HIBRID RANS-LES MODELING WITH SST TURBULENCE MODEL APPLIED TO INTERNAL FLOWS

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Abstract. *This paper present the evaluation of the performance of hybrid technique for turbulence modeling, called Detached-Eddy Simulation (DES), in enclosures flows. Specifically in three-dimensional lid driven cavities. These methodology is a blend of RANS and LES techniques, where the treatment of boundary layer and low turbulence regions has been done using RANS and treatment of the remaining flow has been performed with LES. In DES methodology the base is a RANS turbulence model, which in the region of RANS turbulent length scale is bigger than LES turbulent length scale, switches and behaves as Smagorinsky sub-grid scale model. The three-dimensional flow formed in lid drive cavities has interesting features that allow the proper assessment of a numerical code or turbulence models, such as the formation of several structures and a large basis of experimental and numerical data. The results correspond to flow inside cubical cavities at Reynolds number* 10^4*, in developed turbulence regime. Mean fields and statistical fluctuations of properties flow are presented, as well as, particular characteristics of hybrid model*

Keywords*: turbulent flow, hybrid modeling, SST model.*

1. INTRODUCTION

The most of fluid dynamics problems in environment and industries are related with turbulent flow, moderated and high Reynolds numbers, in complex geometries. For make modifications or solve any problems associated them it must understand the flow process. For this, it is important to know mean statistic, or instantaneous and both results. Thus to use turbulent model is inevitable for both accuracy and numeric convergence results.

The most of problems is impossible to do Direct Numeric Simulation (DNS), so the methodologies RANS (Reynolds Average Navier-Stokes Equations) and LES (Large-Eddy Simulation), have been used. RANS is a good for several problems, mainly in problems using high Reynolds numbers. Due to characteristic as: do not require numeric schemes of high order and very refined mesh, so it shows a lower computational cost than LES. The necessity of fine solutions in flow, which allows computing dynamics, effects of high and low scales. In order to solve problems in industries application was used LES as a natural way. However due to require high mesh resolution in the boundary layer region, so LES is very expensive to industries application and your use is impossible in other application, as estimated by Piomelli and Balaras (2002).

Thus the physical and numerical challenges assembly to numeric solution of turbulent flow with the boundary layer detachment for high Reynolds numbers. Spalart et al. (1997) conceptualize and show a turbulence methodology DES (Detached-Eddy Simulation). This methodology is a result of combination the two methodologies: RANS and LES. The boundary layer and low intensity regions are studies using RANS and other regions of flow is used LES. The DES methodology uses the RANS model as base for the scale turbulence length, which is greater than LES (it's depends on characteristic length mesh) blend and act as a sub-grid model.

The study this methodology is constantly motivated by necessity of become generic, it is shown the follow by chronologic evolution of the methodology development DES.

1997, DES conception based in SA model (Spalart and Allmaras);
1999, Empirical Constant calibration by SA (hybrid based in SA);
2000, DES and calibration, basing on SST model (Shear Strees Transport);
2003, SAS development and presentation (Scale-Adaptative Simulation);
2006, DDES development and presentation (Delayed DES).

Naturally the advance associated by this new technique about turbulence methodology is constant and become universal, in other words, it is able to represents the any kinds of problems. Following is shown the more details about SST model and the DES hybrid methodology.

2. MATHEMATICAL MODEL

The mathematic model that represents the dynamic of fluids newtonian, incompressible and isothermic are the Navier-Stokes equations, as detailed at Bird *et al*. 2002. Certainly, this equations are sufficient for solver turbulent flows, in context of methodology of Direct Numerical Simulations (DNS), however because the large number of equations to be solved simultaneously and the computational capability is impossible to solver engineering problems.

Thus, we have as alternative the solution of part of the flow, using Navier-Stokes equations, and modeling the other part. Using the concept of filtered global equations for turbulence, Silveira Neto *et al.* (2002), represent the methodologies RANS and LES with only a set of equations, which depends of adopted methodology this set represent the Reynolds means equations, that the complement are the fluctuations, or filtered equations, that represent the large scales and the complement are the sub grid scales.

This way, the Reynolds tensor, τ_{ij} (which appear in global filtered process), models the energy transfer between the mean field and theirs fluctuations for RANS or the energy transfer between the large and small eddies for LES. Agreement of Boussinesq hypothesis the tensor, τ_{ij} is associated with deformable rate, turbulent kinetic energy and turbulent viscosity, ν_t, been this last available through of a model.

2.1. SST Model

The model SST (*Shear Stress Transport*) purposed by Menter (1992) joint the best characteristics of classic model to two equations $\kappa-\varepsilon$ (Launder e Sharmam 1974) and $\kappa-\omega$ (Wilcox, 1988), with aim the model the places of boundary layer using $\kappa-\omega$ and the place farther of boundary layer using $\kappa-\varepsilon$. Combine this models RANS simulations is performed through of the equations for turbulent kinetic energy, κ, and modified specific dissipation, ω, and is sum the blend term, as shown in Eqs. (1) and (2):

$$\frac{\partial k}{\partial t}+\frac{\partial u_i k}{\partial x_i}=\frac{P_k}{\rho}-\beta^*\omega k+\frac{\partial}{\partial x_i}\left[\left(\nu+\sigma_k\nu_t\right)\frac{\partial k}{\partial x_i}\right], \tag{1}$$

$$\frac{\partial \omega}{\partial t}+\frac{\partial u_i \omega}{\partial x_i}=\alpha\frac{P_k}{\nu_t}-\beta\omega^2+\frac{\partial}{\partial x_i}\left[\left(\nu+\sigma_\omega\nu_t\right)\frac{\partial \omega}{\partial x_i}\right]+2\left(1-F_1\right)\sigma_{\omega 2}\frac{1}{\omega}\frac{\partial k}{\partial x_i}\frac{\partial \omega}{\partial x_i}, \tag{2}$$

where u_i is velocity fiels and (α,β,σ) are constants available from constants of $\kappa-\omega$ and $\kappa-\varepsilon$ models, agreement the expression $\phi=F_1\phi_{\kappa\omega}+(1-F_1)\phi_{\kappa\varepsilon}$ and ν is molecular viscosity. The blend function F_1, that appear in the last term of Eq. (2), control the actuation modes of equations and assumes values between 1, $\kappa-\omega$ model acts, and 0, $\kappa-\varepsilon$ model acts. And is expressed by:

$$F_1=\tanh\left\{\left\{\min\left(\max\left(\frac{\sqrt{k}}{\beta^*\omega y};\frac{500\nu}{y^2\omega}\right);\frac{4\rho\sigma_{\omega 2}k}{CD_{k\omega}y^2}\right)\right\}^4\right\}, \tag{3}$$

where $CD_{k\omega}=\max\left(2\rho\sigma_{\omega 2}\frac{1}{\omega}\frac{\partial k}{\partial x_i}\frac{\partial \omega}{\partial x_i},10^{-10}\right)$ and y is the minor distance to near wall. Finally the production term must be limited by $\tilde{P}_k=\min(P_k,10\varepsilon)$, where P_k defined as:

$$P_k=\nu_t\frac{\partial u_i}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right). \tag{4}$$

The turbulent viscosity is available as the expression:

$$\nu_t=\frac{a_1 k}{\max\left(a_1\omega,SF_2\right)}, \tag{5}$$

where α_1 is a Constant and S is deformations rates. A second function F_2 is defined for blend the values of ν_t in regions of free shear flows.

$$F_2=\tanh\left(\arg_2{}^2\right),\qquad \arg_2=\max\left(\frac{2\sqrt{k}}{\beta^*\omega y};\frac{500\nu}{y^2\omega}\right) \tag{6}$$

2.2. Hybrid Methodology RANS-LES

The modifications that permit the combination between SST-RANS and LES methodology (Smagorinsky) is given by destruction term, $\beta^{*}\kappa\omega$, of transport equation of turbulent kinetic energy. The destruction term has a structure of the destruction term of RANS model, Strelets (2001), and a turbulent length scale for DES (Menter et al., 2003 e Menter e Kuntz, 2004), F_{DES} as shown in expression:

$$\frac{\partial k_i}{\partial t}+\frac{\partial \bar{u}_j k}{\partial x_j}=P_k-\beta^{*}k\omega F_{DES}+\frac{\partial}{\partial x_j}\left[\left(\nu+\sigma_k\nu_t\right)\frac{\partial k}{\partial x_j}\right]. \tag{7}$$

where,

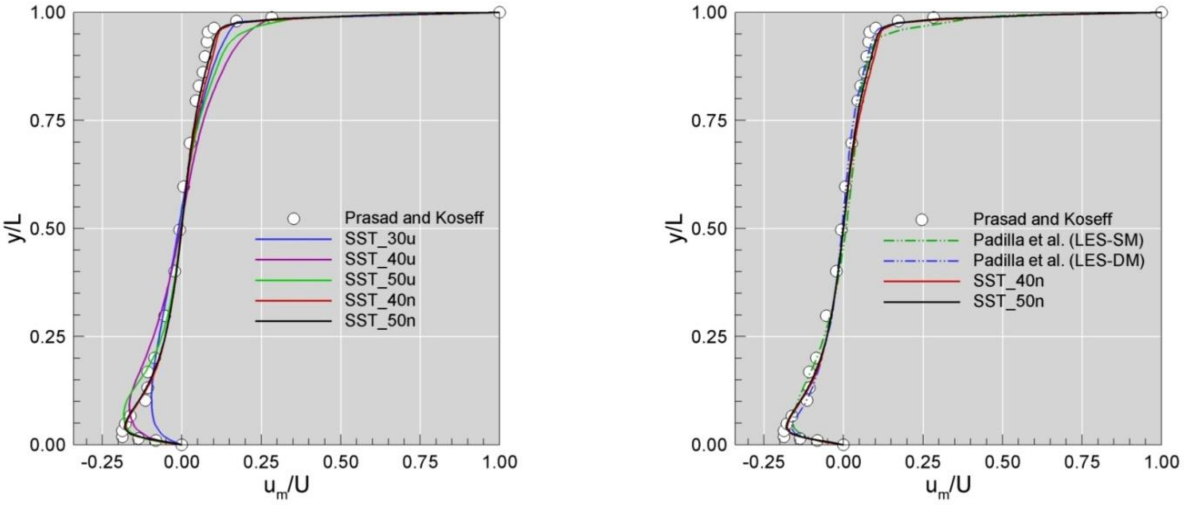
$$F_{DES}=\max\left\{\frac{L_{k-\omega}}{C_{DES}\,\Delta}\left(1-F_s\right),1\right\},\quad F_s=0,F_1;F_2\quad,\ \Delta=\max\left\{\Delta x,\Delta y,\Delta z\right\}\quad, \tag{8}$$

The constant $C_{DES}=0.78$ is set for the model and Δ is the grid characteristic length (place choice as the maximal for the formulation given the RANS mode into the boundary layer). In eq. (7), F_{DES} blend the behavior of modeling RANS and LES for comparison between the turbulent eddies lengths, $L_{\kappa-\omega}$ (eddies for SST-RANS) and $C_{DES}\Delta$ (eddies for LES). The term $(1-F_S)$ were introduced for avoid the detachment problem, because the mesh, due to activate LES mode (Menter e Kunz, 2004), for refinement of the mesh near the walls.

We use for implementations of DES modeling a numerical code developing at finites volumes with second order interpolations schemes and staggered grids (Padilla *et al.*, 2005).

3. RESULTS

The flow formed into 3-dimensional lid driven cavity have interesting characteristics that permit assess the numerical code and turbulent models, for example, formation of several vortex eddies (Migeon et al., 2003) and a good experimental and numerical data base (Prasad and Kossef, 1989; Deshpande and Milton, 1998; Padilla *et al.* 2005). In present study we use a set of cubic cavity of length L, with lid velocity equal U and Reynolds number $Re=UL/\nu=10^{4}$, considered development turbulence regime.

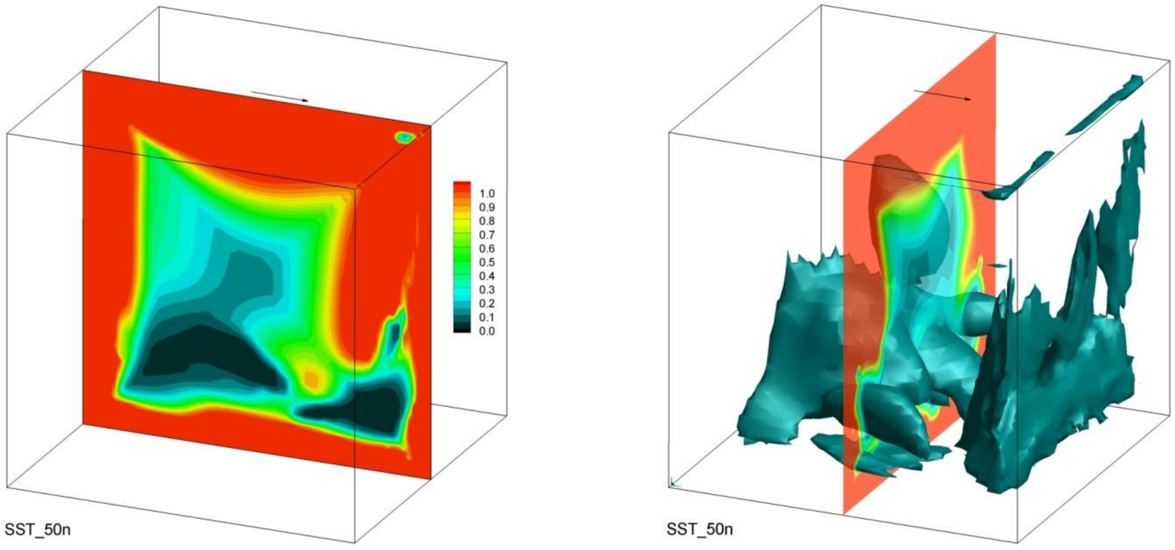


(a) (b)

Figure 1. Mean velocity profiles over mean plane in *y* direction at *x/L*=0.5; (a) refinement mesh effect, (b) comparison with other authors.

Initially, we performed a group of simulations using URANS methodology, with SST model, employing uniform and non-uniform grids, in the last case, the grid is refined near to wall with 5% ratio, this can be observed in Fig. 1. In this figure, it present non-dimensional horizontal velocity profiles u/U, at plane xy located at z/L=0.5, this plane is named mean plane in vertical direction. The influence of mesh refinement, where uniform and non-uniform meshes were used present higher differences, as shown in Fig. 1(a); Note the distributions of non-uniform meshes SST_40n and

SST_50n, due the best field solution near of wall. We observed the minimal diference between the refinement solutions (near of lid cavity) and when compare with experimental references, Prasad e Koseff (1989), and numerical, Padilla et al. (2005), this solutions present good agreement. The numerical references using LES methodology with dynamic model, LES-DM, and Smagorinsky, LES-SM, Fig. 1(b).



(a) (b)

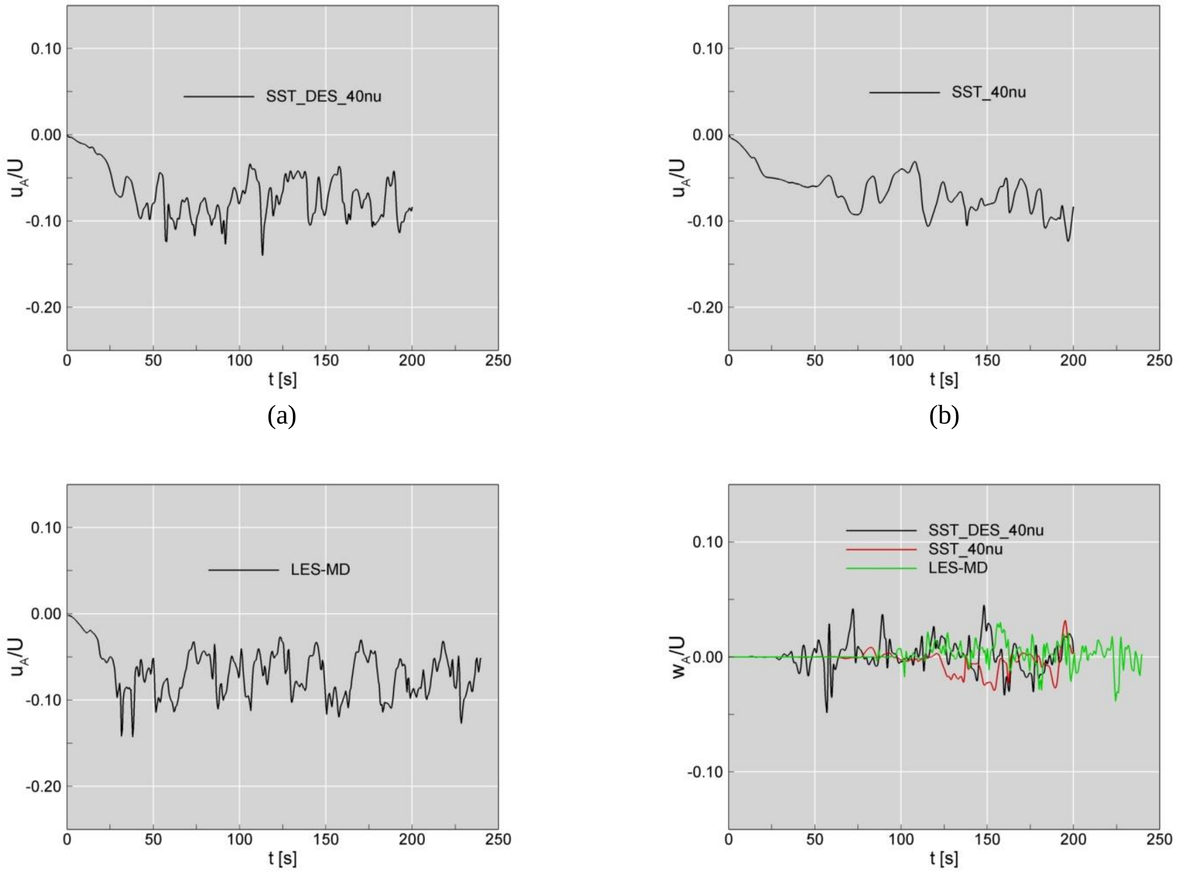
Figure 2. Field of blend function F_1; (a) mean plane, (b) mean transverse plane and iso-surface equal to 0.1.

The SST-URANS model has a associated characteristics to field k and ω, that determines the turbulent viscosity field. The maximal viscosity decrease in function of mesh refinement, e.g., the set cases SST_30u, SST40_u, SST_50u have maximum values, approximately, 68, 30 and 13 times the molecular viscosity. Other important characteristic is the region that act the models k- ω and k-ε, Fig. (2). This region is determined by blend function field F1. At region where $F_1=1.0$ (red) act k- ω model and the places, between 0 and 1, both models act, e.g. in Fig. (2b) is the delimited by iso- surperface $F_1=0.1$, where 10 % of model k- ω act and 90% k-ε model act.

3.1. DES CHARACTERISTICS

Based in Padilla et al., 2005, solutions with URANS and LES, the simulations of DES methodology is performed using mesh with 40^3 volumes, uniform and non-uniform.

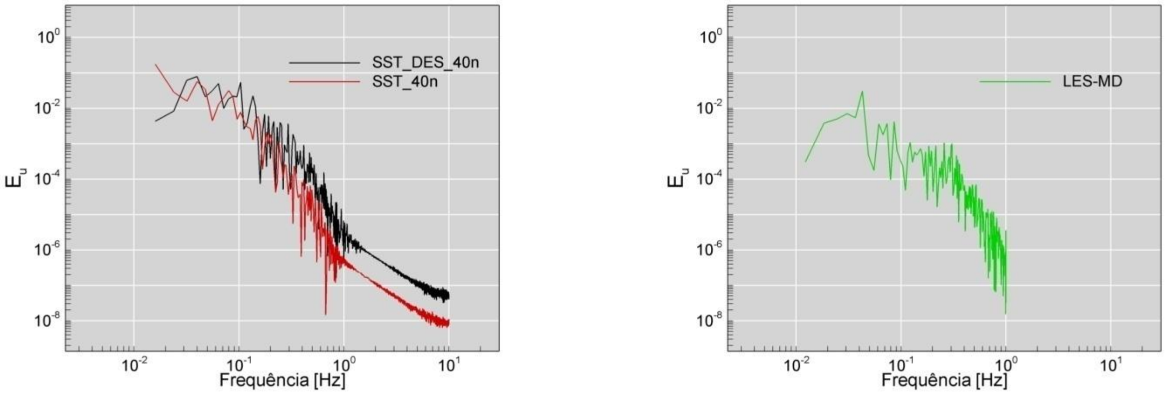
The temporal probes of two velocities components are shown in Fig. (3), comparing the solutions URANS and LES-MD. This probes are placed at point *A* located in $x, y, z/L = 0.5, 0.0076, 0.5$. We can see that frequency of oscillations with DES are less than LES-MD oscillations, however more than URANS. The amplitude is different among the methodologies too. The signal of transversal component velocity, *w/U*, with DES presents peaks higher than LES-MD, this is observed by Viswanathan and Tafty (2006) too.



(a) (b) (c) (d)

Figure 3. Temporal velocity evolution with DES, URANS and LES; (a), (b) e (c) horizontal component, (d) transversal component.

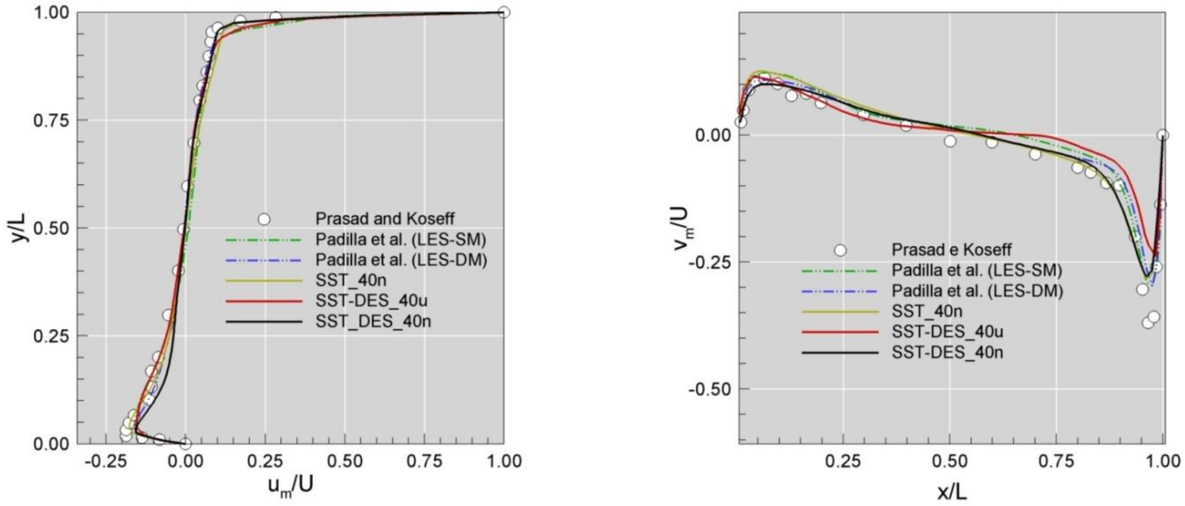
In Fig. 4 are shown the energy Power spectrum of velocity temporal evolution of the Fig. 3, in frequency domain. The energy level for DES solution is higher than URANS with SST model and presents a inertial range more large too. On the other hand, LES-MD solution has a larger inertial range than anothers solutions, demonstrating that solves more scales.



(a) (b)

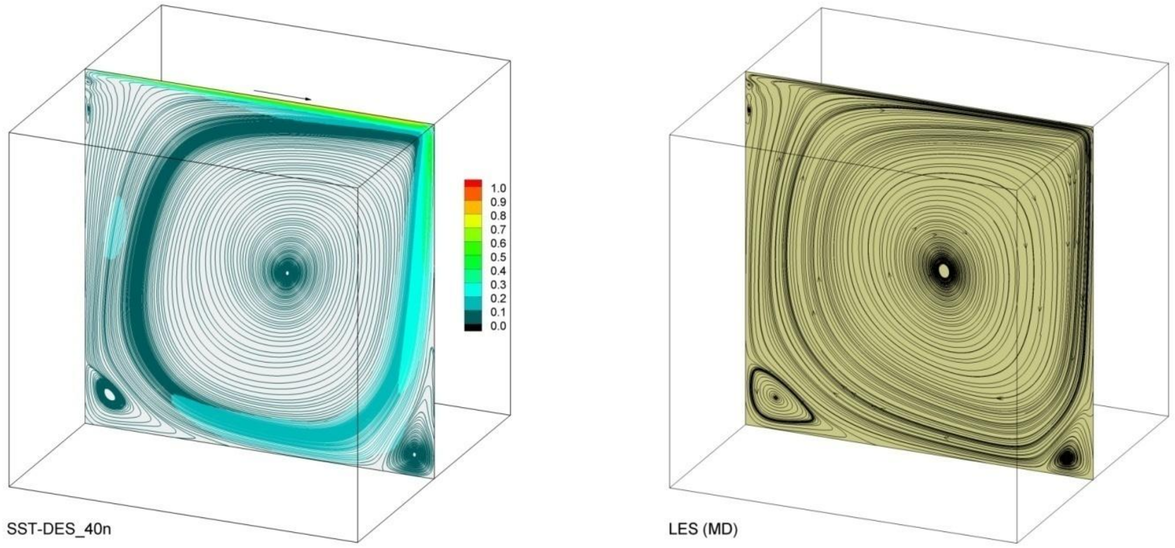
Figure 4. Energy power spectrum of the probes *u/U* of Fig. (3); (a) DES and URANS, (b) LES.

In Fig. 5 we have the comparison of horizontal and vertical mean velocity profiles with reference data and with set of SST_50n . The set SST-DES_40u shown deficiency, principally, in region located in posterior half of cavity shown in *v/U* profile, however present good agreement for *u/U* component when compared with Padilla *et al*. (2005). The set SST-DES_40n show a similar distribution to LES-MD methodology. The images of Fig. 6 ratify that the mean velocity field of set SST-DES_40n and set LES-MD are similar.



(a) (b)

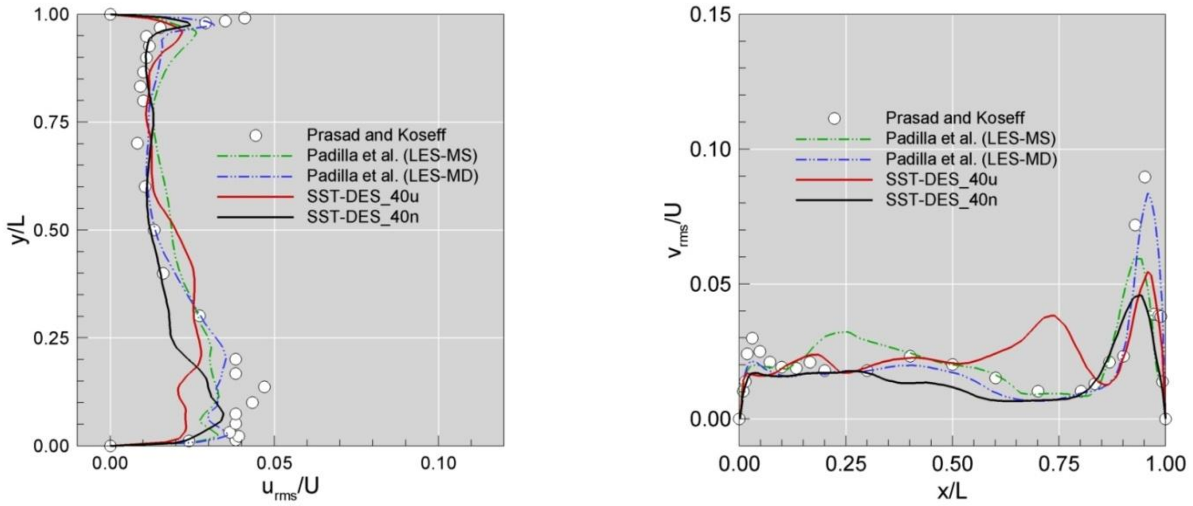
Figure 5. Mean velocity profiles over mean plane; (a) *y* direction in *x/L=0.5* and (b) *x* direction in *y/L=0.5*.



(a) (b)

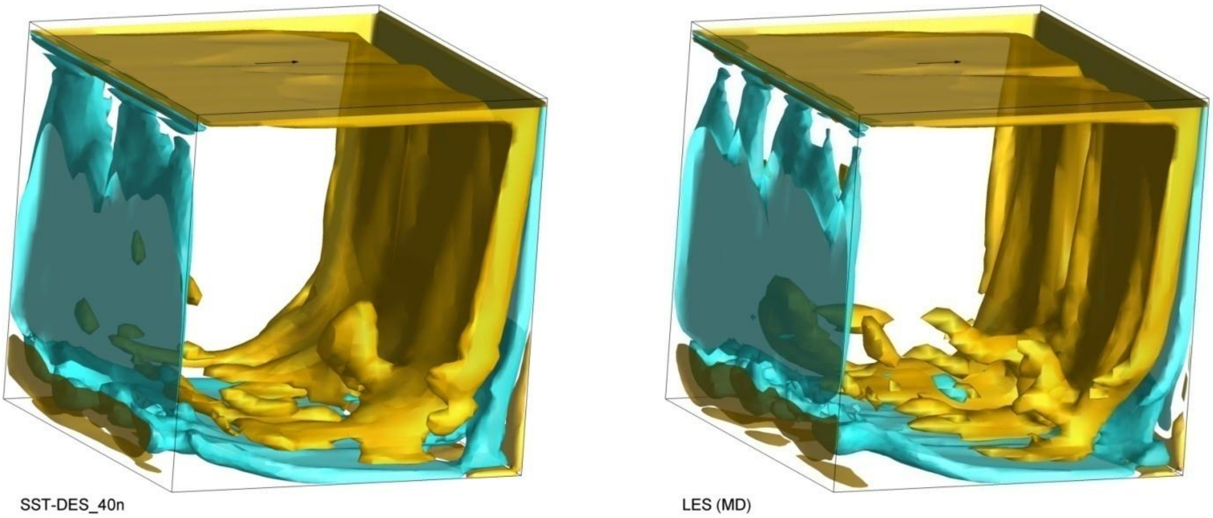
Figure 6. Mean flow at mean plane for methodologies: (a) DES, (b) LES.

The statistics fluctuations (Fig. 7) are associated with two velocity components analysed and permit to make a comparison of dynamic characteristics flow. Considering that field fluctuations of sets SST-DES are the same order as LES fluctuations. However the set SST-DES well-behaved in mean field velocity, in turbulent intensity profiles present lower results than LES and experimental results near the posterior and inferior walls, but not near the upper wall (Fig. 7a), where the comparison has good agreement. Maybe this results can be better using refinement meshes. The representation of characteristics eddies in mean plane is better when use the refined mesh near the walls and depends of turbulence model too.



(a) (b)

Figure 7. Turbulent intensity profiles over mean plane; (a) *y* direction at *x/L=0.5* and (b) *x* direction at *y/L=0.5.*



(a) (b)

Figure 8. Padrão do escoamento médio para as metodologias: (a) DES, (b) LES.

The important characteristic of flow dynamic of lid driven cavity is the Taylor-Gortler, which are structures formed between the posterior and inferior walls, that in turbulent present fluctuations and separated. The hybrid methodology SST-DES show this kind of eddies with good quality, Fig. 8, being that the physical characteristics of captured structures are intermediate between LES-MD and LES-MS.

4. CONCLUSIONS

Results of turbulent flows simulations in cubic lid driven cavitys using URANS and DES with SST turbulence model methodologies, are presented with the aim of assessing the qualities of the hybrid technique of turbulence model. The capability of reproduce the eddies dynamic present for SST-DES model is related to the activation mode LES, that happens when the turbulent length scale attached is lower than the turbulent length scale of URANS mode, allowing most turbulent effects is solved. This way, the comparision among several mean and instantaneous fields have good agreement, qualitatively and quantitatively, with results obtained using LES-MD methodology.

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