VERIFICATION OF THE MERGING OF IMMERSED BOUNDARY AND PSEUDO-SPECTRAL FOURIER METHODOLOGIES FOR FLOWS WITH HEAT TRANSFER

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Abstract. *A new numerical methodology combining Fourier pseudo-spectral and immersed boundary methods - IMERSPEC – has been developed for fluid flow problems modeled using the Navier-Stokes, mass and energy equations, for incompressible flows. The numerical algorithm consists in a Fourier pseudo-spectral methodology using the collocation method, where every kind of thermal boundary condition can be modeled using an immersed boundary method (Multi Direct Forcing Method). The IMERSPEC methodology was presented by Mariano et al. (2010). A new model for boundaries conditions of first, second and third were developed, implemented and verified, using synthesized solutions for Navier-Stokes and energy equation. Preliminary results are presented in the present paper.*

Keywords*: boundary condition 1, Fourier pseudo-spectral method 2, immersed boundary method 3 energy equation 4*

1. INTRODUCTION

In the last two decades a lot of effort has been spent by the fluid dynamic scientific community to address two crucial but conflicting key issues in the science of computational fluid dynamics (CFD). These are associated with the need to model increasingly complex boundary conditions in one hand, and, at the same time, requiring high accuracy Ferziger and Peric (1996). The great majority of engineering and geophysical fluid flow problems are characterized by very complex geometries that arise mainly from the irregular domain frontiers. This is often associated with the presence of moving and deformable geometries.

The immersed boundary methodology (IBM) has been developed since 1970 by several researchers Peskin (1972), Goldstein et al. (1993), Lima e Silva et al. (2003), Mittal and Iaccarino (2005). This method reach a portion of the requirements described above; specifically, it can handle complex and moving geometries, using Cartesian mesh.

The IBM was applied by Mariano et al. (2010) to solve the flow over a driven cavity and over a backward facing step. The cavity flow was also simulated by Botella and Peyret (1998), using a Chebyshev collocation method. This work was developed and applied for isothermal flows. Flows with heat transfer effects was simulated using immersed boundary methodology for boundary conditions of first, second and third type by Jungwoo and Haecheon (2004), Jeong et al. (2010), Wang et al. (2009), Pan (2006) and Young et al. (2009).

In special Wang et al. (2009) used the multi-direct forcing scheme to ensure the temperature Dirichlet boundary conditions at the immersed boundary and the finite difference scheme was applied to solve heat transfer problems, the results showed second-order spatial accuracy when applied to solve the Taylor-Green vortices.

Within the family of spectral methods (Canuto et al. 2007), the classical Fourier pseudo-spectral collocation method (FPSM) is extremely high accuracy and its low computational cost. Moreover, the pressure terms in the Navier-Stokes equations can be lumped together with the non-linear term, for incompressible flows. So, the FPSM does not requires the solution of a pressure Poisson equation. It results in an unusually fast time stepping procedure. These classical methods, however, are in general not applicable for over complex geometries. The Fourier collocation method, in particular, can only be used in flows with periodic boundary conditions.

The goal of the present work is to show a new methodology for incompressible flows with heat transfer and with three kinds of boundary conditions. It has been looked to combine the accuracy and low computational cost of the classical Fourier pseudo-spectral method Canuto (2007) with flexibility in handling complex geometries allowed by the immersed boundary methods. Was introduced, specifically, the *IMERSPEC* method Mariano et al. (2010), which combines a classical Fourier pseudo-spectral method, with an IBM. The main goal is to take into account the effects arising from the presence of complex boundaries. Also, any spatial derivative is computed with spectral accuracy. In the present paper, a new model for boundaries conditions of first (Dirichlet), second (Newman) and third (Robin) types were developed, implemented and verified, using synthesized solutions for the Navier-Stokes and energy equations.

2. MATHEMATICAL MODELING

The presented methodology is based on the merging process of the IBM with a classical FPSM. The equations in physical space, the pseudo-spectral and immersed boundary methods are described in Mariano et al. (2010). In this paper, only the models for several kinds of boundary conditions will be presented.

The mathematical model for incompressible flows of Newtonian fluids, with heat transfer is established with the energy, the Navier-Stokes and mass equations. These equations present source terms that model the boundary conditions for momentum and heat transfer. These equations are presented below:

$$\vec{\nabla}\cdot\vec{V} = 0 , \tag{1}$$

$$\rho\frac{\partial\vec{V}}{\partial t} = \overrightarrow{RHS} + \vec{f} \ , \tag{2}$$

$$\rho C_p \frac{\partial T}{\partial t} = RHS_T + f_T \ , \tag{3}$$

$$\overrightarrow{RHS} = -\rho\vec{\nabla}\cdot\left(\vec{V}\vec{V}\right) - \vec{\nabla}p + \mu\nabla^2\vec{V} + \vec{f}_{ss} , \tag{4}$$

$$RHS_T = -\vec{\nabla}\cdot\left(\vec{V}T\right) + \kappa\nabla^2 T + f_{Tss} , \tag{5}$$

where $\vec{f}(\vec{x},t)$ is the source term for Navier-Stokes equations and $f_T(\vec{x},t)$ is the term source for the energy equation respectively. The terms $\vec{f}_{ss}(\vec{x},t)$ and $f_{Tss}(\vec{x},t)$ are the source term related to the synthetized solutions for Navier-Stokes and energy equations. These source terms model the boundary conditions as well as others kind of physical effects.

The main goal of the present work is to present the verification of the new model for the IMERSPEC methodology Mariano et al. (2010) extended for flows with heat transfer effects. In the following topic the models for $f_T(\vec{x},t)$ source terms are presented. Particularly, a model for the energy source term is proposed for the boundary conditions of first, second and third types.

2.1 Boundary conditions for the energy equation

Three types of boundary conditions will be proposed, which alters the force term, f_{Tss} , at Eq. (5), Dirichlet (first type), Eq. (6), Newman (second type), Eq. (7) and Robin (third type), Eqs. (8), (9) and (10) boundary conditions:

$$f_T(\vec{x}) = \int_\Gamma F_T\left(\vec{X}\right)\delta\left(\vec{x}-\vec{X}\right)d\vec{X} = \int_\Gamma \rho C_p \frac{T^{n+1}(\vec{x}) - T^{*n+1}(\vec{x})}{\Delta t} d\vec{X} \tag{6}$$

$$\nabla^2 f_T^{n+1}(\vec{x}) = \vec{\nabla}\cdot \underbrace{\int_\Gamma \frac{\rho C_p}{\Delta t}\left(\begin{matrix}\dfrac{\vec{q}''^{\,n+1}\left(\vec{X}\right)}{k_f}+ \\ \vec{\nabla}T^{*n+1}\left(\vec{X}\right)\end{matrix}\right)\delta\left(\vec{x}-\vec{X}\right)d\vec{X}}_{\vec{I}^{\,n+1}(\vec{x})} = \vec{\nabla}\cdot\vec{I}^{\,n+1}(\vec{x}). \tag{7}$$

$$\nabla^2 f_T^{n+1}(\vec{x}) + \vec{\nabla}\cdot\vec{I}_1^{\,n+1}(\vec{x}) = \vec{\nabla}\cdot\vec{I}_2^{\,n+1}(\vec{x}), \tag{8}$$

where

$$\vec{I}_1^{\,n+1}(\vec{x}) = \lambda\int_\Gamma F_T^{n+1}\left(\vec{X}\right)\vec{n}\left(\vec{X}\right)\delta\left(\vec{x}-\vec{X}\right)d\vec{X} \ , \tag{9}$$

and

$$I_2^{n+1}(\vec{x}) = \frac{1}{\Delta t}\int_\Gamma\left[\begin{matrix}\lambda\rho C_p T_\infty\vec{n}\left(\vec{X}\right) - \\ \left(\begin{matrix}\rho C_p\vec{\nabla}T^{*n+1}\left(\vec{X}\right) + \\ \lambda\rho C_p T^{*n+1}\left(\vec{X}\right)\vec{n}\left(\vec{X}\right)\end{matrix}\right)\end{matrix}\right]\delta\left(\vec{x}-\vec{X}\right)d\vec{X}. \tag{10}$$

3. RESULTS

3.1 The Green Taylor problem with thermal effects, with immersed boundary

A synthesized or manufactured solution consists to determine a source term, given an analytical solution to the velocity, temperature and pressure field. The following equations, proposed by Green-Taylor, and a similar analytical solution for the energy equation, Eq. (3), were used in the present work:

$$u^{an}(\vec{x},t)=U_r \sin\left(\frac{ax}{L_r}\right)\cos\left(\frac{by}{L_r}\right)e^{-\frac{\nu t}{L_r^2}\left(a^2+b^2\right)}, \quad (11)$$

$$\vartheta^{an}(\vec{x},t)=-U_r \frac{a}{b}\cos\left(\frac{ax}{L_r}\right)\sin\left(\frac{by}{L_r}\right)e^{-\frac{\nu t}{L_r^2}\left(a^2+b^2\right)}, \quad (12)$$

$$p^{an}(\vec{x},t)=-\rho U_r^2 \frac{a^2}{2b^2}\left[\cos\left(\frac{by}{L_r}\right)^2-\sin\left(\frac{by}{L_r}\right)^2\right]e^{-2\frac{\nu t}{L_r^2}\left(a^2+b^2\right)}, \quad (13)$$

$$T^{an}(\vec{x},t)=T_r \sin\left(\frac{ax}{L_r}\right)\cos\left(\frac{by}{L_r}\right)\sin\left(\frac{\alpha t}{L_r^2}\right). \quad (14)$$

where, u^{an}, ϑ^{an}, p^{an} and T^{an} are, respectively, the analytical solution for the velocity components, the pressure and the temperature. The terms *a* and *b* are constants, L_r is a characteristic length, α is the diffusion coefficient of internal energy, *x* and *y* are the components of the coordinates system, *t* is the time, ρ and ν are the fluid density and the cinematic viscosity respectively. It should be observed that when *u*, ϑ and *p* in Equation (2) (Navier-Stokes equations) are replaced by Eqs. (11), (12) and (13), results in a source term $\vec{f}_{ss}(\vec{x},t)=\vec{0}$. This is the main characteristic of the Green-Taylor analytical solution. The source term $f_{Tss}(\vec{x},t)$, due to the synthetized solution for the energy equation is obtained replacing Eqs. (11), (12) and (14) at Eq. (3) results:

$$f_{TSS}(\vec{x},t)=\frac{\rho C_p T_r}{L_r^2}\sin\left(\frac{ax}{L_r}\right)\begin{bmatrix}\alpha\cos\left(\frac{by}{L_r}\right)\cos\left(\frac{\alpha t}{L_r^2}\right)+\\ U_r a L_r\cos\left(\frac{ax}{L_r}\right)e^{-\frac{\nu t}{L_r^2}(a^2+b^2)}\sin\left(\frac{\alpha t}{L_r^2}\right)+\\ \alpha a^2\cos\left(\frac{by}{L_r}\right)\sin\left(\frac{\alpha t}{L_r^2}\right)+\\ \alpha b^2\cos\left(\frac{by}{L_r}\right)\sin\left(\frac{\alpha t}{L_r^2}\right)\end{bmatrix}. \quad (15)$$

In order to compare the numerical simulation with analytical solution, the initial and boundary conditions were determined using this analytical solution. This procedure results in periodical boundary conditions, which is appropriated as a reference for the Fourier pseudo-spectral method.

The dimensionless constants *a*=1 and *b*=1 was taken.

The main goal of this simulation is to perform the verification of the proposed methodology and its numerical implementation. For that, the velocity and temperature fields were obtained numerically. The pressure field was recuperated as a post-processing procedure, as shown by Mariano et al. (2010). Then, the velocities components, the temperature and the pressure fields were compared with the analytical solution (Eqs. (11), (12), (13) and (14)) and the norm L_2 was obtained using the following equation:

$$L_{2\theta}=\sqrt{\frac{\sum_{i}^{N_x}\sum_{j}^{N_x}\left(\theta_{ij}^{num}-\theta_{ij}^{an}\right)^2}{N_x N_y}}, \quad (16)$$

where θ^{num} and θ^{an} stand, respectively, for the numerical and the analytical solution of the generic field, θ, *i.e.*, u, ϑ, p and T. The global error of the simulation can be determined using the norm L_2, calculated over the entire domain at each time. The solution and the domain present a periodical behavior. In order to use the boundaries conditions, modeled by the immersed boundary methodology, it is possible to insert, inside this domain, an immersed boundary over which all kind of boundary condition can be modeled and simulated using the forcing method.

This case is characterized by the presence of an immersed boundary Γ inside the complete domain Ω; a continuous initial condition. Figure 1 shows the temperature field and the immersed boundary inside the global domain.

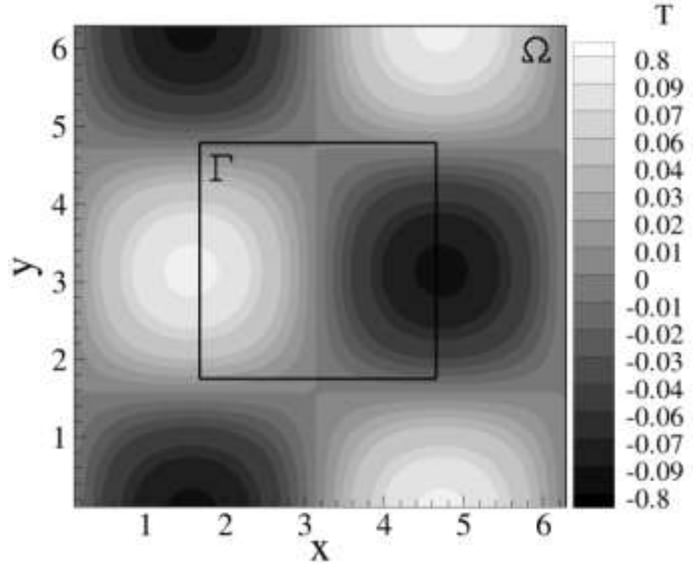

Figure 1. The temperature field; immersed boundary Γ inside the complete domain Ω.

The global error is shown in Fig. 2, for the boundary condition of first type and mesh N_xxN_y=8x8, changing the time step (Δt). It can be observed that the error decreases as the time step is also decreased. Moreover, for this mesh N_xxN_y=8x8, this methodology converges and it is possible to obtain round-off truncation error. For the boundary conditions of second and third types the results were similar.

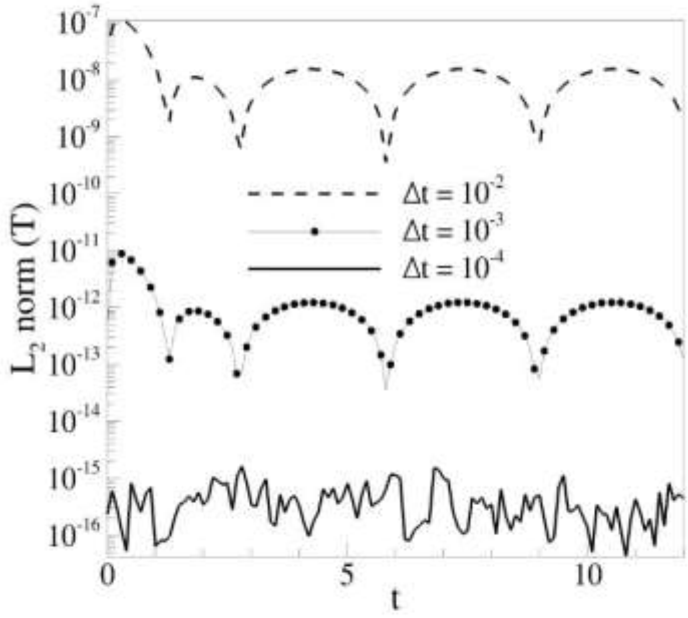

Figure 2. Errors (L_2) of the temperature for Green-Taylor problem, with thermal effect and immersed boundary for first type, N_xxN_y=8x8, changing Δt.

The Figure 3 shows the error for the boundary conditions first, second and third type. It can be seen that the boundary conditions keep the accuracy of the spectral method when the IBM is used. Its reaches round-off truncation error.

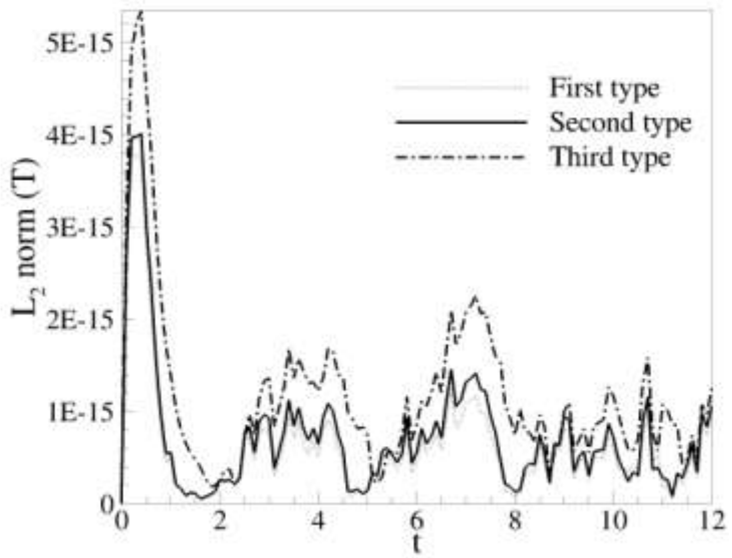

Figure 3. The errors (L_2) of the temperature for N_xxN_y=64x64 collocations nodes.

4. CONCLUSION

A new kind of IBM for fluids flows with thermal effects was proposed. Mathematical model for thermal boundary conditions of first, second and third types were proposed. They were implemented in a pseudo-spectral numerical code. The implementation was verified using a synthetized analytical solution for the Navier-Stokes and energy equations. The simulations that are presented in the present paper show that the proposed methodology is very accurate, at least for the simulated problem.

5. ACKNOWLEDGEMENTS

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