FRICTION LOSSES IN ABRUPT CONTRACTIONS AND EXPANSIONS FOR POWER-LAW AND BINGHAM FLUIDS

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Abstract. *Non-Newtonian fluids are widely used in the petroleum industry and there is a huge class of these materials that can be modeled either as a power-law fluid, when the material exhibits pseudo-plastic behavior, or as a Bingham fluid, when the material possesses an yield stress. Pressure losses in piping systems result from a variety of sources that can be roughly divided into the wall friction, change in the flow direction, and change in the cross section of the duct. Here we use the commercial software Polyflow to measure the friction losses in two situations very common in pipe systems: the abrupt contraction and the abrupt expansion in a tube. Two kinds of non-Newtonian materials are tested, a power-law fluid and a Bingham material, and compared with the Newtonian result. The main objective is investigate how the power-law index,* n*, and a dimensionless yield stress,* τ_0'*, (similar to the Bingham number) influence these pressure losses. For the entrance region, we have found that as* n *decreases, friction loss also decreases, therefore a pseudoplastic fluid has a lower friction factor than a Newtonian one. In the same geometry, an increase in the dimensionless yield stress decreases the friction factor. For the abrupt contraction and expansion geometries, there is an interesting change of relative position among the different power-law fluids after a critical Reynolds number. For the viscoplastic Bingham material, an increase of the dimensionless yield stress increases the friction losses.*

Keywords: *friction losses, viscoplastic materials*

1. INTRODUCTION

The importance of non-Newtonian materials is becoming more and more recognized in the petroleum industry. One of the reasons is due to the increasing necessity of determining more accurately certain procedures that have a traditional Newtonian calculation counterpart. It was not uncommon, in the past, to use Newtonian results as an approximation for non-Newtonian flows. Another reason is that non-Newtonian materials are more complex, exhibiting features that are not present in Newtonian fluids world such like pseudoplasticity, viscoplasticity, elasticity, thixotropy. These features can be used in the design of new apparatus and to conceive new methodologies to optimize standard processes. In the recovery of oil, drilling mud removal, among other procedures, non-Newtonian fluids are widely used. But, perhaps, one of the most important applications is related to heavy oils with non-Newtonian behavior.

A very relevant problem to the petroleum industry is the calculation of pressure loss in fittings and components of piping system where non-Newtonian materials flow. The energy consumption of the piping system and the pump selection procedure are dependent on such analysis. Despite of this fact, there are few works in the literature that investigate non-Newtonian pressure loss. Most of these works are experimental investigations. In this case, a change on the material is given by a change in a concentration of a set of particles that is added to a Newtonian solvent. With a new concentration, the material is characterized through its viscosity as a function of the shear rate. Generally the viscosity curve is fitted by a chosen model (power-law, Sisko, Carreau) and the parameters of this model are used to understand the principles of friction loss in the fittings considered. The friction loss results are generally giving by a resistance coefficient parameter of the fitting, K, as a function of the Reynolds number.

In the literature, there are mainly two equations used to fit these curves. These equations are used independently of the kind of fitting considered. The one presented by Kittredge and Rowley (1957) is given by

$$K = A(Re)^{-B} \tag{1}$$

and the $two\ Ks$ method presented by Hooper (1981), where

$$K = \frac{K_1}{Re} + K_\infty \tag{2}$$

In general, for the two Ks method, the first part proportional to the inverse of the Reynolds number is related to the laminar regime, while the value K_∞ is the constant level associated to the turbulent regime. The advantage of these equations is their simplicity due to the fact that they have only two parameters.

Turian et al. (1998) investigated the friction losses in some fittings with slurries in a Newtonian fluid. They study not only laminar, but also turbulent flows with different concentration of two kinds of slurries: Titanium Dioxide, Laterite, and Gypsum. The model, used to fit the rheological viscosity curve was the Sisko model, given by

$$\eta = m\dot{\gamma}^{n-1} + \eta_\infty \tag{3}$$

where m and n are the consistency and exponent index of the power-law part and η_∞ is the asymptotic viscosity value for high values of the shear rate. The fittings investigated were: two kinds of valves gate, and globe; three elbows: with curvatures of 45^0, 90^0, and 180^0 degrees, a Venturi tube, and an abrupt expansion and contraction. They found that the two Ks method fits well the data. For the laminar regime, the values found for K_1 are independent of the diameter, while the values found for K_∞ are different, for different diameters.

The components tested by Bandyopadhyay and Das (2007), on their experimental apparatus, were orifices, globe and gate valves, and elbows of 45^0, 90^0, and 135^0 degrees. The tested fluid was a dilute solution of a sodium salt of Carboxy Methyl Cellulose of high viscous grade. They fitted the viscosity function with a power-law equation. The main result of the article was to propose correlations for the different fittings as a function of the Reynolds number and a fitting parameter (such as the degree of the elbow curvature or the ratio of valve opening). The Reynolds number was calculated with an effective viscosity, μ_{eff}, given by

$$\mu_{eff} = m^* \left(\frac{8V}{D}\right)^{n^*-1} \tag{4}$$

The effective viscosity provides a product $fRe = 64$ independently of the material under investigation, in accordance to Metzner and Reed (1955) and Soares et al. (2003). The other important contribution was the experimental investigation of the influence of the diameter size on the friction losses in fittings when a non-Newtonian fluid is the main fluid of the hydraulic system. For this purpose, they have carried out their experiments with a tube of a small diameter, $d = 0.0127m$, and compared their results with other, from the literature, with tubes of regular diameter sizes.

The work of Polizelli et al. (2003) analyzes the laminar and turbulent non-Newtonian flows of aqueous solutions of sucrose and Xanthan Gum. Although the viscosity function was better fitted by a Hershell-Buckley model, the yield stress was too small and sometimes negative. Therefore, the power-law equation was chosen to fit the data. The tested fittings were elbows of 45^0, 90^0, and 180^0 and butterfly and plug valves. They used the two Ks method for the different fittings finding a good agreement between predicted and experimental values.

Pinho et al. (2003) investigated the friction losses in sudden expansion $1 : 2.61$ for shear-thinning power-law fluids in the laminar regime. They give also results for the size of the recirculation at the corner and the relative intensity of the stream function at the core of the recirculation. They found a change on the relative position between the curves of friction losses. For low Re a more shear-thinning fluid has higher values for K, while for high values of the Reynolds number, the opposite happens.

2. Physical formulation

2.1 Conservation equations

The velocity and pressure fields are defined by the governing equations that impose conservation of mass and momentum for an incompressible fluid, together with the appropriate boundary conditions. In the present work, the main hypothesis considering the flow conditions are

1. The fluid is incompressible.
2. Steady-state laminar regime.
3. The flow is axisymmetric.
4. Body forces are conservative.
5. Isothermal flow.

With these hypothesis, the conservation of mass is given by

$$\frac{1}{r}\frac{\partial}{\partial r}(rv) + \frac{\partial u}{\partial r} = 0, \tag{5}$$

while the conservation of momentum is given by

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r}\right) = \frac{1}{r}\frac{\partial}{\partial r}(rT_{xr}) + \frac{\partial}{\partial x}(T_{xx}) \tag{6}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial r}\right)=\frac{1}{r}\frac{\partial}{\partial r}(rT_{rr})-\frac{T_{\theta\theta}}{r}+\frac{\partial}{\partial x}(T_{rx})=0 \tag{7}$$

Where u and v are respectively the axial and radial components of the velocity field $\mathbf{u}$ and the quantities T_{xx}, T_{xr}, T_{rx}, T_{rr} and $T_{\theta\theta}$ are the components of the stress tensor $\mathbf{T}$.

2.2 Constitutive equations

In order to close the set of unknowns and equations given by Eqs. (5), (6), and (7), additional equations are needed. Generally they come from an assumption on the behavior of the material when it is subjected to a certain state of stress. This constitutive equation is, therefore, a relation between the stress tensor $\mathbf{T}$ and kinematics. In the present work, the material models are under a class called Generalized Newtonian Fluids (GNF) that obey the following constitutive equation

$$\mathbf{T}=-p\mathbf{1}+2\eta\mathbf{D} \tag{8}$$

where p is the mechanical pressure, η is the viscosity, and $\mathbf{D}\equiv 0.5\left(\nabla\mathbf{v}+\nabla^T\mathbf{v}\right)$ is the symmetric part of the velocity gradient. For a Newtonian fluid, the viscosity $\eta=\mu$ is constant. For a GNF the viscosity is a function of the deformation rate $\dot\gamma$, a scalar that measures the intensity of the tensor $\dot{\boldsymbol{\gamma}}\equiv\nabla\mathbf{v}+\nabla^T\mathbf{v}=2\mathbf{D}$, given by

$$\dot\gamma=\sqrt{\frac{1}{2}\mathrm{tr}\left(\dot{\boldsymbol{\gamma}}^2\right)} \tag{9}$$

where $\mathrm{tr}()$ indicates the trace operator of a tensor.

The two viscosity functions considered here are the power-law fluid and the Bingham material. As explained previously, the power-law constitutive equation is used when the operational flow conditions are in a range where viscosity decays (or goes up) with a constant slope in a log-log plot of viscosity as a function of the shear rate.

$$\eta=m\dot\gamma^{n-1} \tag{10}$$

where m is the consistency index, while n is the exponent index.

A Bingham viscosity function is used when the fluid has an yield stress, τ_0. Additionally when τ_0 is transposed, the fluid flows as a Newtonian one of viscosity μ_p.

$$\begin{cases}\eta=\frac{\tau_0}{\dot\gamma}+\mu_p & \text{if} \quad \tau\geq\tau_0\\ \dot\gamma=0 & \text{if} \quad \tau<\tau_0\end{cases} \tag{11}$$

In fact, the implemented constitutive model was an approximation of the Bingham model, introduced by Papanastasiou (1987), given by

$$\eta=\frac{\tau_0}{\dot\gamma}\left[1-exp(-\xi\dot\gamma)\right]+\mu_p, \tag{12}$$

transforming Relation (11) into a function. The parameter ξ is called a regularization parameter and, in the present work $\xi\dot\gamma_c=1000$, where $\dot\gamma_c$ is a characteristic deformation rate, as recommended by You et al. (2008) for a good representation of a Bingham response. In the present work, $\dot\gamma_c$ is the shear rate at the wall of the fully developed region (in the tube with the smaller diameter of the problem).

A dimensionless yield stress number, τ_0', can be constructed as

$$\tau_0'=\frac{\tau_0}{\mu_p\dot\gamma_c+\tau_0} \tag{13}$$

2.3 General analysis

A simple conservation of mechanical energy, between an upstream position 1 and a downstream position 2, in a piping system under the hypothesis considered, leads to the well known equation

$$\frac{P_1}{\rho_1}+\alpha_1\frac{\bar V_1^2}{2}+gZ_1=\frac{P_2}{\rho_2}+\alpha_2\frac{\bar V_2^2}{2}+gZ_2+h_t, \tag{14}$$

where P_i, $\bar V_i$, Z_i are, respectively, the values of: the mechanical pressure, the mean velocity, and the height in the gravitational direction, at position i. The present analysis considers a modified pressure that incorporates the work done by gravitational forces. The coefficients α_i are related to non-uniformity of the velocity profile and are defined as

$$\alpha_i=\frac{1}{\bar V^3 A}\int_A V^3 dA. \tag{15}$$

For uniform velocity profiles, $\alpha = 1$. In the case of a laminar fully-developed Newtonian flow, since the profile is quadratic, $\alpha = 2$. The quantity h_t represents the total energy, per unit of mass, lost from point 1 to point 2.

The procedure followed in the present investigation is to split h_t into two parts: one part that considers the loss that would occur in a straight pipe in a fully-developed regime and a second part related to the change (associated to the fully-developed counterpart) in the flow profile caused by the accident considered.

Figure 1 depicts a scheme to illustrate the procedure and the calculation of a generic coefficient of local loss, K. Therefore, from Eq. (14) we have that

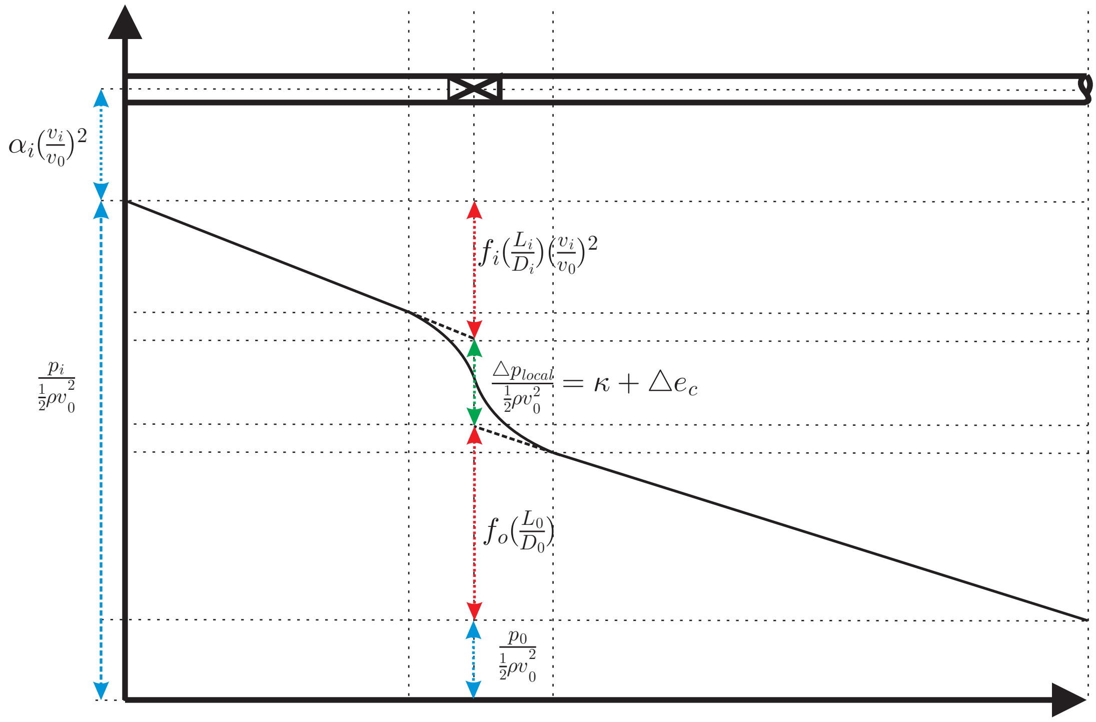

Figure 1. Dimensionless pressure loss in a hypothetic piping system. Calculation of of a generic coefficient of local loss, K.

$$h_t = \sum_{j=1}^{nj} f_j \frac{L_j}{D_j}\frac{\bar{V}_j^2}{2} + \sum_{p=1}^{np} K_p \frac{\bar{V}_p^2}{2} = \frac{P_1 - P_2}{\rho} + \alpha_1\frac{\bar{V}_1^2}{2} - \alpha_2\frac{\bar{V}_2^2}{2}, \tag{16}$$

where nj is the number of straight tubes and np is the number of accidents between positions 1 and 2. Quantities L_j, D_j, and $\bar{V}_j$ are the length, diameter and mean velocity of tube j, while $\bar{V}_p$ is a chosen characteristic mean velocity associated to accident p. Since we are investigating laminar losses, the friction factor associated to tube j is $f_j = \frac{64}{Re_j}$. K_p is local friction coefficient of accident p. The Reynolds number, in this work, is defined as

$$Re = \frac{8\rho\bar{V}^2}{\tau_{wd}} \tag{17}$$

where τ_{wd} is the shear stress at the wall of the fully-developed part.

3. Results

3.1 Abrupt contraction

3.2 Power-law fluid

The results for the streamlines near the corner of the abrupt contraction are shown in Fig.2. We show three fluids, a shear-thinning fluid with power-law exponent $n = 0.5$, a Newtonian fluid, and a shear-thickening one with $n = 1.5$. Special attention is given to the size of the recirculation. We can notice the expected result that as the Reynolds number increases, the size of the recirculation decreases. This happens because as inertia becomes more important the effect of the presence of the abrupt contraction upstream to its position becomes weaker. The effect of the power-law exponent can be explained as in Thompson et al. (1999) by the following. As n increases, the viscosity of the fluid increases for the same shear rate. Therefore, the fluid resists more a high shear rate. For small recirculations, we have high shear rates, since the fluid goes from the bigger tube to the smaller one in a thinner space. Hence, if the fluid resists more to high shear rates, the fluid starts its path to the smaller tube before, anticipating the point where the fluid near the wall detaches from it, increasing the size of the recirculation and decreasing the average change on velocity per unit space.

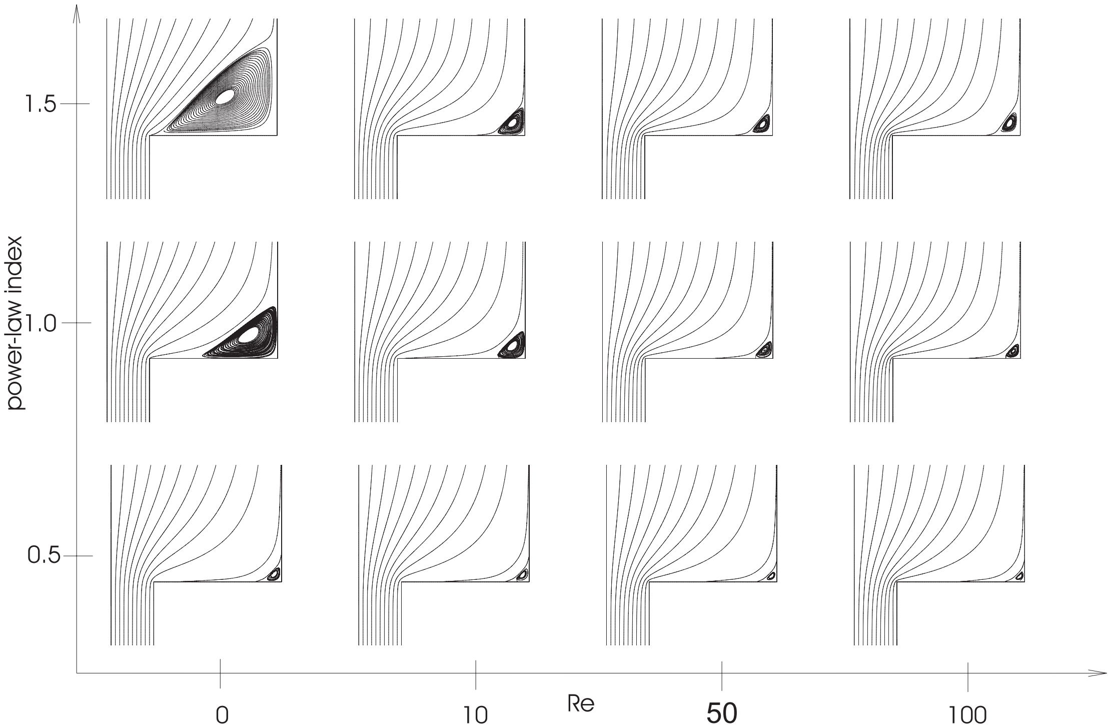

Figure 2. Streamlines near the corner of the $4:1$ abrupt contraction as a function of the Reynolds number for a shear-thinninf fluid ($n = 0.5$), a Newtonian one, and a shear-thickening one ($n = 1.5$).

The local friction coefficient as a function of the Reynolds number, for different power-law fluids is depicted in Fig.3. These curves have a very interesting non-monotonic behavior. For low values of Re, a more shear-thinning fluid has a higher K, while for high values of Re the opposite occurs. Interestingly, it seems that there is a single point where these curves change from one relative position to the other. Said differently, there is a Reynolds number which corresponds to a value of a local friction coefficient which is independent of the power-law index. An identical phenomenon was reported by Pinho et al. (2003) for the case of shear-thinning fluids in an abrupt expansion. Interestingly, the same two-parameter Eq.(2) was used with excellent curve fittings for the different power-law exponents including shear-thinning and shear-thickening branches. Table **??** shows the values for K_1 and K_∞ used for the power-law fluids in Fig.3.

As in the entrance flow, we obtain equations for K_1 and K_∞ as functions of the power-law index from the data of Table **??**. Figure 4 shows how these functions fit the K_1 and K_∞ data. The master curve obtained from Eq.(2) is given by

$$K_n^{AC} = \frac{18.03 - 40.77\log(\mathrm{n})}{Re} + 0.06 + 0.24n \qquad (18)$$

The numerical results are, at this time, fitted using Eq.(18) and the new curve fittings are shown in Fig.**??**. The agreement is, again, very satisfactory.

3.2.1 Viscoplastic material

Figure 5 shows the local friction coefficient K as a function of the Reynolds number for a Bingham material for different levels of the dimensionless yield-stress, τ_0'. We can see that for low values of Re the different viscoplastic fluids have a very similar behavior, following a linear tendency on a log-log graph. However, as the Reynolds number is increased, the curves tend to detach from each other with the Newtonian fluid having lower values. Comparing this behavior with the power-law ones, we can see that there is no change of behavior at a critical value and the similarity with shear-thinning behavior can only be advocated for values of Re below the critical power-law value. The curve fittings of these curves are also shown in Fig.5, using the same Eq.(2) as in the previous cases. Table **??** shows the different K_1 and K_∞ for different Bingham materials in the $4:1$ abrupt contraction. The agreement is incontestable.

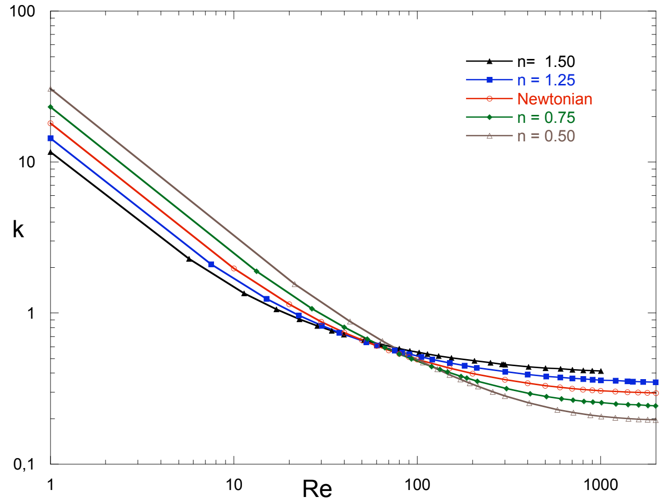

Figure 3. Local friction coefficient of the $4:1$ abrupt contraction as a function of the Reynolds number, for different power-law fluids wtih $n = 0.5$, 0.75, 1, 1.25 and 1.5. Points are obtained from numerical data while curves come from a curve fitting using the two Ks method.

4. Final remarks

The present work have analyzed the local friction losses for two problems, the entrance flow from an uniform profile and a $4:1$ abrupt contraction, for power-law fluids and Bingham materials. The generalized Reynolds number employed was such that in the fully-developed region $fRe = 64$.

In the case of the entrance flow, there was a similar behavior between power-law shear-thinning fluids and viscoplastic materials. There are examples of other phenomena where these two class of materials behave similarly.

On the other hand, for the abrupt contraction, the behavior of the two fluids was qualitatively different. Besides that, an interesting change of relative position between the K-curves happens at a critical value of Reynolds number. Below this critical value, the more shear-thickening is the fluid, the less is the value of K. Above Re_c, the opposite happens, a more shear-thinning corresponds to lower values of K. One possible reason for this kind of behavior could be related to growth of the recirculation size. When this parameter is higher, there is a competition between the extra energy consumption to maintain the recirculation and the save of energy consumption due to creation of smoother paths for the fluid material. This possibility needs further investigation.

We have presented values for K_1 and K_∞ proposed by Hooper (1981) for the four cases: entrance flow with power-law fluids, entrance flow with Bingham materials, abrupt contraction with power-law fluids, and abrupt contraction with Bingham materials. The curve fittings were excellent, showing that this two parameter equation can also be used in laminar flows of different non-Newtonian viscosity functions. This result shows that is a good approximation to consider

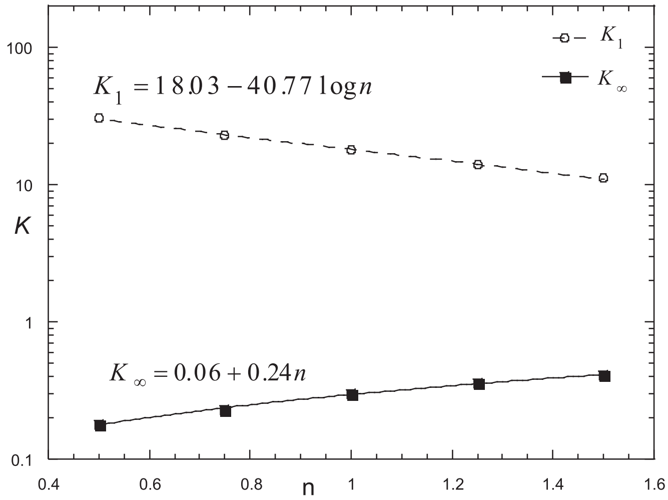

Figure 4. Curve fitting for the values of K_1 and K_∞ as function of the power-law index for the $4:1$ abrupt contraction.

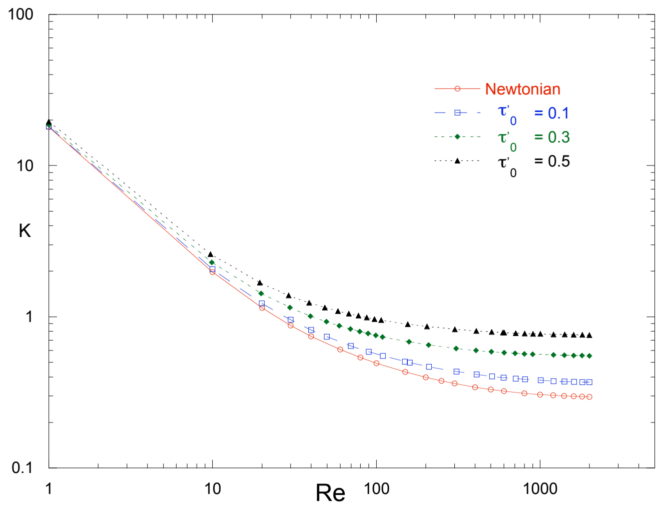

Figure 5. Curve fitting for the values of K_1 and K_∞ as function of the dimensionless yield stress for the $4:1$ abrupt contraction.

that K is a linear function of the inverse of the Reynolds number.

An important result was the compilation of the data obtained for K_1 and K_∞ in single equation as a function of the Reynolds number and rheological parameter under consideration. In other words we found four functions for the entrance flow from an uniform profile (ENT) and the $4:1$ abrupt contraction (AC) for power-law fluids and Bingham materials (with Papanastasiou's approximation) as functions of n and τ_0', respectively. These functions are

$$K_n^{ENT}(Re,n) = \frac{3.11\exp(3.26n)}{Re} + 0.52n + 0.09 \tag{19}$$

$$K_{\tau_0'}^{ENT}(Re,\tau_0') = \frac{75.48 - 92.22\tau_0'}{Re} + 0.66\tau_0' + 0.63 \tag{20}$$

$$K_n^{AC}(Re,n) = \frac{18.03 - 40.77\mathrm{log(n)}}{Re} + 0.24n + 0.06 \tag{21}$$

$$K_{\tau_0'}^{AC}(Re,\tau_0') = \frac{17.70 - 2.36\tau_0'}{Re} + 0.92\tau_0' + 0.28 \tag{22}$$

These equations show that the sensibility of the friction coefficient to the Reynolds number is linear with its inverse irrespectively of the two materials and the two geometries. Concerning the rheological dimensionless parameter, we found that, for high values of Re (K_∞), the friction coefficient is linear with n for a power law fluid and is linear with τ_0' for Bingham fluids. For low Reynolds numbers, the power-law behavior is exponential with n in the entrance flow and obeys a logarithm function in the abrupt contraction. For Bingham materials, a linear behavior is found for low values of Re also, for both geometries. The equations used presented an excellent curve fitting to the numerical data obtained and few parameters to adjust. This result gives confidence to its use for real processes in the petroleum industry.

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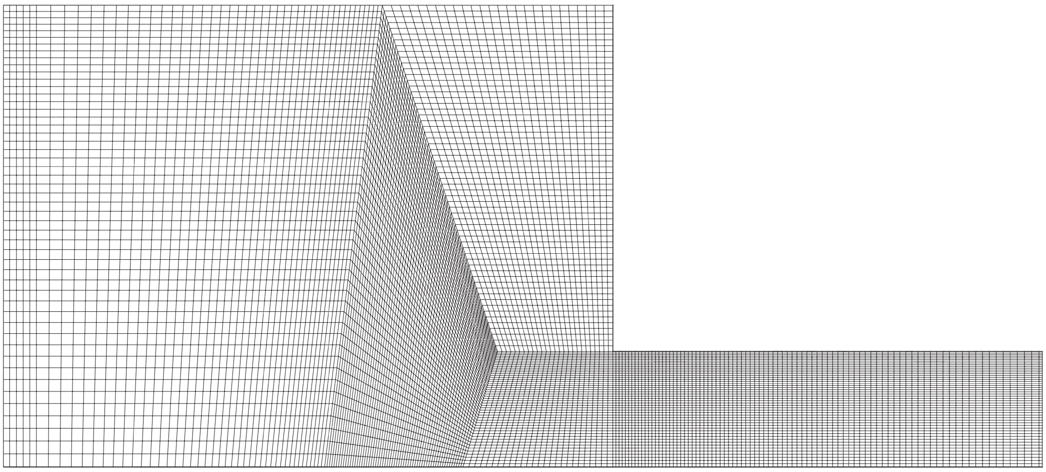

Figure 6. Grid for the 4:1 abrupt contraction problem.