figure 3: Power spectral density of the hot wire signal acquired at the wake of the cylinder.
(a) Cylinder diameter 9.5 mm, (b) cylinder diameter 32 mm.

Using wavelets analysis with the continuous wavelet transform (CWT), Fig. 4 shows the distribution of energy around the vortex shedding frequency in respect to time for the both cylinders, equivalent to the region of larger energy. This slight oscillation around the shedding frequency is not depicted in the power spectral density, since it is averaged over the entire time interval. The presence of the harmonics, however, is barely identified in the wavelet spectrum.

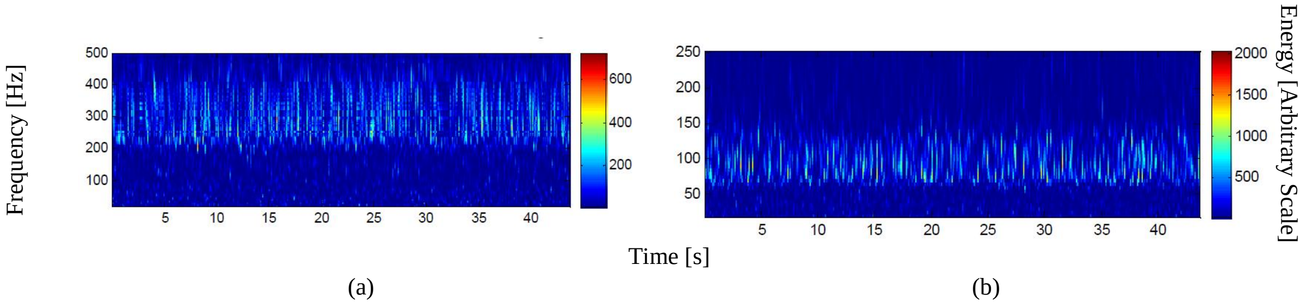


Figure 4: Wavelets spectrum for: (a) Cylinder diameter 9.5 mm, (b) cylinder diameter 32 mm.

The decomposition of the signals in Fig. 2 using the EMD algorithm of HHT, and seventeen IMFs are obtained for the cylinder of diameter 9.5 mm and eighteen IMFs are obtained for the cylinder of diameter 32 mm, Fig. 5 shows the first four components. Each component corresponds to a different oscillation mode with different amplitude and frequency content. The first IMF components have the highest-frequency, and the frequency content decrease with the increase IMF component. This frequency range is very useful to analyze any-frequency oscillation and it is used in HSA to obtain the instantaneous frequency.

From the decomposition of the signal using the Empirical Mode Decomposition by means of the Hilbert spectral analysis is shown on Fig. 5, and it can lead to a better visualization of the fundamental frequency and its subsequent harmonics, with respect to the wavelet analysis.

The Fast Fourier Transform was performed for the first four IMF in Fig. 6 in order to determine the frequency of each IMF components, from the highest to the lowest frequencies. The results show that those four IMFs have the principal contents of process shedding: for the cylinder diameter 32 mm the fourth IMF has the shedding frequency while the third contains the shedding frequency and the two harmonics. The first two decompositions present frequencies with a wide distribution and lower energy compared to the third and the fourth. For the cylinder with diameter 9.5 mm, the vortex shedding frequency and its harmonic are on the first IMF, where the largest content of energy of the signal is presented. For both cylinders, the remaining IMF components have only some lower frequencies and residues of the signal, not providing relevant information of the studied Phenomenon.

5. CONCLUSIONS

In the Hilbert-Huang Transform, the Empirical Mode Decomposition leads to Intrinsic Mode Functions of the signal which reveals the information of oscillations modes of the signal. This features can be possible because this direct and intuitive method, which can be applied in the analysis of vortex shedding process.

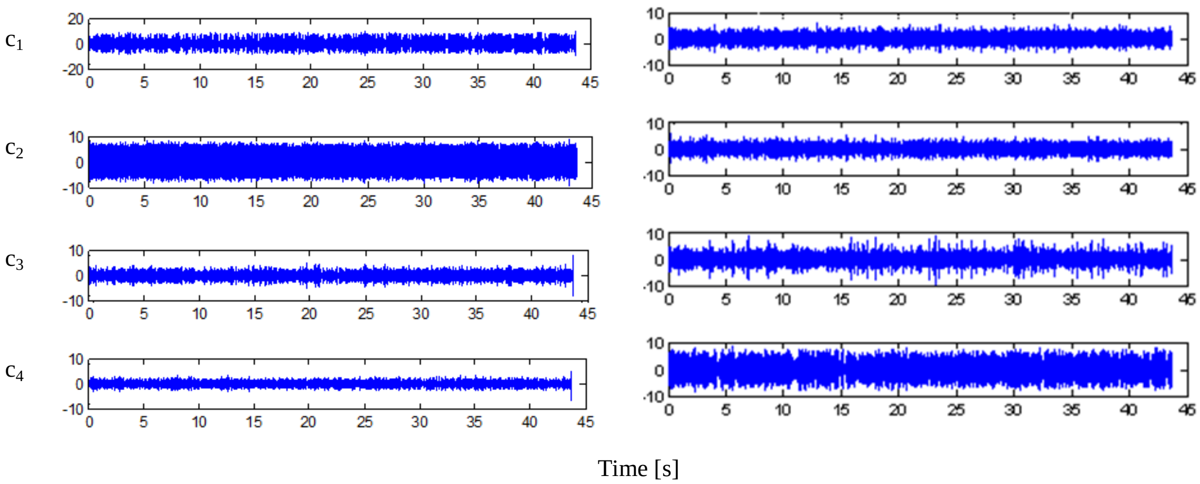
The fact of why the shedding frequency be present in fourth IMF, for cylinder diameter 32 mm, and in first IMF, for cylinder diameter 9.5 mm, may be linked with the blockage ratio, mainly, for the cylinder diameter 32 mm, which has a blockage ratio upper 10 %, which carries some influences on the flow because some corrections in the measures are necessary [Silveira, 2011]. The blockage effect under the cylinder diameter 32 mm is upper than cylinder diameter 9.5 mm, which affect the Strouhal number and, consequently, the vortex shedding frequency.

The gain in use of HHT in respect to Fourier analysis is in the fact that HHT, like wavelets, can be applied to non-stationary signals, simply yet to verify its behavior on PSD or Fourier spectrum. FFT had still to be used to identify the fundamental modes of the studied signal.

This method can be more effective than wavelets, which depends on the wavelet mother to be applied on the signal, which has to be chosen by trial and error, according to the features of the signal analyzed. Since HHT is directly decomposed from original data, the intrinsic physical characteristics are shown clearly by each IMF and its application is simpler than wavelets.

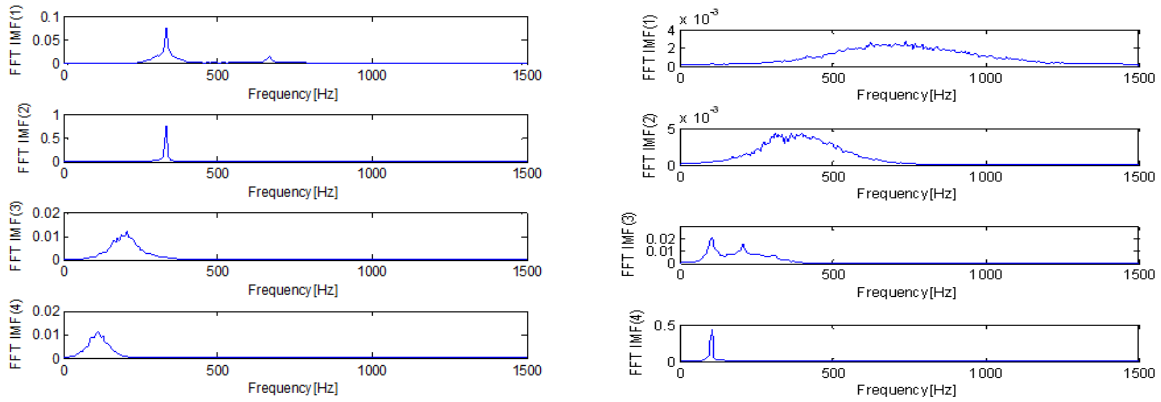
Therefore, the employed methods (Fourier, Wavelets and HHT) are complementary in interpreting the resulting data. In the case of use of FFT in some IMFs, this feature is very important in the methodology of a study since the conditions of applicability are satisfied.

In general, the analysis of the shedding process becomes more comprehensive with the aid of the Hilbert-Huang Transform to wavelet and Fourier transforms. HHT is a new technique and it can be a useful tool for the treatment of signals from turbulent flow studies.



(a) (b)

Figure 5: First four IMF components of velocity data signal.
(a) Cylinder diameter 9.5 mm, (b) cylinder diameter 32 mm.



(a) (b)

Figure 6: Fast Fourier Transform of 1st, 2nd, 3rd and 4th IMF components.
(a) Cylinder diameter 9.5 mm, (b) cylinder diameter 32 mm.

6. ACKNOWLEDGEMENTS

Authors gratefully acknowledge the support by The National Council for Scientific and Technological Development (CNPq), Ministry of Science and Technology (MCT), Brazil.

Rodrigo S. S. Horszczaruk thanks CAPES, Ministry of Education, Brazil, for granting him a fellowship.

7. REFERENCES

Blevins, R.D., 1990. Flow Induced Vibrations, van Nostrand-Reinhold, New York.

Huang, N.E.; Shen, Z.; Long, S.R.; Wu, M.C.; Shih, H.H.; Zheng, Q.; Yen, N.C.; Tung, C.C.; Liu, H.H., 1998. The Empirical Mode Decomposition and the Hilbert Spectrum for Nonlinear and Non-stationary Time Series Analysis, Proceedings of the Royal Society, v. 454, p. 903-995.

Huang, N.E.; Shen, S.S.P., 2005. Hilbert-Huang Transform and Its Applications, Word Scientific, v. 5, Singapore.

Shi, C.X.; Luo, Q.F., 2003. Hilbert-Huang Transform and Wavelet Analysis of Time History Signal, ACTA Seismologica Sinica, v. 16, no. 4, p. 422-429.

Cheng, J.; Yu, D.; Tang, J.; Yang, Y., 2008. Application of the frequency family separation method based upon EMD and local Hilbert energy spectrum method to gear fault diagnosis, Mechanism and Machine Theory, v. 43, p. 712-723.

Can-Yang, H.; Qing-Jun, C.; Tian-Li, H., 2008. Estimation of Local Spectral Density of Seismogram by Orthogonal Hilbert-Huang Transform and Application of the Spectral Density, The 14th World Conference on Earthquake Engineering, Beijing, China.

Yu, D., Cheng, J., Yang, Y., 2003. Hilbert Energy Spectrum and Its Application to Gear Fault Diagnosis, Journal of Human University, v. 30, N°. 4, p. 47-50.

Ziada, S., Oengören, A. 1993. Vortex Shedding in an in-line Tube Bundle with Large Tube Spacings, Journal of Fluids and Structures, Vol. 7, p. 661-687.

Silveira, R.S., 2010, Estudo Experimental do Efeito da Razão de Bloqueio em Canal Aerodinâmico sobre o Número de Strouhal e o Fenômeno da Biestabilidade no Escoamento em Cilindros, Promec-Ufrgs, Porto Alegre, Brasil.

6. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.