OBJECTIVE VORTEX IDENTIFICATION CRITERIA IN CHAOTIC AND TURBULENT FLOWS

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Abstract. *A mathematical definition of a vortex became an important issue in Fluid Mechanics specially after the recognition of the importance of vortical coherent structures on the turbulence dynamics. The birth, evolution, dissipation and death of a vortical coherent structure plays a crucial role on the understanding of turbulence as a phenomenon. The performance of a new set of vortex identification parameters are evaluated for the so called ABC flow (u,v,w)=(A sin z + C cos y, B sin x + A cos z, C sin y + B cos x), which is an example of a laminar Trkalian Beltramian flow exhibiting chaotic behavior. The LES results for the 3-D cavity in the turbulent regime is another flow investigated. Available criteria (such as Q, Delta, lambda 2, etc) selected from the literature are presented and compared with objective (frame indifferent) parameters, based on the non-alignment of the rate-of-strain tensor and its covariant convected time derivative evaluated at some relevant planes in the flow.*

Keywords: *Vortex, Coherent vortical structure, Frame invariance.*

1. INTRODUCTION

Vorticity and vortex form an interesting duality. Vorticity has a mathematical definition but its physical interpretation is not well understood. On the other hand, a vortex is recognized from subjective and intuitive basis and does not have a rational definition. Although the word vortex is frequently used when one wants to describe, understand, and explain flow patterns in fluid dynamic problems, the connection of this word to an entity which is unambiguously identified is still controversial. The mission of the fluid mechanicist which is involved in such identification is, therefore, to propose a mathematical definition of a concept which was constructed by its use, and not by a definition or a convention, during centuries of analysis of fluid mechanics problems.

Some intriguing questions that someone who is invited to investigate the subject asks are "Is a vortex an entity that should be defined by a kinematic or a dynamical criterion?". "Should a vortex be defined by Lagrangean or Eulerian quantities?". "The definition of a vortex should be Galilean invariant or objective? Should it have subjective thresholds or must be problem-independent?". "Should we seek for a definition looking at the velocity gradient only or do we have to observe acceleration gradients also?". Those questions encompass the opposite poles that are considered by the different authors who have investigated this matter and the diferences between their conclusion on how a vortex can be described.

In the study of fluid mechanics, its well known that a typical turbulent flow is dominated by compact regions, with the same roll-up time as the flow characteristic time scale, known as coherent structures. The vorticity dynamics equations governs its evolution, their interactions and the coupling with turbulence. A vortex definition should identify those structures, follow then from its formation since dissipation and classify according to the morphology. Despite the advances in understanding and modeling of phenomena related to turbulent flows in recent decades, some gaps continues without a full definition with respect to vortex representation. Due to the strong rotational characteristics of turbulence in fluids, the correct description of coherent areas is of great importance in the classification and capture the evolution of the flow and can provide some guidelines in the comprehension of turbulence itself.

2. OPPOSITION IN VORTEX IDENTIFICATION

Concerning the proposed criteria in literature, some opposite ideas with respect to the formulation of vortex identification criteria can be observed from different authors. Those ideas can help the understanding on the subject and

A first bi-polar strength present in the literature is the CAUSE X MANIFESTATION one. The approaches considered to identify a vortex can be, on one side, based on dynamics or force related quantities, entities related to the cause of the patterns of a flow. On the other side, the identification can be based on the manifestation, or the kinematics that is presented by the flow. One good example of this last statement is the identification of vortices as regions that presents circular streamlines. It is worth mentioning, however, that even when the Şcause" branch is used, the approach generally ends with some manifestation mathematic criterion.

A second opposition is the LAGRANGEAN X EULERIAN approaches. In fact, is not very clear in the literature if a vortex should be defined as a region in space which has certain instantaneous properties or a set of fluid particles that undergoes a particular trajectory in time. Although the Lagrangian branch can represent the "geometry" of the vortex, described in the topology of particle trajectories, some subjective variables as the total integration time and the time step in the integration can change vortex visualization.

Another unclear issue in the literature is related to kind of transformation of the frame of reference which leads the entities, necessary to the calculation of the considered criterion, invariant. Is the opposition GALILEAN X GENERAL FRAME invariance. There are authors that consider the criterion should be Galilean invariant while others are more restrictive and advocate the objectivity of the criterion, in other words, that the criterion should be invariant to any kind of rigid transformation.

The other controversial issue is related to the introduction of thresholds in the criterion. It is a SUBJECTIVE X OBJECTIVE bi-polarization. Although there are some advantages, associated to the flexibility of the criterion, since the threshold can depend on the problem considered, this flexibility implies a subjective criterion, i.e. depends on the value of the threshold inputed by the fluid mechanicist analyzing the problem.

3. CLASSICAL CRITERIA FOR VORTEX IDENTIFICATION

The classical criteria for vortex identification, known as the Q-criterion (Hunt et al., 1988), the Δ-criterion (Chong et al., 1990), and λ_2-criterion (Jeong and Hussain, 1995) are the most widely used criteria in the literature. A great number of works that concern vortex identification or propose to evaluate coherent structures in a given turbulent flow commonly apply those criteria.

3.1 Hunt et al. (1988) criterion

The criterion proposed by Hunt et al. (1988) is intrinsically related to a competition between vorticity and rate-of-strain where, in the case of a vortex, vorticity wins. Hunt et al. (1988) define a vortex as a connected region in space where

$$Q = \frac{1}{2}[\|\mathbf{W}\|^2 - \|\mathbf{D}\|^2] > 0 \tag{1}$$

where $\mathbf{W}$ and $\mathbf{D}$ are, respectively the skew-symmetric and symmetric parts of the velocity gradient and the operator $\| \; \|$ indicates the Euclidean norm of a tensor. Therefore, the competition between rate of rotation and rate of deformation is translated by the difference between the the Euclidean norm of each part, symmetric and skew-symmetric, of the velocity gradient. A vortex is identified where vorticity dominates the rate of deformation.

It is worth noticing that the Q-criterion is strictly related to the Vorticity number introduced by Truesdell (1953), defined as

$$N_k = \frac{\|\mathbf{W}\|}{\|\mathbf{D}\|} \tag{2}$$

and interpreted as a Şmeasure of the quality of the vorticity". Its possible to observe that $Q > 0$ is equivalent to $N_K > 1$. In summary, the Q-criterion is not generally frame indifferent, since it is dependent on the vorticity. It is an Eulerian approach. It does not give a clear picture of how it can be extended to compressible flows, one can keep the same difference or work with the new second invariant. It has a non-subjective definition.

3.2 Chong et al. (1990) criterion

A second criterion was formulated by Chong et al. (1990). This criterion is based on the fact that, when vorticity vanishes, the eigenvalues and eigenvectors of the velocity gradient are (the same as the rate-of-strain) real, since the velocity gradient, in this case is symmetric. If we gradually increase the vorticity, there is a threshole which is eventually achieved, where there will be a real and two complex conjugates eigenvalues. Therefore, the importance of vorticity changes the nature of the eigenvalues of the velocity gradient and produce a rotation like behavior. The so-called Δ-criterion is given by a region where

$$\Delta = \frac{III_{\mathbf{L}}^2}{2} + \frac{Q^3}{27} > 0 \tag{3}$$

where $III_{\mathbf{L}}$ is the third invariant (determinant) of the velocity gradient. The Δ-vortex is a larger region than a Q-vortex, since $Q > 0$ is equivalent to $\Delta > 0$. This also shows that, to produce complex eigenvalues, the vorticity intensity measured by its norm, may not overcome the rate-of-strain intensity with the same measurer.

3.3 Jeong and Hussain (1995) criterion

Another very important criterion in the literature was proposed by Jeong and Hussain (1995). This criterion is based on a pressure minimum at the vorticity plane. The gradient of the Navier-Stokes equation can be separated into a symmetric and skew-symmetric parts. The skew-symmetric part is related to the evolution of vorticity, while the symmetric part is connected to the evolution of the rate-of-strain. The symmetric part of the equation is given by

$$\frac{D}{Dt}(\mathbf{D}) - \nu\nabla^2\mathbf{D} + \mathbf{WW} + \mathbf{DD} = -\frac{1}{\rho}\mathbf{P} \tag{4}$$

where $\nabla^2\mathbf{D}$ is the Laplacian of $\mathbf{D}$ and $\mathbf{P}$ is the pressure Hessian. According to Jeong and Hussain (1995), the principle of minimum pressure, can be corrected by discarding the terms related to unsteadiness of the flow and to viscous forces, this condition is satisfied, for an incompressible Newtonian fluid, when

$$\lambda_2^{\mathbf{D^2}+\mathbf{W^2}} < 0 \tag{5}$$

where $\lambda_2^{\mathbf{D^2}+\mathbf{W^2}}$ is the intermediate eigenvalue of tensor $\mathbf{D^2} + \mathbf{W^2}$. It is interesting to notice that, although expressed by kinematic quantities, this criterion is based on dynamical arguments.

4. OTHER IMPORTANT CRITERIA

4.1 Tabor and Klapper (1994) criterion

Tabor and Klapper (1994) presented a systematic study on the stretching and alignment dynamics in general flows and came up with an interesting kinematic tensor very relevant to the present work. This tensor,Ω, measures the rate of rotation of the eigenvectors of $\mathbf{D}$, defined as

$$\Omega = \dot{e}_i^{\mathbf{D}} e_i^{\mathbf{D}} \tag{6}$$

where $e_i^{\mathbf{D}}$ is an eigenvectors of $\mathbf{D}$. They used, in that study, the Relative-rate-of-rotation tensor, $\bar{\mathbf{W}}$, the difference between the vorticity tensor and Ω. Therefore, a Q_s-criterion criterion can be constructed as

$$Q_s = \frac{1}{2}[\|\mathbf{W} - \Omega\|^2 - \|\mathbf{D}\|^2] > 0 \tag{7}$$

It is worth noticing that the relation between Q and the vorticity number, N_k is analog to the relation between Q_s and the Şstress-relieving" parameter, R_D, proposed by Astarita (1979), defined as

$$R_D = -\frac{tr\bar{\mathbf{W}}^2}{tr\mathbf{D}} \tag{8}$$

$Q_s > 0$ is equivalent to $R_D > 1$.

4.2 Kida and Miura (1998) criterion

The criterion proposed by Kida and Miura (1998) follows the same principle considered by Jeong and Hussain (1995). The difference lies on the fact that the pressure minimum is calculated at the plane defined by the eigenvector correspondent to the smallest eigenvalue of the pressure Hessian. The vortex core is defined as a region where the skew-symmetric part of the velocity gradient projected on this plane overcomes its symmetric part.

4.3 Zhou et al. (1999) criterion

The so-called λ_{ci}-criterion was introduced by Zhou et al. (1999). It is based on the Δ-criterion of Chong et al. (1990). When $\Delta > 0$, the velocity gradient has two complex eigenvalues $\lambda_{cr} + i\lambda_{ci}$. The imaginary part λ_{ci} is identified as the swirling strength of the vortex. The criterion consists of a $\lambda_{ci}^2 > \delta$, where δ is a threshold generally chosen as percentage of its maximum value. When $\delta = 0$, $\Delta > 0$ and $\lambda_{ci} > \delta$ are equivalent.

4.4 Haller (2005) criterion

Another criterion based on a non-local vortex definition was presented by Haller (2005). He considered a vortex as a set of fluid trajectories that avoid the so-called hyperbolic domain, a domain defined as a region in space where the fluid defies, in a certain sense, the trend suggested by the rate-of-strain. To define the hyperbolic domain Haller (2005) uses (half of) the second Rivlin-Ericksen tensor, $\mathbf{A}_2$, the covariant convected time derivative of the rate of deformation tensor, $\mathbf{A}_1 = 2\mathbf{D}$, defined as

$$\mathbf{A}_2 = \dot{\mathbf{A}}_1 + \mathbf{A}_1\mathbf{L} + \mathbf{L}^T\mathbf{A}_1 \tag{9}$$

where $\mathbf{L}$ is the transpose of the velocity gradient. For flows where the first invariant of $\mathbf{D}$ vanishes ($I_\mathbf{D} = 0$, isochoric flows) and the third invariant of $\mathbf{D}$ is a non-zero quantity ($III_D \neq 0$), he defines an elliptical cone on the basis of the eigenvectors of D, (e_1, e_2, and e_3) as

$$d\xi_3^2 = ad\xi_1^2 + (1+a)d\xi_2^2 \tag{10}$$

where $d\xi = d\xi_1 e_1 + d\xi_2 e_2 + d\xi_3 e_3$ is an infinitesimal vector and a is the ratio between the greatest and smallest eigenvalues of $\mathbf{D}$. The hyperbolic domain is a region in space where the second Rivlin-Ericksen tensor is positive definite in the elliptical cone defined by Eq.(10).

4.5 Chakraborty et al. (2005) criterion

Chakraborty et al. (2005) proposed a further step on the analysis of Zhou et al. (1999) by adding to the swirling strength criterion, an inverse spiraling compactness, measured by the ratio $\frac{\lambda_{cr}}{\lambda_{ci}}$. This ratio can be seen as local version of the non-local quantity introduced by Cucitore et al. (1999).

5. NEW SET OF VORTEX IDENTIFICATION CRITERIA

It is very common, in many physical and mathematical situations, the identification of the necessity to compare the diagonal components of a matrix with its off-diagonal ones. One simple idea is to measure this competition by an overall ratio index. A parameter which has in the numerator and the denominator, the intensities of one and other sides of this balance: diagonal and off-diagonal components of the strain acceleration tensor, evaluated in the strain basis. Here, we have developed two methods for an anisotropic comparison between the diagonal and off-diagonal components of a matrix. Following Haller (2005) and Thompson (2008) we use the matrix associated with the second Rivlin-Ericksen tensor, Eq.(9), the first method which will be called here line-method is to compare, in the diagonal components of the tensor $\mathbf{A}_2^{\mathbf{A}_1}$, acceleration tensor on the basis of the strain tensor, $\mathbf{L}$, the part of each component that comes from the diagonal and off-diagonal component of tensor $\mathbf{A}_2^{\mathbf{A}_1}$

$$AR_i^A = \frac{(A|_{ii})^2}{(A^2)|_{ii}} \tag{11}$$

where $\mathbf{A} = \mathbf{A}_2^{\mathbf{A}_1}$. An isotropic version was also formulated, based on the same idea provided in above relations

$$IR = \frac{A_{ii}A_{ii}}{[AA]_{jj}} \tag{12}$$

6. ABC FLOW

The ABC flow is a classical flow due to its chaotic behavior even for laminar flows (Dombre et al., 1986).

7. Lamb vector and helicity density

The local geometrically orthogonal decomposition of the velocity vector $\mathbf{v}$ with respect to the vorticity vector $\mathbf{w}$ introduces two quantities of crucial importance in vorticity dynamics: the vector $\mathbf{w} \times \mathbf{v}$, known as Lamb vector, and the scalar $\mathbf{w} \cdot \mathbf{v}$, known as helicity density. The two interesting non-trivial cases are when the helicity density or the Lamb vector vanishes. When $\mathbf{w} \cdot \mathbf{v} = 0$ and $\mathbf{w} \times \mathbf{v} \neq 0$, the flow is called *complex lamellar flow*. It exists if and only if

$$\mathbf{v} = \lambda \nabla \xi \tag{13}$$

where ξ = const are equi-potential surfaces orthogonal to the streamlines everywhere (potential flow, also called lamellar flow, is obtained when $\lambda = 1$). When $\mathbf{w} \cdot \mathbf{v} = 0$ and $\mathbf{w} \times \mathbf{v} = 0$ the streamlines are parallel to the vorticity lines, or

$$\mathbf{w} = \zeta \mathbf{v} \tag{14}$$

which implies that the velocity is an eigenvector of the curl operator. This kind of flow is called Beltrami (or helical) flow. If ζ is constant, the flow is specifically called Trkalian.

When the Lamb vector is a complex lamellar field or

$$\mathbf{w} \times \mathbf{v} = g\nabla h \tag{15}$$

there exist a set of surfaces h =const, called Lamb surfaces which are orthogonal to the Lamb vector everywhere. It can be shown that the existence of the Lamb surfaces imply the integrability of the system and therefore this kind of flow cannot be chaotic. Therefore, A Beltramian flow is a candidate of a chaotic flow. However, if $\nabla\zeta \neq 0$ the velocity is still integrable, since the velocity will be on the surfaces normal to $\nabla\zeta$. Therefore, the only possibility of an incompressible chaotic steady flow is when $\nabla\zeta = 0$, where the flow is Trkalian.

Arnold (1965), seeking steady inviscid chaotic flow, proposed a Trkalian flow where $\zeta = 1$ or $\mathbf{w} = \mathbf{v}$. The ABC flow in cartesian coordinates is given by

$$u = A\sin z + C\cos y \tag{16}$$

$$v = B\sin x + A\cos z \tag{17}$$

$$u = C\sin y + B\cos x \tag{18}$$

7.1 Results for the ABC flow

Figure 1 shows the values of the isotropic normalized ratio that compares linear acceleration deformation, in the sense provided by the covariant convected time derivative, to angular acceleration gradient (in the same sense). Higher values correspond to hyperbolic-like behavior.

Also all three fields of anisotropic index associated to a line-method are shown in Fig. 1. Since the orientation of the index is different depending on the point considered, we have decided to produce indexes based on the comparison between the three anisotropic indexes of each method. What is shown in the first row of the second column is related to the highest (among three) value of the tendency to evolve persistently the same material line. Figure 1 show the contours of the Q-criterion and Q_s-criterion.

All the criteria are normalized in order to obtain the same basis for comparison so if a certain region presents values below 0.5, this region remains in a vortical region, according to the criterion.

8. THE 3D CAVITY

8.1 LES

Large Eddy Simulation methodology is about filtering of the equations of movement and decomposition of the flow variables into a large scale (resolved) and a small scale (unresolved) parts. The filtering process is applied on the governing equations for separate the fields that contains the large and sub-grid scales. After performing the volume averaging, the filtered Navier-Stokes equations become

$$\frac{\partial\bar{\mathbf{U}}}{\partial t} + \nabla\cdot(\overline{\mathbf{U}\mathbf{U}}) = -\frac{1}{\rho_0}\nabla\bar{p} + \nu\nabla^2\bar{\mathbf{U}} + f_B \tag{19}$$

Developing the non-linear transport term and introducing the sub-grid scale (SGS) stresses $\tau = \overline{\mathbf{uu}} - \bar{\mathbf{U}}\bar{\mathbf{U}}$, the filtered Navier-Stokes equations can be rewritten as

$$\frac{\partial\bar{\mathbf{U}}}{\partial t} + \nabla\cdot(\bar{\mathbf{U}}\bar{\mathbf{U}}) = -\frac{1}{\rho_0}\nabla\bar{p} + \nu\nabla^2\bar{\mathbf{U}} - \nabla\cdot(\tau) + f_B \tag{20}$$

The dynamic sub-grid scale model was used with the Large Eddy Simulation to obtain the sub-grid scales. In this sub-grid model the proportionality coefficient is computed as a function of time and space. As a consequence, some difficulties on finding a correct constant value in heterogeneous meshes, as in the Smagorinskyt's model are avoided. The expression that defines the turbulent viscosity, μ_T can be written as

$$\mu_T = C\Delta^2\|\mathbf{D}\| \tag{21}$$

where C is the proportionality coefficient, calculated in ANSYS CFX along time and space as a function of the velocity fluctuations and $\|\mathbf{D}\|$ the rate of strain tensor and Δ is the length scale of the grid filter.

8.2 Description

The first experimental results for lid-driven cavity flows was published in the work of Koseff and Street (1984), showing the three-dimensionality of the problem. The main characteristic of this kind of flow is the secondary vortices observed in the upper corners and a primary one along the complete space. Results from Migeon el al. (2003) considered parallel unsteady, three dimensions lid-driven cavity and have shown the development of Taylor-like vortices. Recent works from Ku et al. (1987) and Babu and Korpela (1994) show the comparison between two and three-dimensional simulations and, agreeing with the work of Koseff and Street (1984), their results shown a great difference in the development of vortical structures in both cases. Direct numerical simulations approach were performed in the work of Leriche and Gavrilakis

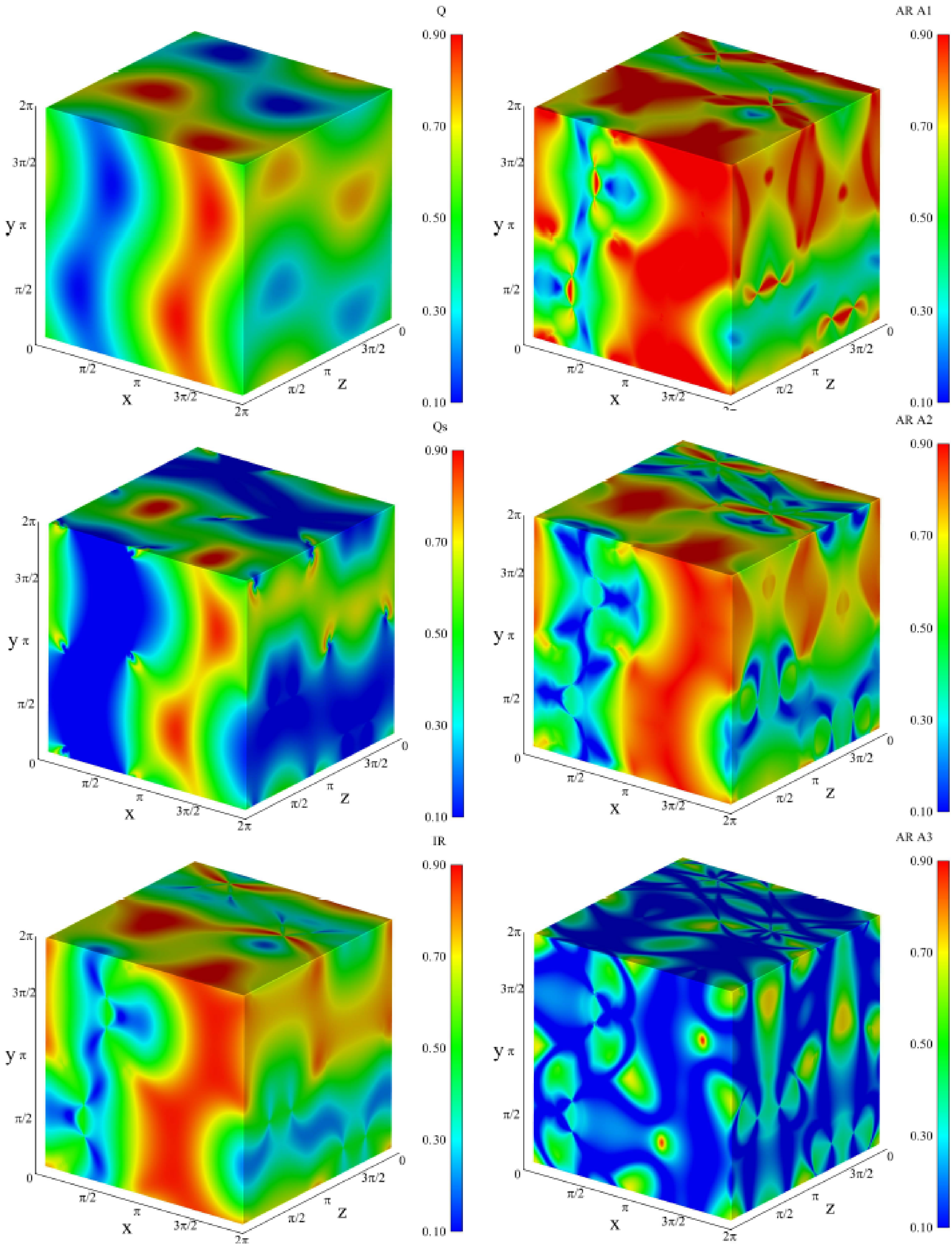

Figure 1. Vortex identification criteria evaluated in ABC flow.

(2000) and large-eddy simulation by Zang et al. (1993), Deshpande and Milton (1998), Hassan Barsamian (2001). These works compared the results with the experimental data from Koseff and Street (1984) and show statiscal characteristics and the evolution of coherent structures. Recent work from Padilla (2008) showed the power spectra and the streamlines for parallel and non-parallel cavities for Reynolds up to 3000. In this paper will be present a comparison between many vortical structures identification criteria in a parallel lid-vortex cavity for Reynolds number equal to 10000.

8.3 Results for the 3D cavity

Figure 2 shows the isosurfaces of 0.5 for all parameters evaluated in the previous case.

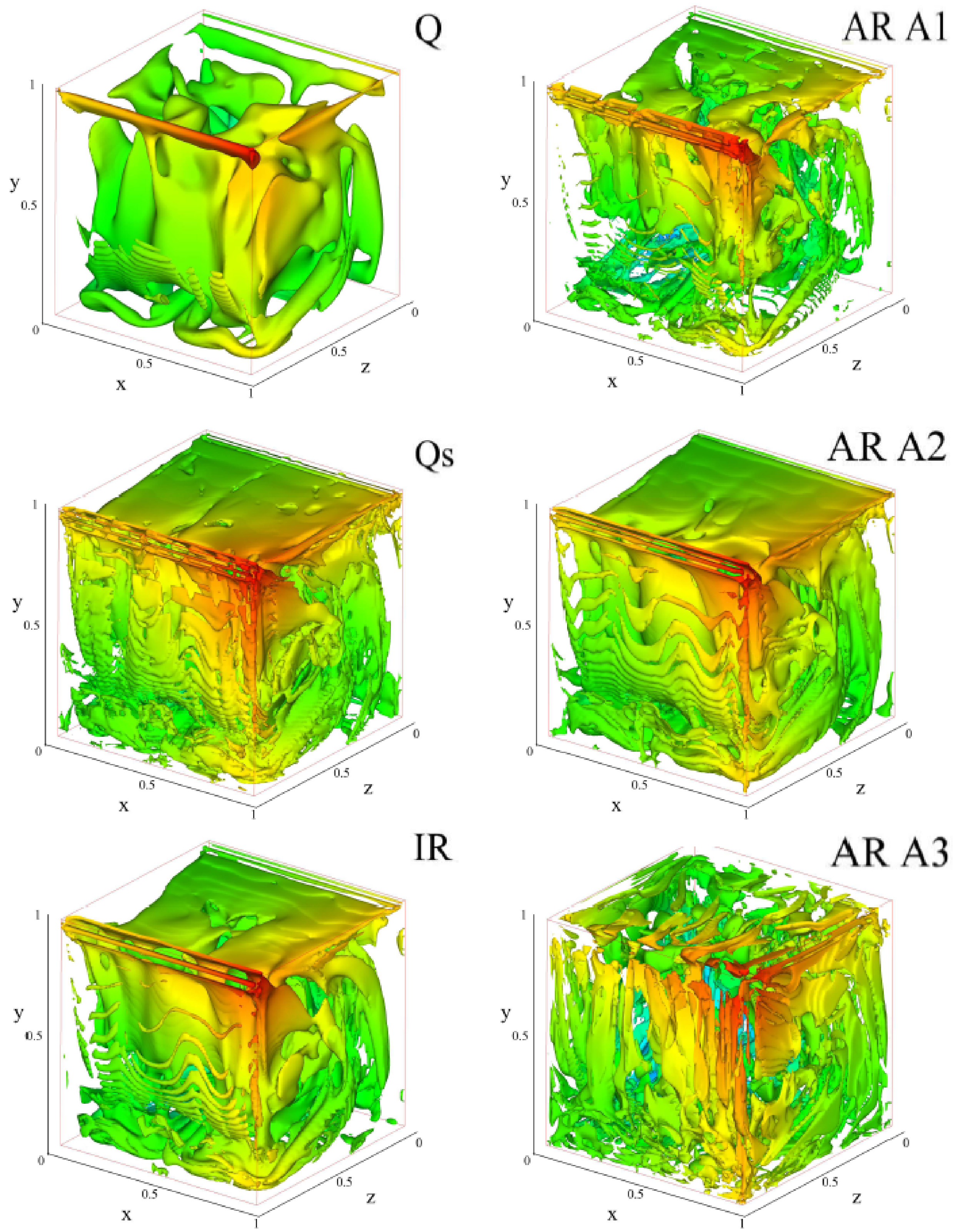

Figure 2. Vortex identification criteria evaluated in 3D turbulent cavity for Reynolds number equal to 10000.

9. FINAL REMARKS

We have presented a theoretical analysis to capture directional tendencies of stretching material elements. These directional quantities are able to delineate coherent structures that are present in turbulent flows. Besides that they are strongly related to flow-type classification criteria, giving an anisotropic version of previous criteria in the literature. The theoretical entities introduced are applied in a accompanied paper. We have presented also two application of the theory developed in an accompanied paper concerning flow classification. The general results are complex in nature and the full interpretation are in order.

10. ACKNOWLEDGEMENTS

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12. Responsibility notice