BIVARIATE ANALYSIS BY FINITE MIXTURE MODEL OF A BISTABLE TIME SERIES ON THE FLOW AFTER TWO ROWS TUBE BANK

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Abstract. *This work presents a study of a turbulent bistable flow on two rows of a triangular tube array. Velocity time series are obtained with the constant temperature hot wire anemometry technique in an aerodynamic channel and are used as input data in a finite mixture model to classify the observed data according to a family of probability density functions (PDF). In this context, the bistable phenomenon is considered as an incomplete data problem and PDF-estimation is performed using the observed data. The bistability phenomenon is analyzed as a bivariate function, and the number of clusters in the fitting process was considered equal to two, where an expectation-maximization algorithm was applied to estimate the maximum log-likelihood function according to a normal PDF with aid of a Monte Carlo method, to obtain the probability distribution function which is more likely to have produced the observed data. The experimental time series obtained show the bistable nature of the flow, with changes in the mean velocity levels from one flow mode to the other. The simulations with the bivariate Gaussian mixture model and the expectation-maximization algorithm show a goodness of fit in both flow modes, what suggests that the bistable phenomenon can be treated as a Gaussian process in the studied geometry.*

Keywords*: turbulent flow, circular cylinders, hot wires, bivariate probability density function, finite mixture models.*

1. INTRODUCTION

Banks of tubes or rods are found in several engineering applications, like in the nuclear and process industries being the most common geometry used in heat exchangers. The turbulent flow impinging on circular cylinders placed side-by-size presents a floppy and random phenomenon that changes the flow mode. This behavior is called in the literature as bistable flow and is characterized by a wide near-wake behind one of the cylinders and a narrow near-wake behind the other, which generates two dominant Strouhal numbers, each one associated with one of the two wakes formed: the wide wake is associated with a lower Strouhal number and the narrow wake with a higher one.

Païdoussis (1982) comment that heat exchangers have faced "spectacular failures", which involves almost immediate failure of components, due to the flow-induced vibration.

The need for more efficient heat exchangers leads the operating conditions of these equipments to become critical, due to the reduction of the aspect ratio of the tube banks (pitch-to-diameter ratio) and the increasing of the flow velocity. As a consequence of the reduction of the flow area in the narrow gaps between the tubes, which causes velocity fluctuations, and the constant change of the flow direction, static and dynamic loads will be increased (Endres et al., 1995). According to Blevins (1990), the dynamic loads of the turbulent flow over small aspect ratio tube banks are characterized by broad band turbulence, without a defined shedding frequency. For large aspect ratio tube banks, the dynamic loads are basically associated with vortex shedding process.

Zdravkovich and Stonebanks (1990) comment that the leading feature of flow-induced vibration in tube banks is the randomness of dynamic responses of tubes, and even if the tubes are all of equal size, have the same dynamic characteristics, are arranged in regular equidistant rows and are subjected to an uniform steady flow, the dynamic response of tubes is non-uniform and random.

When two circular cylinders are placed side-by-size and submitted to a turbulent cross-flow, an interesting phenomenon occurs: the flow that emanates through the gap between the cylinders is biased towards the rear surface of one of the cylinders, and has a narrow wake. This can be called as a flow mode. When a floppy and random behavior of the gap flow changes intermittently the flow mode, from one cylinder to other at irregular time intervals, in literature, this is the so called *bistability* phenomenon.

Triangular geometry was chosen in this work due to the fact that several engineering applications have been using it, and also because bistability has been found more recently in tube banks with square arrangement (Olinto et al., 2006, Olinto et al., 2009).

As flow induced vibration and structure-fluid interaction are very dependent of the arrangement or configuration of the cylinders, new studies are necessary to improve its understanding, since bistability can be an additional excitation mechanism on the tubes. Furthermore, tube arrangement is also equally important in the characterization of fluid flow and heat transfer of a tube bank.

Finite mixture models are statistical tools applied in many knowledge areas to perform the PDF-estimation of an incomplete data problem, and recently the mixture of skew distributions has been found to be effective in the treatment of heterogeneous data with high asymmetry across subclasses. Through this approach bistable phenomenon can be considered as an incomplete data problem and experimental time series can be used as observed data. A maximum likelihood estimation (MLE) can be performed to know what the probability distribution function is more likely to have produced the observed data.

Flow visualization techniques also help in the comprehension of the phenomena studied in laboratory conditions.

THE BISTABLE EFFECT

According to Sumner et al. (1999), the cross steady flow through circular cylinder with same diameter (D) placed side-by-side can present a wake with different modes depending on distances between its centers, called pitch (P). Different flow behaviors can be found for different pitch-to-diameter ratios P/D. For intermediate pitch ratios (1.2 < P/D < 2.0), the flow is characterized by a wide near-wake behind one of the cylinders and a narrow near-wake behind the other, as shown schematically in Fig. 1a and Fig. 1b. This phenomenon generates two dominants vortex-shedding frequencies, each one associated with a wake: the wide wake is associated with a lower frequency and the narrow wake with a higher one. The switching of the gap flow, which is biased toward the cylinder, from one side to other at irregular time intervals, is therefore known as a flip-flopping regime or bistable flow regime (Bearman and Wadcock, 1973). Figure 1 presents a link between the wakes patterns (Figs. 1a and 1b) and a velocity measurement technique, performed by the hot wire anemometry technique (Fig. 1c). The velocity signals are measured downstream the cylinders, along the tangent to their external generatrices, where one switching mode can be observed (modes 1 and 2).

Previous studies show that this pattern is independent of Reynolds number, and it is not associated to cylinders misalignment or external influences, what suggest an intrinsically flow feature.

According to Kim and Durbim (1988) the transition between the asymmetric states is completely random and it is not associated with a natural frequency. Through a dimensional analysis, they concluded that the mean time between the transitions is on order 10^3 times longer than vortex shedding period, and the mean time intervals between the switches decreases with the increasing of Reynolds number. The authors conclude that there is no correlation between the bistable feature and the vortex shedding, due to the fact that Strouhal numbers are relatively independent from the Reynolds numbers (Žukauskas, 1972).

Indrusiak et al. (2005), studying the velocity and pressure fluctuations of transient turbulent cross-flow in a tube bank with square arrangement and a pitch-to-diameter ratio of 1.26, determined experimentally the presence of a biased and bistable flow mode inside a tube bank, behind the third row of tubes. Transient and steady state flows were studied, by means of accelerating and decelerating the centrifugal blower of the aerodynamic channel. Wavelet and wavelet packet multiresolution analysis, together with continuous wavelet transform were applied, and the results show that the flow through tube banks has an unsteady three-dimensional nature.

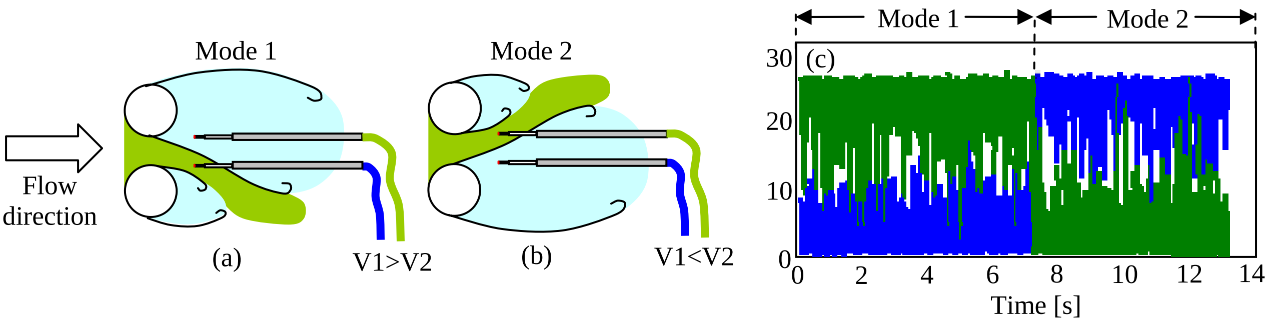

Figure 1. Bistability scheme for (a) mode 1 and (b) mode 2, and the respective characteristic signals (c).

3. OBJECTIVES

The purpose of this paper is to study the bistable flow after two rows tube bank in triangular arrangement to better comprehend the switching of the gap flow and to classify the data according to a representative PDF in a bivariate mixture model approach.

4. METHODOLOGY

Time series of axial and transversal velocity obtained with the constant temperature hot wire anemometry technique in an aerodynamic channel are used as input data in a finite mixture model, to classify the observed data according to a family of probability density functions (PDF).

4.1. Experimental Technique

The measurements are performed in an aerodynamic channel made of acrylic, with a rectangular test section of 0.146 m height, width of 0.193 m and 1.02 m of length (Fig. 2a). The velocity of the flow and its fluctuations are measured by means of the DANTEC *StreamLine* constant hot-wire anemometry system, with aid of a double hot wire probe (type DANTEC 55P71 Special), with straight/slant wires (the straight wire is placed perpendicularly to the flow, and the slant forms a 45° with the axial plane). The probes are aligned along the tangent to the external generatrices of a cylinder (Fig. 2b). The circular cylinders, with external diameter of 25.1 mm, are made of Polyvinyl chloride (PVC), and are rigidly attached to the top wall of test section. The probe support is positioned with a 3D table placed 200 mm downstream the end of the channel (Fig. 2c). The mean error of the flow velocity determination with a hot wire was about +/- 3%. The Reynolds number of the experiment is 21,000, computed with the tube diameters and the gap velocity of 12.9 m/s, and the pitch-to-diameter ratio is P/D=1.26.

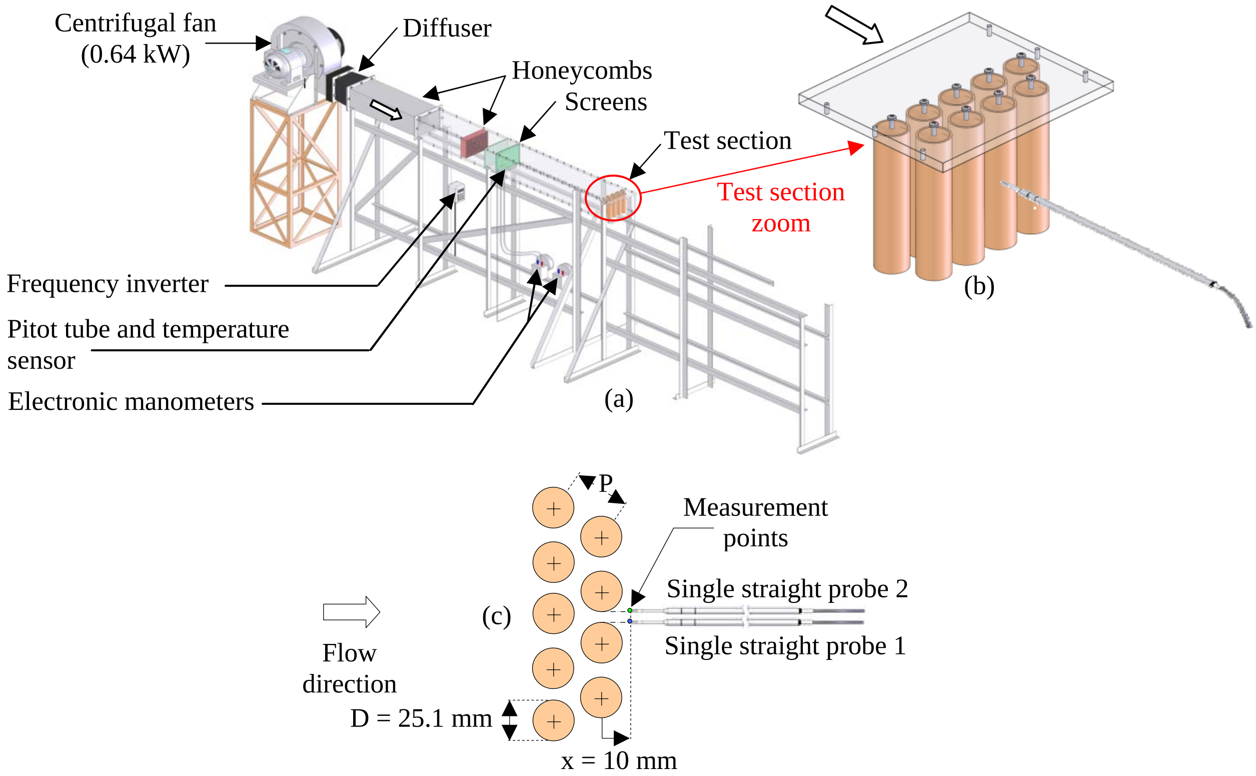

Figure 2. Schematic view of (a) the aerodynamic channel, (b) test section and (c) probe position.

4.2. Finite Mixture Models

The time series obtained are used as input data to classify the probability density functions of the phenomenon under study. A finite mixture model is applied to classify the observed data according to a family of probability density functions (PDF). In this context, the bistable phenomenon is considered as an incomplete data problem and PDF-estimation is performed using the observed data (time series), where the number of clusters was considered equal to two. An expectation-maximization (EM) algorithm (Dempster et al., 1977) together with a Monte Carlo (MC) method is applied to estimate the maximum log-likelihood function (Wei and Tanner, 1990) according to a Gaussian PDF (McLachlan and Peel, 2000), to know what the probability distribution function is more likely to have produced the observed data.

4.3. Water channel

Flow visualizations were performed in a water channel which has a settling chamber with a honeycomb that acts as a flow straightener, a nozzle, a 30 m long open channel (10 m upstream, 20 m downstream the test section) with 0.5×0.6 m rectangular cross section, a vertical gage to control the water level, and a discharge tank with the return pipe to close the circuit. The experiments were conducted inside a test section placed in the visualization section of the water channel. The tube bank was completely submerged, so that the upper plate was at 0.08 m below the water surface to avoid the effect of gravitational waves. The cylinders are rigidly attached to the base plate, and are built with commercial PVC tubes, with diameter of 0.06 m and 0.3 m height, covered by a thin white PVC film. A digital camera, placed above the bank, was used for taking digital movies at 30 frames per second in VGA resolution. Flow visualizations were performed with the injection of potassium permanganate directly in the free stream.

5. RESULTS

The time series of axial and transversal velocity measured in the aerodynamic channel are shown in Fig. 2a and Fig. 2b, respectively. Data acquisition is started before the blower is turned on, as in Indrusiak et al., (2005), with a transient flow from 5 to 15 seconds. Thereafter, the flow mode 1 (wide near-wake) is established, until 40 seconds, where the gap flow changes its direction to the mode 2 (narrow near-wake) lasting for the rest of the time acquisition. The axial and vertical signals (Figs. 2a and 2b) present similar behaviors, where an increase of velocity is observed in both signals, during the switching of the gap flow. Figures 2c and 2d show in details the time series of Figs. 2a and 2b, respectively, from 20 to 60 seconds. The velocities PDF present the predominance of two major states of energy, with different shapes (Fig. 2e and Fig. 2f).

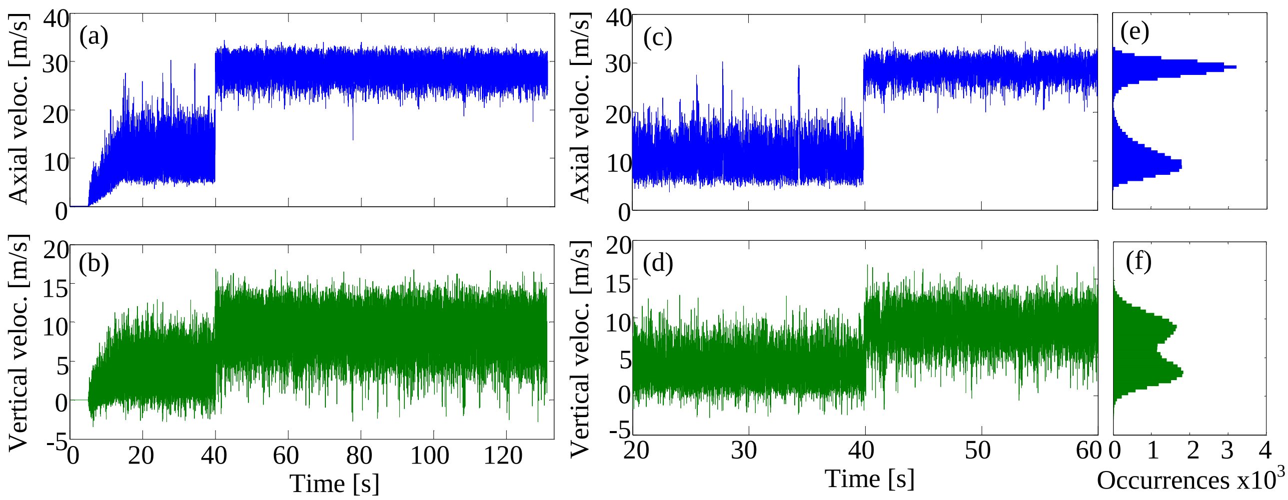

Figure 3: Time series of (a) axial velocity and (b) vertical velocity. Details of time series from 20 to 60 seconds: (c) axial velocity and (d) vertical velocity. PDF of the detailed time series: (e) axial velocity and (f) vertical velocity.

A joint analysis of axial and vertical velocity components is presented in Fig. 4, where the bivariate velocity PDF shows the presence of two prominences. The higher velocity mode seems to be present in larger quantity in this case.

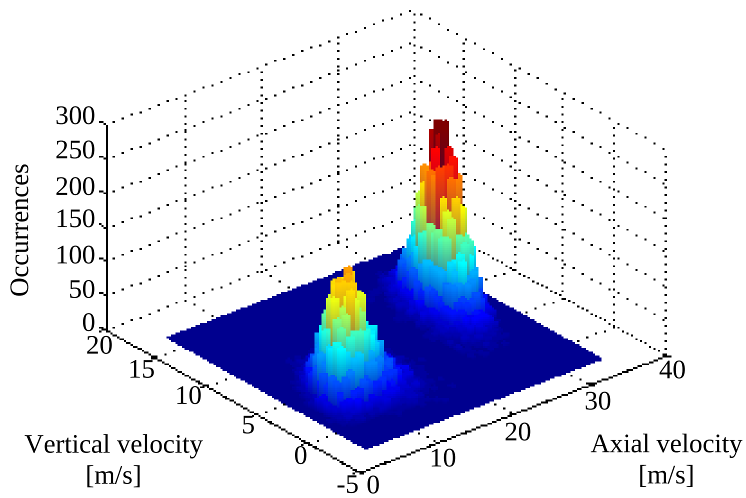

Figure 4: Bivariate velocity PDF of joint analysis of axial and vertical velocity components.

Figure 5 presents the results of the orders pair of data, with both axial and transversal velocity components, where the points are dispersed over a large area. The two distinct areas are referred to both flow modes (points in blue indicates the flow mode 1, while points in green indicates the flow mode 2). Figure 5 also presents the results of the fitting process with the univariate and the bivariate Gaussian mixture model. The bivariate Gaussian mixture model generates two ellipses, each one for a distinct flow mode. The results of the simulations show a goodness of fit in both flow modes, what suggests that the bistable phenomenon can be treated as a Gaussian process in the studied geometry.

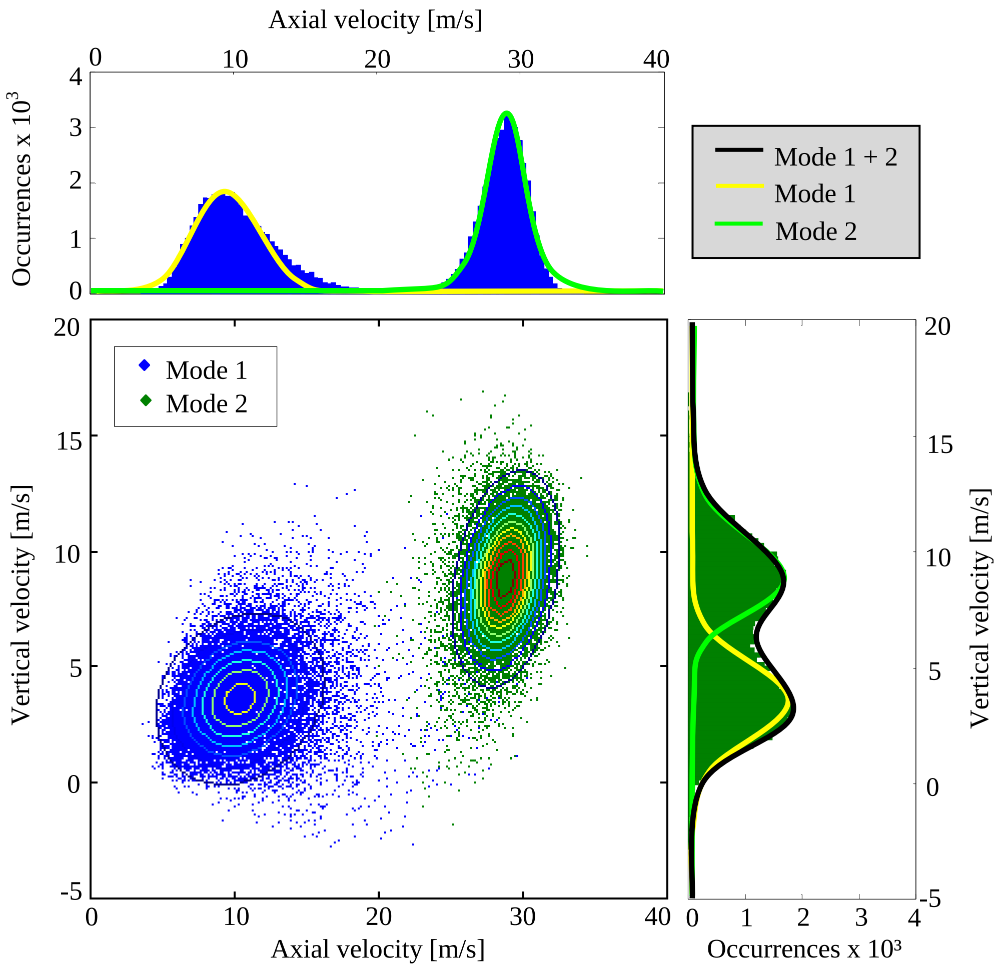

Figure 5: Ordered pair of data points of the flow velocity components with the ellipses generated by the fitting process with the bivariate Gaussian mixture model.

The numerical values of the parameters are: mean values (μ), covariances (σ) and mixture proportions (p). The PDF for the axial velocity show no overlap, while for the vertical velocity, the PDF are partially overlapped.

Table 1 presents the numerical values of the ellipses generated by the fitting process with the bivariate Gaussian mixture model.

Table 1. Numerical values of the fitting process with the bivariate Gaussian mixture model.

Parameter	Mode 1		Mode 2	
	Axial	**Transversal**	**Axial**	**Transversal**
Component Means (μ)	10.38	3.61	28.79	8.82
Component Covariances (σ)	8.38	1.09	2.75	1.02
	1.09	3.40	1.02	4.48
Mixture Proportions (p)	0.49		0.51	

By analyzing in details the axial velocity signal from 26 to 36 seconds of mode 1, is possible to observe two attempts of changing the gap flow direction of short time (Fig. 6). The other change of the gap flow direction refers to transition to mode 2.

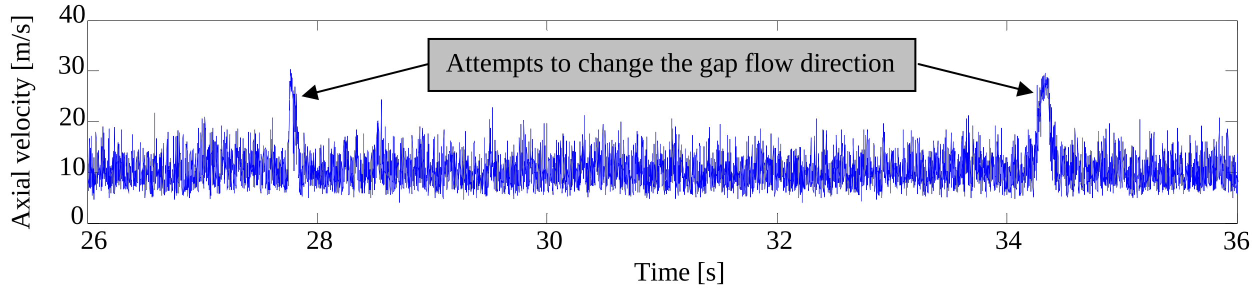

Figure 6: Details of the axial velocity signal from 26 to 36 seconds, showing two attempts to change the gap flow direction.

Figure 7 shows the results of the flow visualization with injection of permanganate potassium direct in the free stream (Fig. 7a and Fig. 7b) with its respective flow patterns (Fig. 7c and Fig. 7d), where the bistable behavior is identified downstream the second row, in the 3rd tube, from left to right. These pictures are separated by a few minutes, long enough for the two flow modes are established. Similar results are observed for P/D=1.6 (De Paula et al., 2009).

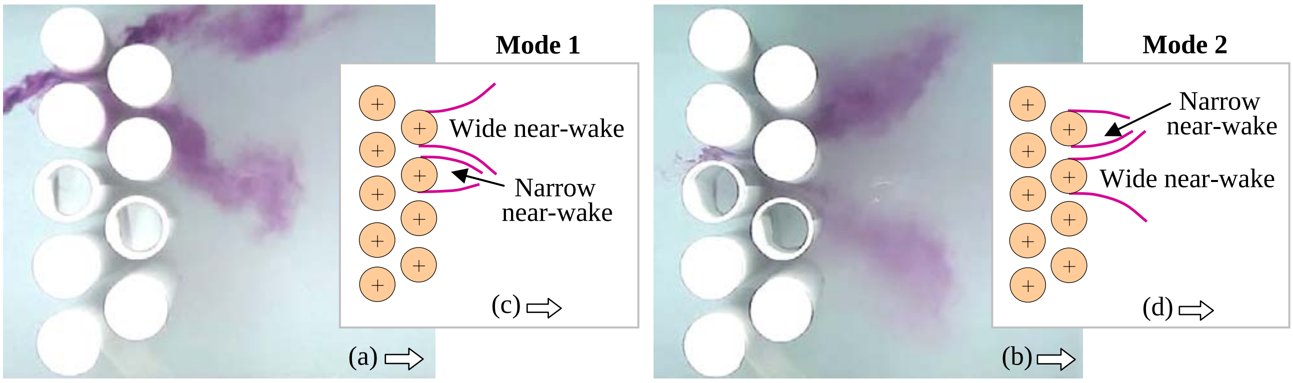

Figure 7: (a, b) Results of flow visualization for modes 1 and 2, with their respective flow patterns (c, d).

6. CONCLUSIONS

This work presents a study about the bistable phenomenon on two rows tube bank. A bivariate finite mixture model tool is applied, where an expectation-maximization algorithm performs the maximum likelihood estimation according to a known PDF with aid of a Monte Carlo method, to know what the probability distribution function is more likely to have produced the observed data. This tool was valuable to determine the numerical values of the shape of the ellipses generated by the fitting process of the bivariate Gaussian mixture model in the measurement plane. Results with one double straight/slant hot wire probe show that in the changes between the flow modes the increase in the axial velocity component is accompanied by increased of the transverse component, and their PDF present the predominance of two major states of energy, with different shapes. The simulations with the bivariate Gaussian mixture model and the expectation-maximization algorithm show a goodness of fit in both flow modes, what suggests that the bistable phenomenon can be treated as a Gaussian process in the studied geometry. From the flow visualizations in water channel, the bistable behavior is clearly identified.

7. ACKNOWLEDGEMENTS

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