Pipeline flow of heavy oil with temperature-dependent viscosity

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Abstract. *The heavy oil produced offshore needs to be transported through pipelines between different facilities. The pipelines are usually laid down on the seabed and are submitted to low temperatures. Although heavy oils usually present Newtonian behavior, its viscosity is a strong function of temperature. Therefore, the prediction of pressure drops along the pipelines should include the solution of the energy equation and the dependence of viscosity to temperature.*
In this work, an asymptotic model is developed to study this problem. The flow is considered laminar and the viscosity varies exponentially with temperature. The model includes one-dimensional equations for the temperature and pressure distribution along the pipeline at a prescribed flow rate. The solution of the coupled differential equation is obtained by second-order finite difference.
Results show a nonlinear behavior as a result of coupled interaction between the velocity, temperature, and temperature-dependent material properties.

Keywords: *heavy oil, laminar flow, pipe flow, numerical simulation*

1. Introduction

The heavy oil produced offshore needs to be transported through pipelines between different facilities. Unfortunately, their high viscosities make it difficult to transport them. The pipelines are usually laid down on the seabed and are submitted to low temperatures, as show in Fig. 1. Although heavy oils usually present Newtonian behavior, its viscosity is a strong function of temperature. Therefore, the prediction of pressure drops along the pipelines should include the solution of the energy equation and the dependence of viscosity to temperature.

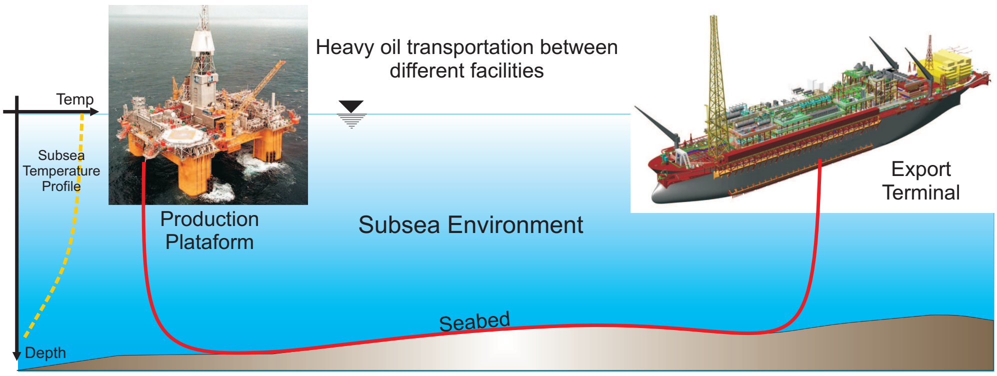

Figure 1. Simplified schematic of a subsea transport pipeline.

Relevant contributions to the laminar flow with high viscosity variations for low Reynolds number were made by Ockendon & Ockendon (1977) who studied the two-dimensional steady flow of a Newtonian fluid driven by a constant mass flux in a rectangular channel. The channel walls were assumed to be suddenly heated or cooled. Effects of heat dissipation were neglected. Asymptotic descriptions for the velocity and temperature fields have been derived for algebraic and exponential variation of viscosity with temperature. Extended asymptotic studies for circular pipe flows of hot polymer melts including viscous dissipation and solidification near cooled walls were performed by Richardson (1986). With the viscosity depending on the temperature and the shear rate, multi-valued relationships between flow rate and pressure-drop are found. Whitehead & Helfrich (1991) considered a slot flow with cooled walls and a viscosity depending linearly on temperature. They treated the problem in the framework of the Hele-Shaw approximation where the velocity, temperature and viscosity were averaged across the gap. For sufficiently large viscosity contrasts and a given pressure drop three steady state solutions for the velocity were found as well. Furthermore, a stability analysis as well as experiments for pipe and slot flow were performed. This work for cross-averaged flow structures was continued by Helfrich (1995). Wylie & Lister (1995) studied the linear stability of steady flows to two-dimensional and three-dimensional disturbances in a channel flow with cooled walls and viscosity depending on temperature. The bifurcations observed in previous studies were confirmed. Lange & Loch (1995) also developed analytical pipe flow models of a highly viscous fluid driven by a pressure gradient and affected by heat loss through the wall. They used a simplest model where the temperature was cross-section averaged. Giessler et al. (2007) developed an one-dimensional model describing the laminar flow of a highly viscous fluid representing glass melt in a pipe. Where the flow was strong influence by Lorentz force. Taking into account the full nonlinear temperature dependence of the viscosity and the electrical conductivity.

In this work, an asymptotic model is developed to study heavy oil transport in deep water where exist high rate heat transfer. The flow is considered laminar and the viscosity varies exponentially with temperature. The model includes one-dimensional equations for the temperature and pressure distribution along the pipeline at a prescribed flow rate.

2. Mathematical Formulation

The heavy oil with constant density ρ has a strong temperature dependence viscosity, $\mu(T)$, as show in Fig. 2 which can be determined experimentally and conveniently represented by the form:

$$\mu(T) = \mu_0 e^{A/(T+B)} \tag{1}$$

The constant parameters μ_0, A and B depend on the fluid. Where B is typically negative. This expression is given by a limited ranges of T. This representation fails when $T \rightarrow |B|$, but this singularity is irrelevant for our consideration because in piping flow process take place at temperatures much higher than $-B$. Moreover, we assume that the fluid is thermally conducting with an effective thermal conductivity k and heat capacity c_p.

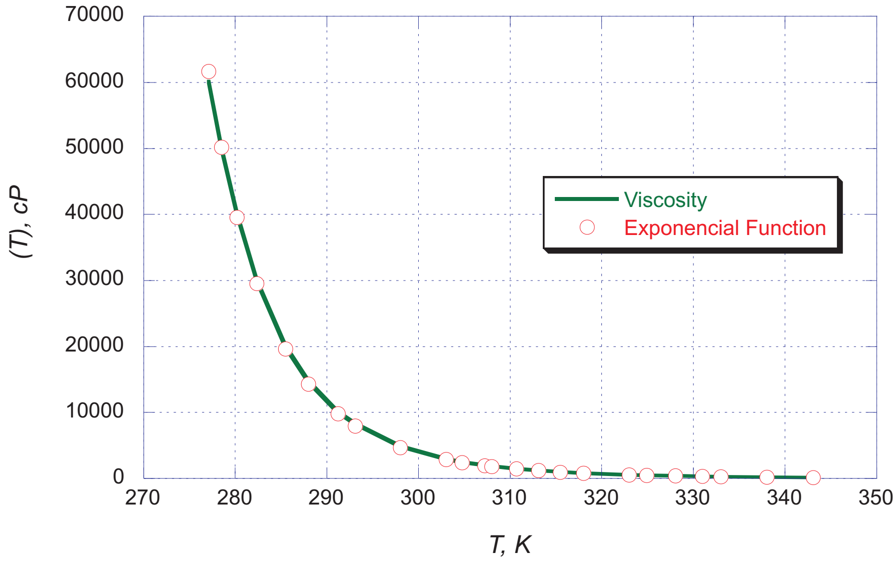

Figure 2. The liquid viscosity expressed in exponential function.

2.1 Equation of motions and energy

The physical situation is shown in Fig. 3. We consider a laminar and steady flow of heavy oil that is assumed to be Newtonian liquid flowing into a pipeline with radio R and length $L \gg R$ driven by gradient pressure ∇p and gravity g.

To avoid unnecessary complications, we shall assume that the material is incompressible, and the inertia forces are negligible. In practical applications, the flow is held at low-Reynolds number. Body forces are unimportant. Applying those considerations, the resulting momentum equation is given by following expression:

$$0 = \rho\vec{g} + \nabla \cdot (-p\underline{\underline{I}} + \mu(T)[\nabla\vec{V} + (\nabla\vec{V})^T]) \tag{2}$$

where p is the pressure and $\vec{V}$ is the velocity.

The resulting energy equation without internal heat generation and after neglecting the viscous dissipation rate:

$$\rho c_p(\vec{V} \cdot \nabla)T = k\nabla^2 T \tag{3}$$

In flow where the particles paths (streamlines in the steady-state problem) are almost parallel, scaling arguments can be used to simplified the equations, above indicated, to ordinary differential equations for pressure and temperature that captures the most important features of the problem. With the previous condition $L \gg R$ the lubrication approximation is

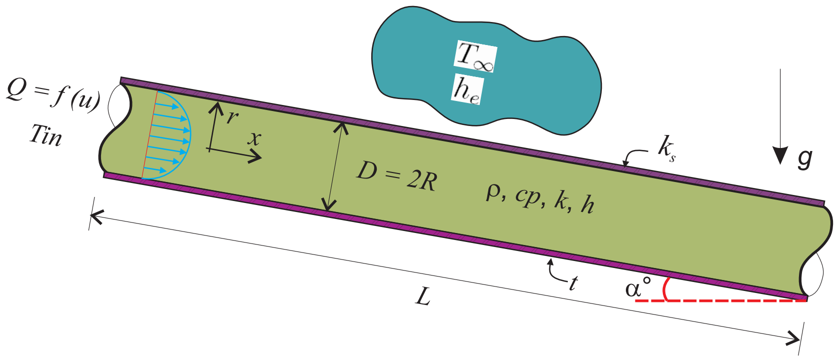

Figure 3. Sketch of the considered problem and its co-ordianate system.

valid and this assumption enables us to reduce the 3D to 1D cross-section averaged temperature $T_o(x)$ and a single cross-section averaged velocity u (Whitehead & Helfrich, Lange & Loch, Giessler et al.). According the conservation mass equation for incompressible fluid, u does not depend on x. With our simplifying assumptions the momentum equation integrated over the pipe length becomes.

$$\rho g \int_0^L \sin\alpha dx - \Delta p = \frac{8\mu_0}{R^2} u \int_0^L e^{A/(T_o+B)} dx \tag{4}$$

where terms on the left-hand side represent the driven forces and the single term on the left-hand side is the friction force that depend of the temperature distribution $T_o(x)$. The energy equation provide us the relation between u and $T_o(x)$. The simplified one-dimensional heat equation is given by:

$$\rho c_p u \frac{dT_o}{dx} = k \frac{d^2 T_o}{dx^2} - \frac{2}{R} U (T_o - T_\infty) \tag{5}$$

The boundary condition imposed is:

$$T_o = T_{in} \quad \text{for} \quad x = 0 \tag{6}$$

$$\frac{dT_o}{dx} = 0 \quad \text{for} \quad x = L \tag{7}$$

where U represent the global conductance and T_∞ is the free stream temperature. The global conductance is given from a resistance series argument that consider the external conductance h_e, the wall resistance and the internal conductance h:

$$\frac{1}{U} = \underbrace{\frac{R/r_e}{h_e} + \frac{R}{k_s} ln \frac{r_e}{R}}_{1/h_{eff}} + \frac{1}{h} \tag{8}$$

where h_{eff} is the effective external coefficient, k_s and r_e are the tube wall conductivity and outer radius.

2.2 Dimensionless variables, parameters and equations

It is convenient to define dimensionless variables in terms of a characteristic velocity u, related to the flow rate $Q = u(\pi D^2/4)$, the pipeline diameter D, a characteristic viscosity μ_o and the physical parameters k, c_p, ρ as well as A and B already introduced. We write:

$$\text{Dimensionless temperature as} \quad \theta = \frac{T_o - T_\infty}{T_{in} - T_\infty}, \tag{9}$$

$$\text{Dimensionless length as} \quad X = \frac{x}{L}, \tag{10}$$

$$\text{Reynolds number} \quad Re = \frac{\rho u D}{\mu_o}, \tag{11}$$

$$\text{Prandtl number} \quad Pr = \frac{c_p \mu_o}{k}, \tag{12}$$

$$\text{Peclet number} \quad Pe = RePr, \tag{13}$$

$$\text{Nusselt number} \quad \hat{N}u = \frac{UD}{k}, \tag{14}$$

$$\text{Froude number} \qquad Fr = \frac{u}{\sqrt{gL}}, \tag{15}$$

$$\text{Dimensionless pressure drop as} \qquad \Delta P = \frac{\Delta p}{0.5\rho u^2}, \tag{16}$$

$$\text{Viscosity function constants} \qquad S_\mu = \frac{T_{in} - T_\infty}{A} \quad \text{and} \quad Q_\mu = \frac{T_{in} + B}{A}; \tag{17}$$

The momentum equation (4) in dimensionless form is obtained as follow:

$$\frac{2}{Fr^2}\int_0^1 \sin\alpha dX - \Delta P = \frac{64}{Re}\left(\frac{L}{D}\right)\int_0^1 e^{1/(S_\mu\theta + Q_\mu)} dX \tag{18}$$

and the energy equation (5) in dimensionless form:

$$\frac{d^2\theta}{dX^2} - \underbrace{RePr}_{Pe}\left(\frac{L}{D}\right)\frac{d\theta}{dX} - 4\hat{N}u\left(\frac{L}{D}\right)^2\theta = 0 \tag{19}$$

and its respectively boundary condition:

$$\theta = \theta_{in} \qquad \text{for} \qquad X = 0 \tag{20}$$

$$\frac{d\theta}{dX} = 0 \qquad \text{for} \qquad X = 1 \tag{21}$$

The nature of those equations consist of an integral and a second order differential equation which determine the nondimensional flow rate given by Re and the temperature distribution $\theta(X)$. The equation (18) expresses the balance between the driving forces on the left-hand side and the length-integral viscous friction on the right-hand side. The equation (19) represent the balance between heat diffusion, heat advection and wall heat loss.

The set of governing equations (18-19) are coupled by the temperature $\theta(X)$ which appears in the friction term of the momentum equation because we are considering temperature-dependent viscosity. The another coupling is due to the appearance of Re in the heat equation.

For an horizontal pipeline in laminar regime the equation (18) make sense when the viscosity is not a function of temperature. Because the resultant expression is the Darcy - Weisbach equation in term of pressure loss.

$$\Delta P = \frac{\Delta p}{(0.5\rho u^2)} = \underbrace{\frac{64}{Re}}_{f}\left(\frac{L}{D}\right) \tag{22}$$

where f is the Darcy - Weisbach factor.

2.3 Numerical Method

The set of equations was solved using the following procedure. For a given regime of Re we first solve the equation (19) with its boundary conditions in order to obtain the temperature distribution $\theta(X)$. This equation was discretized using finite-difference approximations. After that, the resulting $\theta(X)$ is used to compute the integral equation (18) with the help of the composite trapezoidal rule.

3. Results

3.1 Validation

Before examining analysis of flow of high liquid viscosities with temperature dependence into a pipe, it is instructive to compare the present one-dimensional (1D) pipe flow model with two-dimensional (2D) axisymmetric simulations using the commercial software Fluent that predict the dependence of u and T on two coordinates, namely the streamwise coordinate x and the radial coordinate r. The mesh used has a total of 10000 quadrilateral cells. We checked the mesh by comparison with another refined mesh calculating the velocity profile along the pipeline for a given inlet velocity.

The liquid viscosity expressed in exponential function is shown in Fig. 2. Where we can obtain those three parameters: μ_0, A and B. Typical values of the various dimensional parameters and the resulting values for the dimensionless quantities are given in Tab. 1.

The main purpose of the validation is to estimate the internal conductance h or well know as the heat transfer coefficient. Usually h is given, for the flow system of interest, as an empirical correlation of Nusselt number as a function of the relevant dimensionless quantities, such as the Re, Pr and the geometric ratio L/D. For very large temperature differences, the viscosity variations may result in a large distortion of the velocity profiles that is necessary to account for this by introducing an additional dimensionless group, μ_b/μ_w, where μ_b is the viscosity at the arithmetic average bulk

Table 1. Table of geometric values, liquid properties, boundary conditions and its respectively dimensionless parameter

ρ (kg/m^3)	μ_{ref} $(kg/m-s)$	g (m/s^2)	c_p $(J/Kg-K)$	k $(W/m-K)$	k_s $(W/m-K)$	h_{ext} (W/m^2-K)
850	1.3×10^5	9.81	500	0.55	0.35	10
L (m)	$D = 2R$ (m)	t (m)	$D_{ext} = 2r_e$ (m)	T_{in} (K)	T_∞ (K)	μ_o (cP)
50	0.16	2.54×10^{-2}	0.211	333	277	1.3×10^{-2}
A	B	S_μ	Q_μ	L/D	Pr	Re
1600	-173	0.035	0.065	312.5	260.31	[0.0475 - 475]

temperature and μ_w is the viscosity at the arithmetic average wall temperature. For circular tubes with nearly constant wall temperature and for laminar flow an empirically relation is founded in Bird (2002):

$$Nu = 1.86\left(RePr\frac{D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_w}\right)^{0.14} \quad (23)$$

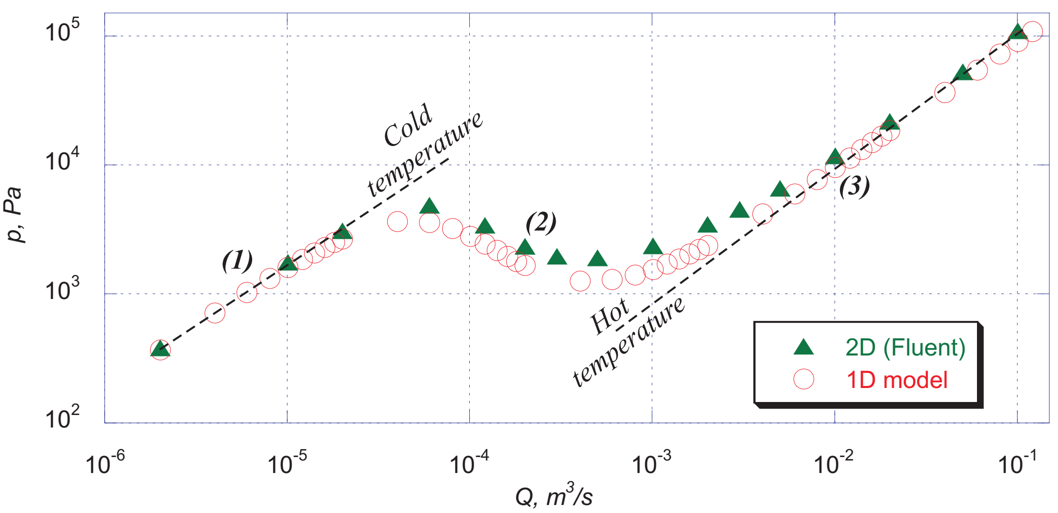

Figure 4. Pressure drop Δp as a function of flow rate Q.

Figure 4 shows the pressure drop as a function of flow rate for a horizontal pipe, ($\alpha = 0^o$). The curve can be divided in three branches: (1) At low flow rates, the temperature along the pipeline falls very quickly and most of the length of the pipe is at low temperature and the pressure drop is proportional to the flow rate. We can see in Fig. 5 the temperature distribution decreases very fast at the entrance of the pipe. (2) At intermediate flow rates, the temperature drop from high to low takes place along the entire length of the pipeline. In this region, as the flow rate rises, the section of the pipeline at high temperature (and consequently at low viscosity) increases and pressure drop falls. The third branch (3) occurs at high flow rate. Most of the pipeline is at high temperature and thermal effects are not important. The pressure drop is again proportional to the imposed flow rate.

The pressure drop along the pipeline for three different flow rate is showed in Fig. 6. The value of those flow rate have been chosen according to see what happen with the drop pressure into the branch (2) with $Q = 1.2 \times 10^{-4} m^3/s$ and out of this branch. In branch (1) with $Q = 2.0 \times 10^{-5} m^3/s$ and branch (3) with $Q = 5.0 \times 10^{-4} m^3/s$ the drop pressure along the pipeline is almost linear but into the branch (2) It is not anymore linear and the reason is the temperature dependent viscosity. As we can see at right side term of Eq. (4) when the temperature value along the pipeline is constant the relation between Δp and L is linear.

The comparison between results obtained with the present 1D model and the comercial program Fluent shows a good agreement in all of those results.

Results in dimensionless parameters are show in Fig. 7 as a function of ΔP vs Re along the laminar regime. Here, we can see the influence of the parameter L/D where is possible to see three different branches. The dot green lines represents the pressure that divides de branches.

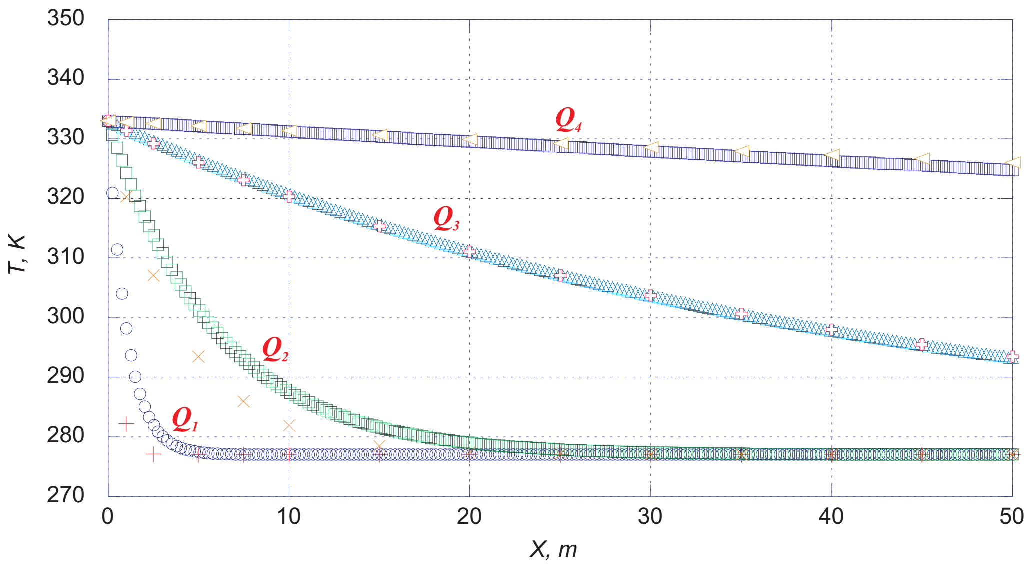

Figure 5. Temperature distribution T along the pipe axis x at different flow rate: $Q_1 = 2.0 \times 10^{-6}$, $Q_2 = 2.0 \times 10^{-5}$, $Q_3 = 2.0 \times 10^{-4}$, $Q_4 = 2.0 \times 10^{-3}$ measured in m^3/s.

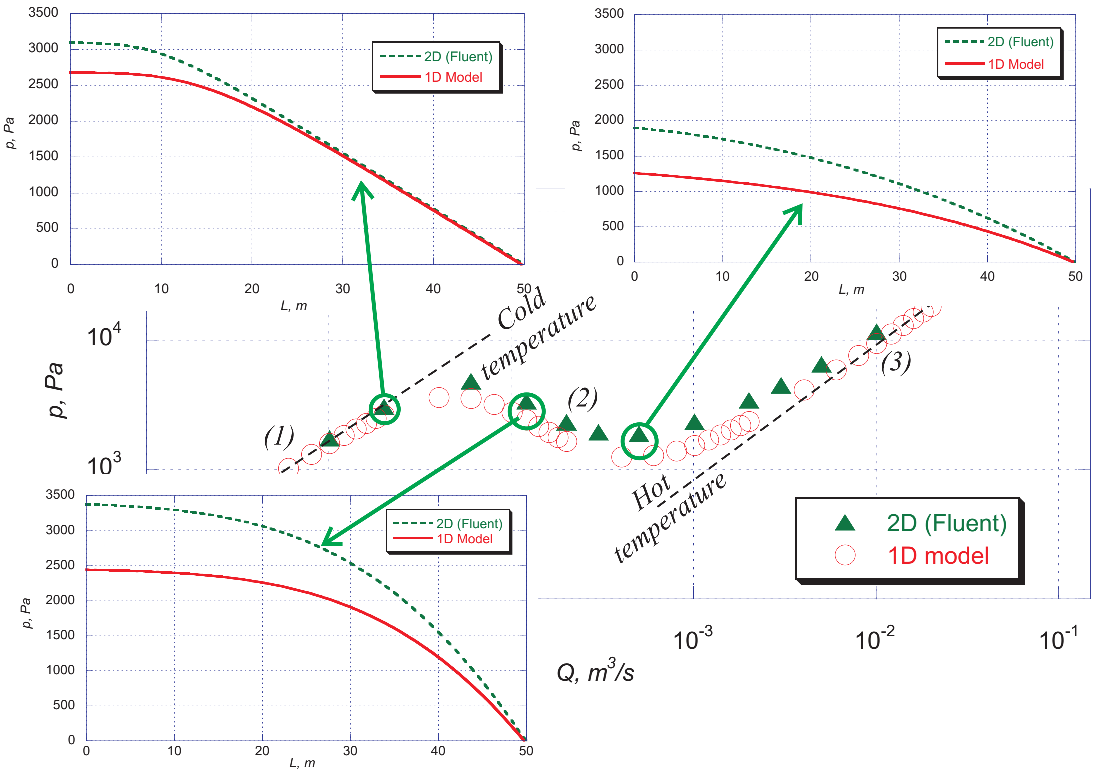

Figure 6. Pressure drop Δp along the pipeline for three different flow rate: $Q = 2.0 \times 10^{-5}; 1.2 \times 10^{-4}; 5.0 \times 10^{-4}$ in m^3/s

4. Final remarks

We presented an asymptotic model that is able to study heavy oil transport in deep water where exist high rate heat transfer. The flow is considered laminar and the viscosity varies exponentially with temperature. The results shows how the model can predict the relation between Δp against Q that is not linear by presence of the temperature dependent viscosity.

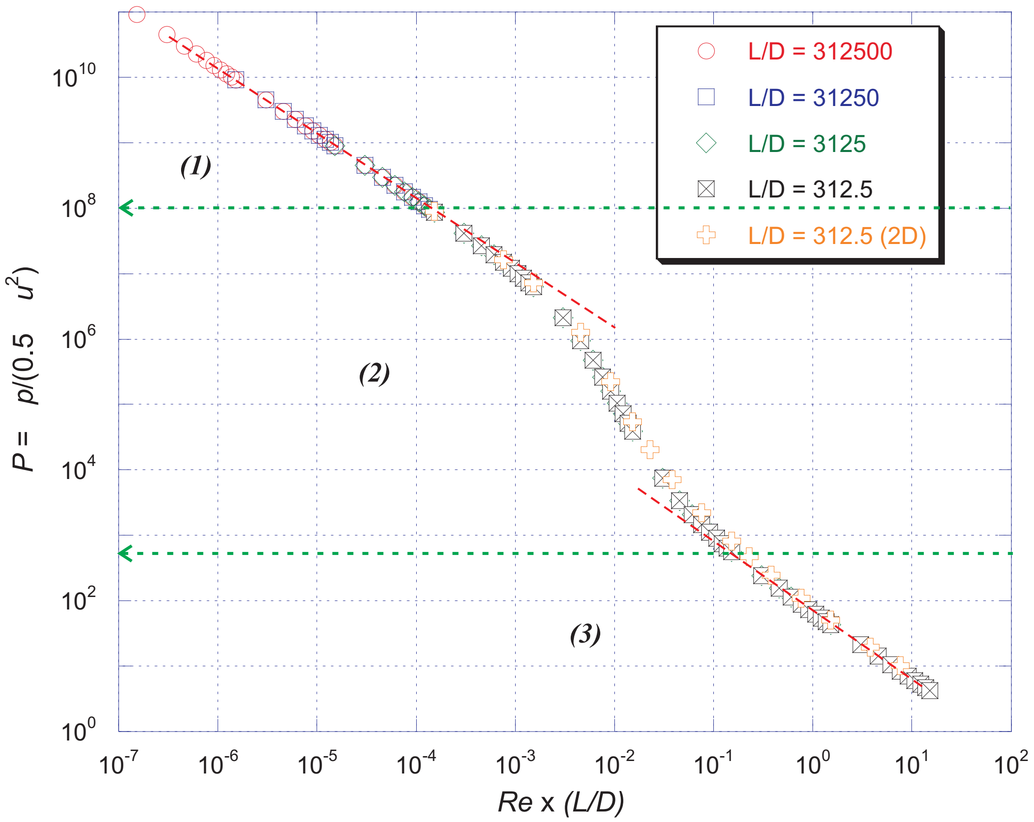

Figure 7. The dimensionless parameter ΔP as a function of $Re \times (D/L)$ where exhibit the influence of the parameter L/D

5. Acknowledgements

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