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MATHEMATICAL MODEL OF THE AMAZON STIRLING ENGINE

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Abstract. *The Excellency Group in Thermoelectric and Distributed Generation (NEST, for its acronym in Portuguese) at the Federal University of Itajubá, has designed a Stirling engine prototype to provide electricity to isolated regions of Brazil. The engine was designed to operate with residual biomass from timber process. This paper presents mathematical models of heat exchangers (hot, cold and regenerator) integrated into second order adiabatic models. The general model takes into account the pressure drop losses, hysteresis and internal losses. The results of power output, engine efficiency, optimal velocity of the exhaust gases and the influence of dead volume in engine efficiency are presented in this paper. The objective of this modeling is to propose improvements to the manufactured engine design.*

Keywords*: Stirling Engine, Mathematical Modeling, Biomass, Distributed Generation*

1. INTRODUCTION

Brazil has a great potential of biomass for use in electricity generation. Furthermore, much of its territory is covered by forests, which are habited or habitable regions, but face the problem of power shortages (Wilke and Silva,2004). This is the reason why an alternative such as the Stirling engine, which is characterized as being external combustion and capable to operate with almost any heat source, is presented as a good alternative for energy supply in these isolated regions.

This paper addresses the issue of modeling of Stirling Amazon engine designed by NEST to operate with biomass. The Stirling Amazon prototype is an alpha type engine with a generating capacity initially designed for 8 KW, composed of five basic parts in its construction: the hot heat exchanger, the cold heat exchanger, the regenerator, the expansion chamber and the compression chamber. The complete mathematical modeling developed was coded in FORTRAN and integrates models of heat exchangers, cold, hot and regenerator to an adiabatic model of second order.

Nomenclature		**Subscripts**	
As	Surface area for heat transfer	c	Compression
A_{mr}	Surface area of regenerative matrix	ck	Interfaces Compression-Cooling
A_{pe}	Cross-sectional area of the piston expansion	e	Expansion
At	Cross-sectional area	ee	Exhaust Input
Cm	Specific heat of material	eef	Water Input
C_{pg}	Specific heat of the working gas	eh	External horizontal
Dheh	External diameter horizontal hydraulic	en	Input
Dhi	Regenerator diameter wire	ev	External vertical
Dmst	Minimum length of the external surfaces of the tubes	fc	Exhaust
D	Diameter	ff	Water
ΔTml	Logarithmic med. temperature difference	h	Heating
f1	Friction factor for smooth pipes	h	Referring to the horizontal tubes
K	Thermal conductivity	he	Interfaces Heating-Expansion
L	Length of vertical pipes	ih	Internal horizontal
$\dot{m}$	Mass flow	iv	Internal vertical
M_{mr}	Mass mesh regenerative	k	Cooling
m_w	Number of mesh	kr	Interfaces Cooling - Regeneration
Pr	Prandtl number	r	Regeneration
P	Pressure of working gas	rh	Interfaces Regeneration-Heating
Q	Heat exchanged	s	Heat wall heat exchanger
Rt	Total thermal resistance	sal	Output
S	Perimeter of duct	se	Exhaust outlet
St	Number of Stanton	sef	Water output
T	Temperature	sf	Cold wall heat exchanger
U	Total coefficient of heat transfer	w	Wall

V_{mpe}	Half the piston velocity of expansion	v	Referring to the vertical tubes
Vr	Average velocity of working gas in the regenerator		
Λ	Longitude reduced		
ρ	Density		
ν	Kinematic Viscosity		
ξ	Friction factor		
ν_e	Kinematic viscosity of the exhaust		
ω	Engine Velocity		
W	Power		

2. MOTOR STIRLING AMAZON

The Stirling engine developed by NEST, is an alpha type engine originally designed to operate with air as a working fluid. The fuel is the residual biomass from the timber industry in Brazil (Paula, *et al*. 2006). This is based on an engine which uses the crank mechanism of a motorcycle manufactured by DUCATI with a cylinder capacity of 900 cc.

2.1Mathematical model of the Amazon Stirling engine

The complete mathematical model consists of the mathematical models of heat exchangers, hot, cold and regenerator integrated by the second order adiabatic model proposed by Urieli and Berchowitz (1984). See Figure 1.

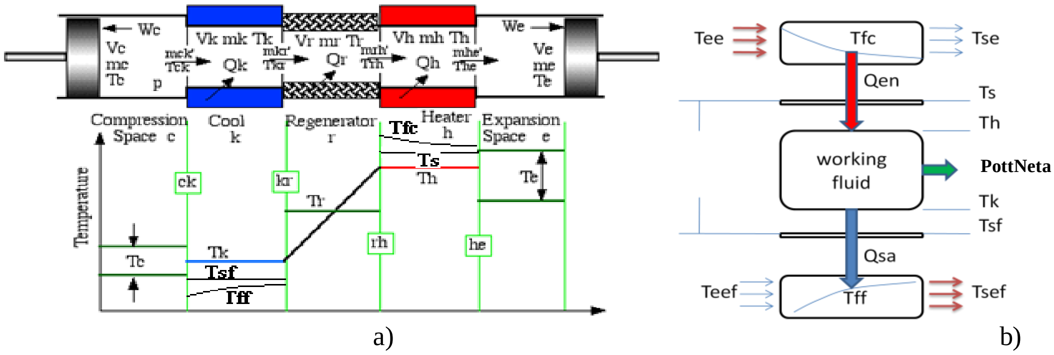

Figure 1. a) Schematic model of the engine (Urieli, 2008) and b) The temperature distribution

2.1.1 Mathematical model of hot heat exchangers

In the particular case of the Amazon engine, the hot heat exchanger has to exchange heat with the kiln exhaust gases; this gas is a biphasic fluid that generates waste deposits in the external surfaces of the exchanger. In this framework, it may not have extended surfaces (like fins) It needs a forced fan to increase the coefficient of heat transfer by convection and cannot get their tubes in a cross flow as it would present a relative large fall of temperature (Podesser, 1999). The fan also ensures the velocity of the exhaust and helps to overcome losses in heat exchangers of preheating the combustion air that entering in the furnace (figure 2).

Figure 2. System of Stirling engine integrated with the biomass furnace

With the ability to control the velocity of the exhaust, an energy and mass balance in the hot heat exchanger was performed. The outside of the heat exchanger is modeled in two parts, the first referred to the vertical tubes and the second to the horizontal tubes getting the total heat delivered to the working fluid inside the exchanger (figure 3).

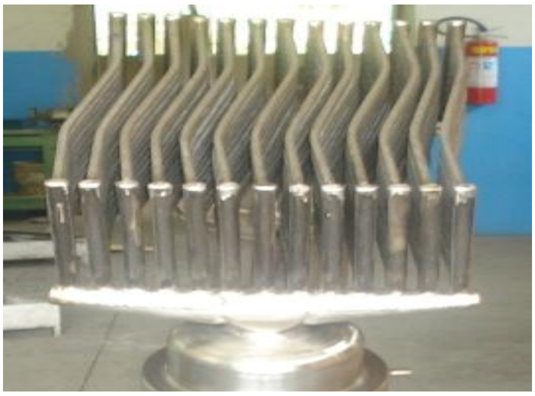

Figure 3. Hot heat exchanger

Heat exchange between the gases inside and outside in this device is governed by Newton's law of cooling in the fluids, and the Fourier Law in the tubes

$$\dot{Q} = U\ As\ \Delta Tml \tag{1}$$

The overall coefficient of heat transfer, U, is calculated with the coefficients of heat transfer by convection in the outer (h_{ev}, h_{eh}), interior(h_{iv}, h_{ih}), and thermal conductivity of the material of tubes.

$$U = (Uv + Uh)/2 \tag{2}$$

For the coefficient of heat transfer by convection (h_{ev}) in vertical tubes on the outside is necessary to seek the Nusselt number (Nu_{ev}) for a perpendicular flow to a bank of tubes using the following equation (Incropera and David, 1996).

$$Nu_{ev} = Fc\ C\ Re_{ev}^{m}\ Pr^{0.36}(\frac{Pr}{Pr_s})^{\frac{1}{4}} \tag{3}$$

For the case of the exchanger studied, the Nusselt number has to be multiplied by a correction factor, Fc, since the number of rows is less than 20.

The Reynolds number (Re_{ev}) is calculated with the maximum velocity (Vmax) that occurs between the outer surfaces of vertical tubes. Thus the coefficient of heat transfer by convection in vertical tubes is:

$$h_{ev} = \frac{Nu_v * K}{D_{ev}} \tag{4}$$

The calculate of the outside of horizontal tubes is done with the relationship of Nusselt number (Nu_{eh}) for parallel flow, where the Reynolds number (Re_{eh}) is calculated with the average velocity of the exhaust.

The horizontal tubes can be considered as flat walls. This is only valid when the boundary of the hydraulic layer does not exceed the half of the distance between the surfaces of adjacent tubes, if the overtaking happens, the behavior is as internal flow. Beyond these conditions, the analysis should be done in the case that the flow is laminar or turbulent, according to (Incropera and David, 1996).

After having the values of the coefficient of heat transfer by convection outside the exchanger, the analysis is made of the working gas passing through the interior of the exchanger.

For the heat transfer coefficient by convection inside the vertical tubes(h_{iv}), a cross-sectional area change is considered, this is due to the gradual incorporation of horizontal tubes along the vertical tubes that changes the velocity of working flow (V_{iv_i}).

$$V_{iv_i} = \frac{A_{pe} * V_{mpe}}{Ati_v + \sum_0^n (i * Ati_h)} \tag{5}$$

With the variable velocity per stretch of the vertical tube, the Reynolds number (Re_{iv}) variable and therefore, the Nusselt number (Nu_{iv}) and coefficient of heat transfer by convection (h_{iv}) variable too. That is shown in Table 1.

Table 1. Nusselt number (Incropera and David, 1996)

Nusselt Number	Conditions
$\dfrac{\left(\frac{f1_i}{8}\right)\left((Re_{iv_i} - 1000)Pr\right)}{1 + 12.7\left(\frac{f1_i}{8}\right)^{1/2}(Pr^{2/3} - 1)}$	Fully developed turbulent flow
$0{,}036 Re_{iv_i}^{0.8} Pr^{\frac{1}{3}} (\frac{Div}{L_v})^{0.055}$	Turbulent flow in the entrance area
$3.66 + \dfrac{0.0668(\frac{D_{iv}}{L_v})}{1 + 0.04[(\frac{D_{iv}}{L_v}) Re_{iv_i} Pr]^{2/3}} Re_{iv_i} Pr$	Laminar flow

For horizontal tubes, the heat transfer coefficients by convection are obtained with the same ratios of vertical tubes.

2.1.2 Mathematical model of the cold exchanger.

Likewise for the hot heat exchanger the portion on the outside which is the cooling water and the portion on the inside which is the working gas can be calculated."

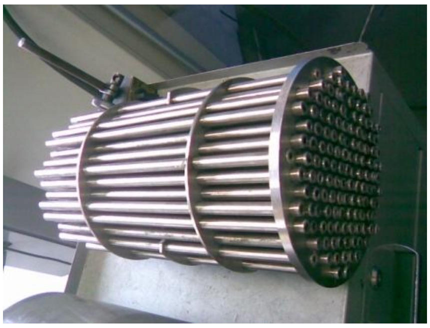

Figure 4. Cold heat exchanger

To calculate the convection heat transfer coefficient outside the tubes of this exchanger, the equation (3) was used for a bank of tubes taking into account the number of steps in the shell. On the inner walls the same ratios used to calculate Nu_{iv_i} in the hot heat exchanger are applied.

2.1.3 Calculation of the efficiency of regenerator

The calculations are made in the regenerator with the dimensionless parameters of Hausen (1929), which are improved by Hargreaves (1991) table 2.

Table 2. Efficiency of regenerator (Hargreaves, 1991)

$\mathbf{Re_r}$	$\mathbf{Nu_r}$	$\mathbf{h_r}$	$\mathbf{\Lambda}$	$\mathbf{\Gamma}$	$\mathbf{\Omega}$	$\mathbf{\eta_r}$
$\dfrac{V_r\, Dhi}{\nu_r(1 - Dhi\, m_w)^2}$	$0.42 Re_r^{0.56}$	$\dfrac{Nu_r K_r}{Dhi}$	$\dfrac{h_r A_{mr}}{\dot{m} C_{pg}}$	$\dfrac{M_{mr} C_M}{m C_{pg}}$	$\dfrac{\Lambda}{\Gamma} + 2 - 2.35\left(\dfrac{\Lambda}{\Gamma}\right)^{\frac{1}{2}}$	$1 - \dfrac{2}{\Lambda + 2 - \Omega}$

2.1.4 Losses

The adiabatic model used needs to include the friction losses and heat transfer. The losses considered in this paper are:

- Loss of pressure drop (Walker, 1980)

$$Pp = \sum \left[\frac{1}{\rho^2}\left(\frac{\dot{m}}{Ac}\right)^3 St\, L\, S\right]_{AR} + \sum \left[\xi \frac{\rho}{2} Ac\, V^3\right]_{Pp} \tag{6}$$

- Losses in the regenerator: the losses by pressure drop (equations 7) and thermal losses (equations 8) considering the efficiency of the regenerator (Organ, 1997).

$$\mathrm{Preg} = \left[\xi \frac{\rho}{2} \mathrm{Ac} \frac{\mathrm{Lre}}{2\,\mathrm{Dhi}} \left[\mathrm{V}\left(\frac{1}{1-\mathrm{Dhi}\,\mathrm{m_w}}\right)^2\right]^3\right]_{\mathrm{Reg}} \tag{7}$$

$$\mathrm{Qreg} = (1-\eta_r)\mathrm{Qri} \tag{8}$$

- Energy loss due to gas spring hysteresis: For an ideal gas, the pressure/volume relationship is either isothermal or adiabatic. In a real gas, there is a certain amount of work that is dissipated (Youssef, *et al.* 2008). Urieli and Berchowitz (1984) propose the following expression

$$\mathrm{Ph} = \sqrt{\frac{1}{32}\omega\gamma^3(\gamma-1)\mathrm{T_w P\,K_w}}\left(\frac{\Delta \mathrm{V}}{\mathrm{V}}\right)^2 \mathrm{A_w} \tag{9}$$

- Thermal losses due to internal conduction between the hot parts and the cold ones of the engine through the different exchangers (Martini, 1983).

$$\mathrm{Qint} = \frac{\mathrm{KA\Delta T}}{\mathrm{L}} \tag{10}$$

where K is the thermal conductance of the material and A the effective area for conduction

- Shuttle conduction: This happens for the expansion piston oscillation across a temperature gradient. It is usually not frequency-dependent for the speed and materials used in the engine. The piston absorbs heat during the hot end of its stroke and gives off heat during the cold end of it stoke (Martini, 1983).

$$\mathrm{Qshutt} = \frac{0.4\mathrm{E^2 KgD\Delta T}}{\mathrm{GLe}} \tag{11}$$

Where E is the stroke of the piston, Kg is the thermal conductivity, D is the piston diameter, G clearance around hot piston and Le equivalent length of the piston

- Pressure drop on the outside of the hot heat exchanger (in the exhaust gases) (Incropera and David, 1996):

$$\Delta \mathrm{Pv} = \mathrm{NX}\left(\frac{\rho \mathrm{Vmax}^2}{2}\right)\mathrm{f} \tag{12}$$

$$\mathrm{Pph} = \sum\left[\frac{1}{\rho^2}\left(\frac{\dot{\mathrm{m}}}{\mathrm{Ac}}\right)^3 \mathrm{St\,L\,S}\right]_{\mathrm{AR}} \tag{13}$$

Where N is the number of rows of heat exchanger and F and X are the friction and correction factors respectively.

- Mechanical losses are assumed at 20%

3. RESULTS

Figure 5 shows the influence of the furnace exhaust gas velocity to the net power (PottNet), engine efficiency (efi) and the own exhaust gas temperature. For low velocity, the output power is low due to limited availability of thermal energy that raises the fact of small gas mass flow. For high velocity, in addition to large mass flow there are also greater pressure losses outside the hot heat exchanger, and the variation of Tse (temperature of the exhaust outlet of the exchanger) decreases, hence net power output of the engine decrease. The next graphics in this work used the velocity that provides the greatest power, it is 9,5 m/s, except the test done for Helium, which took a velocity increase to provide a greater heat feeding into the engine.

The mathematical model of the Amazon engine was run with two working fluids (Figure 6) Helium and Nitrogen. The former, because of its thermodynamic characteristics, has the ability to provide more power than nitrogen, but a larger amount of heat is needed. The largest heat exchange is achieved with a greater temperature difference between the two fluids and an increase in the velocity of the exhaust gases. Both options would need great changes in the initial

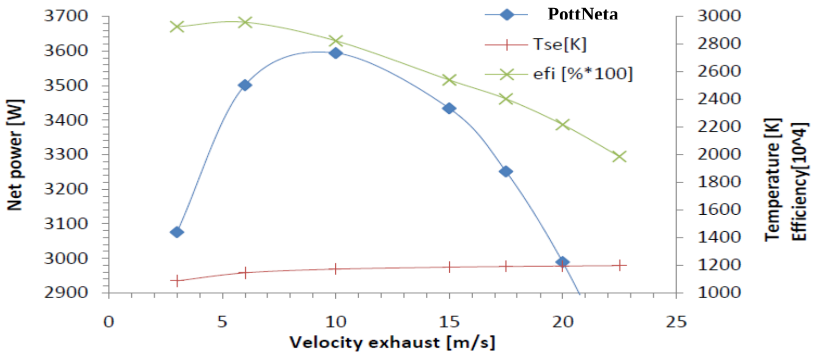

Figure 5. Variation of power output of engine Amazon in terms of velocity of the exhaust

design of the furnace and in the draft system. In figure 6 the pressure losses outside the hot heat exchanger (power consumed by the fan) are not considered.

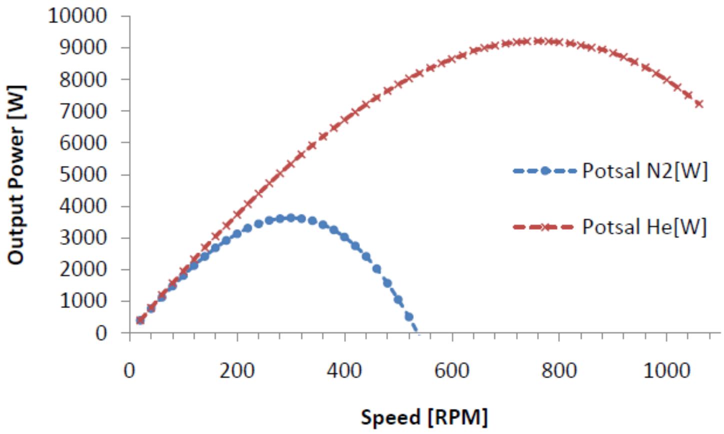

Figure 6. Power output (Potsal) of the Stirling engine Amazon with different working fluids (Helium and Nitrogen)

Figure 7 shows the flows of energy in the motor as a function of the crack angle in the adiabatic model at 300 RPM and nitrogen as a working gas At the angle 360° the amount of energy required and provided by the engine per cycle can be seen. The amount of thermal energy required and rejected per cycle is equal to the energy supplied and consumed in the compression and expansion pistons respectively.

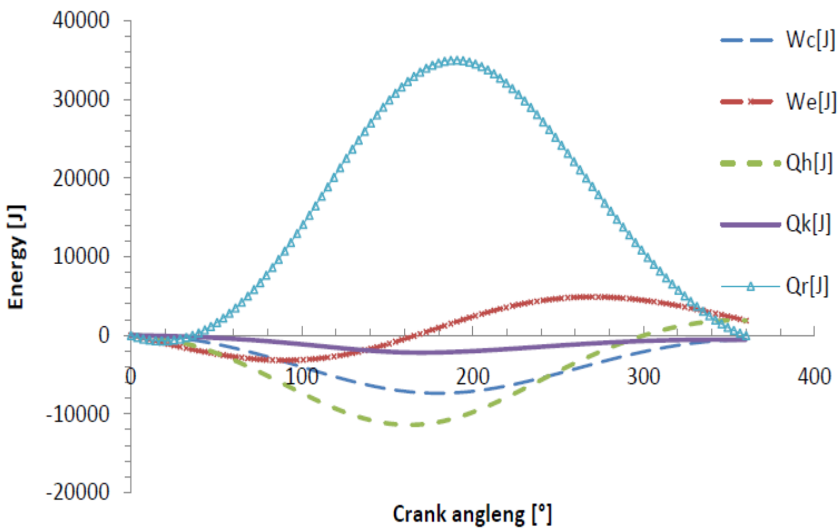

Figure 7. Diagram of energy flows in the engine Amazon in function of the angle in the adiabatic model

Figure 8 shows the total work done by the engine in the adiabatic cycle. The work is performed with a range of volumes between 11950-14400 cm3, which indicates the large amount of dead volume affecting its efficiency.

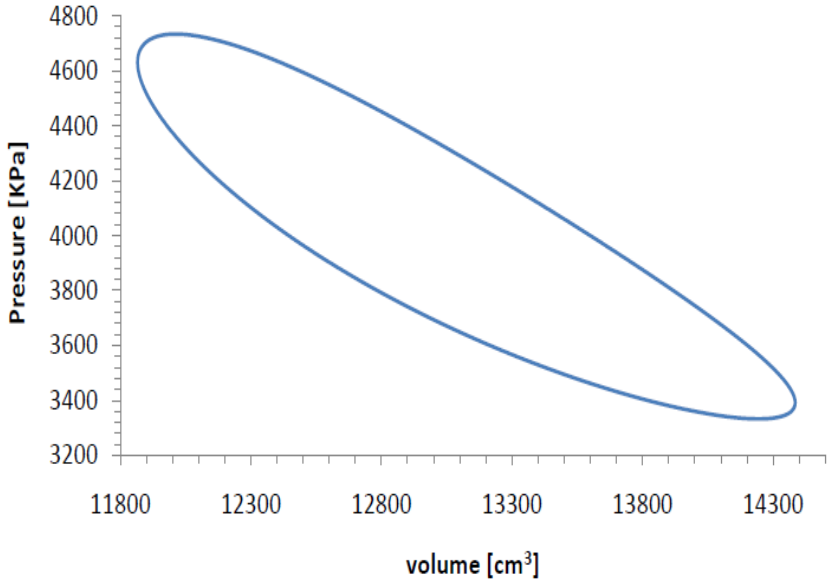

Figure 8. Variation of total pressure of the engine Amazon as a function of volume

The most significant friction losses are presented in the order of: the regenerator, the connector tubes of the cold side, hot and cold heat exchangers, and pressure drop in the outside heat exchanger, while the hysteresis losses are not significant. The losses in the connector tubes of the cold side are due to their low cross-sectional area (see figure 9).The large number of mesh in the regenerator is reflected in its high efficiency (Fig. 12) and high friction losses.

Figure 10 shows the variation of temperatures in the hot and cold heat exchangers. Increase the engine velocity requires more heat input, this requirement generates a greater drop in temperature in the hot heat exchanger. The opposite happens in the cold heat exchanger.

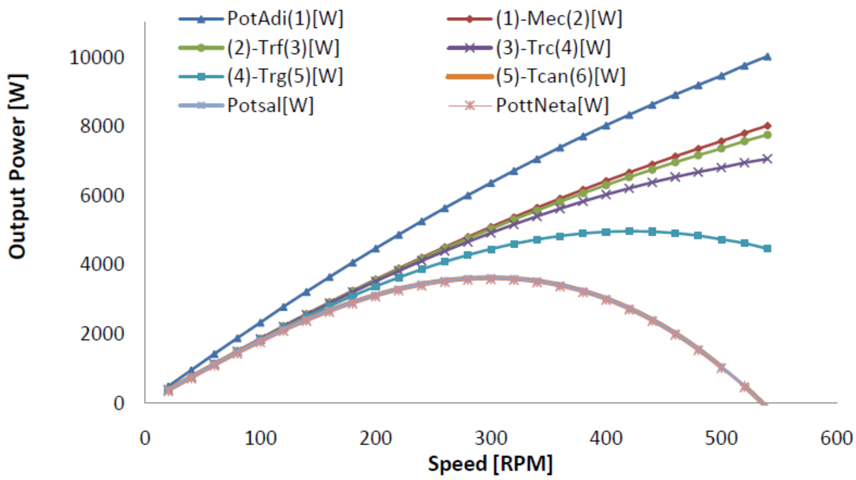

Figure 9. Efficiency variation of the Amazon engine as a function of the RPM with nitrogen as working fluid.

(1)= Output power adiabatic model (PotAdi).
(2)= (1) - mechanical losses (Mec).
(3)= (2) – friction losses in the cold exchanger (Trf).
(4)= (3) – friction losses in the hot exchanger (Trc).
(5)= (4) – friction losses in the regenerator (Trg).
(6)= (5) – lost by friction in the tubes of connectors of cold side (Tcan).
Output Power (Potsal) = (6) - lost by hysteresis in the compression and expansion chambers.

Net Output power (PottNeta) = Output Power (Potsal) - lost to outside pressure in the hot heat exchanger.

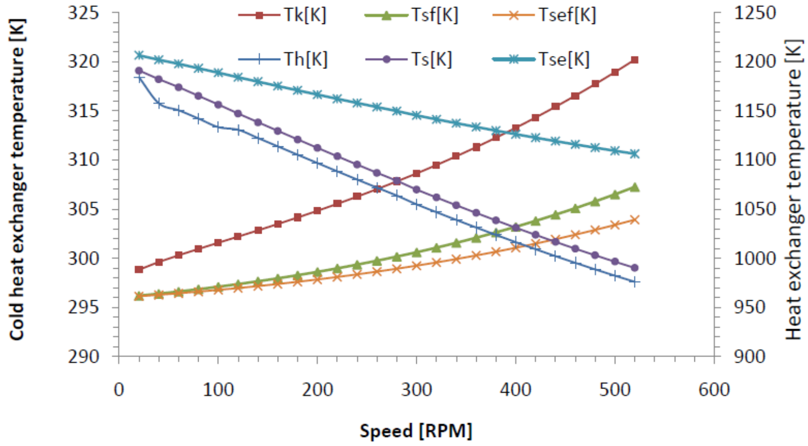

Figure 10. Variation of temperatures in function on engine RPM

The laminar to turbulent regime change inside of the hot heat exchanger, it shown in Figures 11 and 12. In the cold heat exchanger there is turbulent flow since from the first velocity value taken (20 RPM).

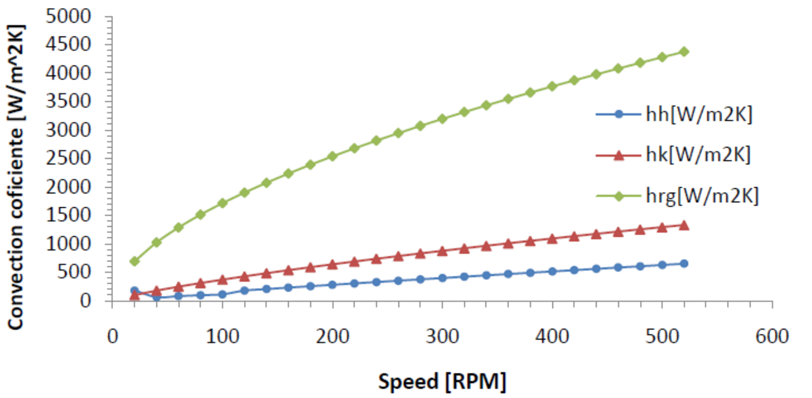

Figure 11. Variation of heat transfer coefficients by convection in heat exchangers (including the regenerator, hrg) depending of the RPM

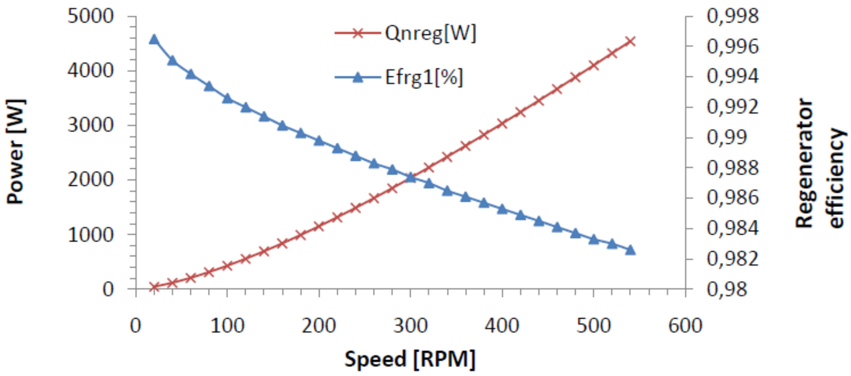

Figure 12. Variation of heat dissipated in the regenerator (Qnreg) and its efficiency (Efrg1) as a function of engine RPM

Figure 13 shows the amounts of heat input and output in function the engine velocity. The heat input in its maximum value has a negative power, where friction losses decrease the heat input requirements, and the heat output continuing to increase.

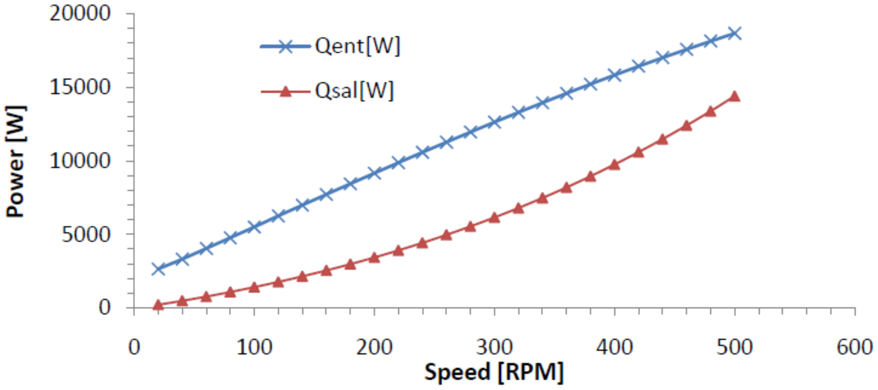

Figure 13. Heat flow in (Qent) and out (Qsal) on the engine in function of engine RPM

The comparison of results with the software PROSA (commercial Program for second order analysis) is shown in Figure 14. The PROSA was run under the same conditions that were used in the model developed. That is, in the hot heat exchanger dead volume of the collecting tubes was added, in the cold side the volume of the connecting tubes was added and wall temperatures were taken from the model proposed in this paper.

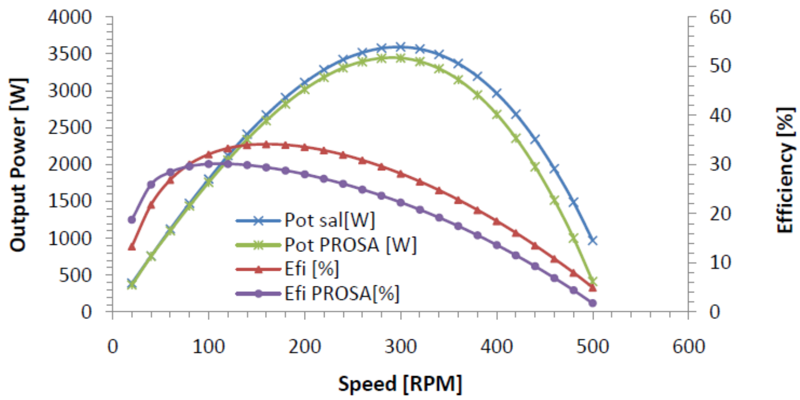

Figure 14. Variation of efficiency and power output of the engine Amazon in function of RPM with the model made (Efi and Potsal) and with the software PROSA (Efi PROSA and Pot PROSA)

4. CONCLUSIONS

The integration of models of heat exchangers, regenerator, accessories and fan forced draft to a second order adiabatic model provides a valuable tool for improving the Amazon Stirling engine project. In particular it deals with the geometry of the heat exchangers allowing a possible optimization, which is the most important and complex in Stirling engines. The disadvantage of this model is that due to the large number of iterations robust computer equipment is required to enable to work with reasonable run times.

The low power generated by the Amazon Stirling engine is mainly due to the heavy losses of the regenerator and the connector tubes of the cold side. These losses can be reduced with little change in the geometric parameters of the engine. An increase in the number of connecting tubes and a decrease in the length of the regenerator, looking for a good balance between dead volume and efficiency losses can help lower the problem of losses.

Because the PROSA has had a hard work in testing and calibration, as well as greater consideration of losses, the results differ somewhat from those obtained by the model developed. However, the PROSA does not allow modeling of the geometric parameters of the hot heat exchanger or the variation of surface temperature of the heat exchangers depending on velocities of the external fluid (exhaust gases and water).

5. ACKNOWLEDGEMENT

The authors thank to FAPEMIG and CNPq, Brazil, for the financial support.

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