Development of Instabilities in Space in Parallel Oil-water Flows

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Abstract. *This paper presents a prospective study on the spatial development of hydrodynamic instabilities in horizontal and slightly inclined stratified and vertical core-annular oil-water flows. In the stratified flow the main goal is to find out the point in space where instabilities become so intense that transition to other flow patterns occurs. In the vertical core-annular flow we expect to numerically simulate the propagation of the interfacial waves and produce a method to predict the development of these waves in space, consequently the development of instabilities too. The idea, for both flow patterns, is based on the theory that considers that a perturbation wave may develop in time as well as in space. For stratified flow pattern we propose that in directional oil wells, for instance, inclination could lead to transition or that in a pipeline a stratified flow pattern artificially generated could breakup into a different topological configuration a few pipe diameters after the injection nozzle, which could be due to spatial development of instabilities. In the vertical core-annular flow it is known that interfacial tension has a significant stabilizing effect (Rodriguez and Bannwart, 2008), but the insertion of such an effect in the numerical model is not trivial. The formulation is based on the one-dimensional two-fluid model for liquid-liquid flows (Trallero 1995, Barnea and Taitel 1994, Rodriguez et al. 2006) and the adopted numerical method is the finite-difference based Method of Characteristics.*

Keywords*: Liquid-liquid flow, hydrodynamic stability, spatial instabilities, method of characteristics, 1-D two-fluid model*

1. INTRODUCTION

Two-phase flow patterns are observed in a wide range of natural and industrial processes. It is composed of two immiscible phases arranged in different geometrical configurations, or flow patterns. Annular and stratified flow patterns are examples of separated flow patterns. The former has being suggested as a rentable alternative for the transport of heavy crude oil and it is the commonest flow pattern of the refrigeration industry and in the natural gas production. The second, as a convenient form to avoid water in oil emulsions in pipelines and of common occurrence in directional oilwells. Those flow patterns are modeled as having parallel phases, *i.e.*, a two-phase parallel flow.

The hydrodynamic stability theory is in the scope of the classical fluid mechanics since the first part of the last century (Schlichting (1979)). The basis for the study of the hydrodynamic stability are presented in Betchov and Criminale (1967), Drazin and Raid (1981) and Lin (1955). Flows that occur in nature should obey the fluid-dynamics equations and be stable. One often analyzes the wavy characteristic of flows, for example the sea waters that propagate in shallow waters. Thus, one can define the study of the hydrodynamic stability/instability as the study of oscillatory motion in fluids. This study is related to the growth, stabilization or decrease of the amplitude of an oscillation of a particular fluidic system, which arises after the injection of a disturbance.

The linear analysis is based on the study of the growth of instabilities over a given basic flow system subject to small disturbances. If the system is unstable to small disturbances, it is assumed that it is also to major disturbances. Small disturbances cause infinitesimal amplitude oscillations, therefore the higher-order derivative terms of the equations of motion may be neglected (applying Taylor's series to the derivatives). So the equations are linearized and the mathematical definition of linear stability analysis is given. In many cases, stability criteria can be derived and used to predict periodic oscillations, chaotic and turbulent flows. Eventually, they can be used to predict the transition or change to a different flow pattern (Wallis, 1969).

The disadvantage of the linear theory is that it only considers infinitesimal disturbances, not taking into account that instabilities can be generated by finite amplitude disturbances even when the basic flow pattern is stable under infinitesimal perturbations, which is known as sub-critical instability. Therefore, a sub-critical instability occurs due to disturbances of finite amplitude and it can only be represented by theories that are nonlinear in nature. A new type of theory, the weakly nonlinear, which can be regarded as a correction of the approximations made in the linear theory has been proposed (Drazin and Reid (1981)).

The stability of parallel two-phase flows has been studied through the use of the method of characteristics (MOC). The MOC is a numerical method for solving systems of hyperbolic partial differential equations of first order or hyperbolic partial differential equations of second order. Hyperbolic equations are those who have real eigenvalues. The method is a variation of the finite difference method, which consists in finding at the time-space plane directions in which the partial differential equations can be reduced to ordinary differential equations. It can reduce the numerical diffusion by allowing a simulation of the propagation of a disturbance wave in the flow accurately and without affecting the flow. The first use of MOC refers to the analysis of transients in single phase flows, for example one can cite the "water hammer" (Assy (1975)). In two-phase flow it has been used in the analysis of interfacial waves in gas-liquid flows, as in Crowley (1992), where an analysis of transition from gas-liquid stratified flow was carried out. The MOC has been used to analyze the stability and propagation of interfacial waves in gas-liquid stratified flow pattern (Barnea and Taitel (1994)).

Although there are a few papers on the stability of separated gas-liquid flow, the correlations and analyzes used in those works shouldn't be readily used in the analysis of liquid-liquid flow. Nevertheless, the method of characteristics has been applied for liquid-liquid stratified flow based on the same simplifying assumption adopted for gas-liquid flow, with no further explanations (Brauner and Maron (1992a) and Trallero (1995).

This paper is a prospective study of the propagation of finite disturbances in oil-water stratified and core annular flow patterns. The main goal is the study of the stability of these flow patterns by means of the implementation of a simplified non-linear approach, which can be used to extend and reinforce the classical linear stability analysis. Firstly, it is presented the formulation based on the one-dimensional two-fluid model, then the interfacial-wave propagation analysis using the MOC. Finally, some preliminary analyses of the influence of terms that have been usually neglected in previous works on the model predictions are presented. Then conclusions and next steps are discussed.

2. MODELING

2.1. Stratified Flow Pattern

The modeling is based on a stratified flow as shown in Fig. 1. The index 1 indicates the oil phase and the index 2 the water phase.

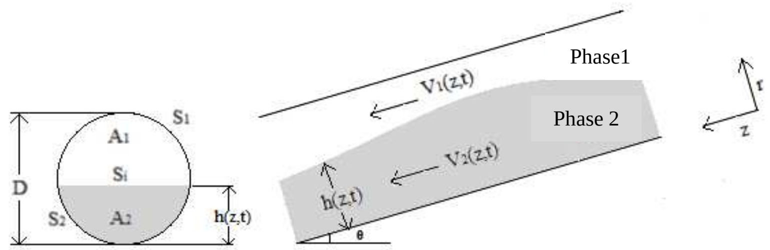

Figure 1. Schematic picture of the stratified flow pattern.

Modeling assumptions: a) isothermal flow, b) no phase change, c) no mass transfer; and d) incompressible fluids. Using the two-fluid model one gets the oil and water phase continuity equations, respectively:

$$\frac{\partial h}{\partial t} - \frac{A_1}{\dot{A_2}}\frac{\partial V_1}{\partial z} + V_1\frac{\partial h}{\partial z} = 0 \quad (1)$$

$$\frac{\partial h}{\partial t} + \frac{A_2}{\dot{A_2}}\frac{\partial V_2}{\partial z} + V_2\frac{\partial h}{\partial z} = 0 \quad (2)$$

The phases' momentum equations are coupled by the Laplace-Young law, which gives:

$$\rho_2\frac{\partial V_2}{\partial t} - \rho_1\frac{\partial V_1}{\partial t} + \rho_2 V_2\frac{\partial V_2}{\partial z} - \rho_1 V_1\frac{\partial V_1}{\partial z} + L\frac{\partial h(z)}{\partial z} - \sigma\frac{\partial^3 h(z)}{\partial z^3} = \tau_i S_i\left(\frac{1}{A_2} + \frac{1}{A_1}\right) - \frac{\tau_{2w}S_2}{A_2} + \frac{\tau_{1w}S_1}{A_1} - (\rho_2 - \rho_1)g\sin\theta \quad (3)$$

where

$$L = (\rho_2 - \rho_1)g\cos\theta \quad (4)$$

In the modeling of the stratified flow pattern, for now, just the radius of the interface in the longitudinal plane is considered..

2.2. Core-Annular Flow Pattern

The modeling is based on a liquid-liquid annular flow as shown in Fig. 2, but with the tube in an upright position (β = 90 °).

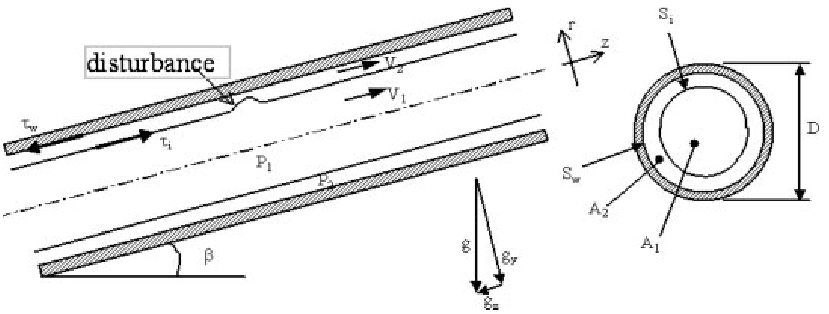

Figure 2. Schematic picture of the core-annular flow pattern.

In addition to the modeling assumptions given before, it is assumed laminar regime in the oil core. Again, using the two-fluid model, one gets the phases' continuity equations:

$$\frac{\partial}{\partial t}\varepsilon_1 + \varepsilon_1\frac{\partial}{\partial z}V_1 + V_1\frac{\partial}{\partial z}\varepsilon_1 = 0 \tag{5}$$

$$-\frac{\partial}{\partial t}\varepsilon_1 + (1-\varepsilon_1)\frac{\partial}{\partial z}V_2 - V_2\frac{\partial}{\partial z}\varepsilon_1 = 0 \tag{6}$$

and the momentum equation coupled by the Laplace-Young law:

$$(1-\varepsilon_1)\rho_1\left[\frac{\partial}{\partial t}(V_1\varepsilon_1) + \frac{\partial}{\partial z}(K_1V_1^2\varepsilon_1)\right] - \varepsilon_1\rho_2\left[\frac{\partial}{\partial t}\big(V_2(1-\varepsilon_1)\big) + \frac{\partial}{\partial z}\big(K_2V_2^2(1-\varepsilon_1)\big)\right] + \varepsilon_1(1-\varepsilon_1)\frac{\partial}{\partial z}\left\{\frac{2\sigma}{D\sqrt{\varepsilon_1}}\left[1 + \frac{D^2}{32\varepsilon_1}\left(\frac{\partial\varepsilon_1}{\partial z}\right)^2 - \frac{D^2}{8}\left(\frac{\partial^2\varepsilon_1}{\partial z^2}\right)\right]\right\} = -\frac{\tau_i S_i}{A} + \frac{4\tau_w\varepsilon_1}{D} + \varepsilon_1(1-\varepsilon_1)g(\rho_2-\rho_1) \tag{7}$$

3. STABILITY ANALISYS – SIMPLIFIED NON-LINEAR ANALYSIS (METHOD OF CHARACTERISTICS)

3.1. Stratified Flow Pattern

To use the MOC starts with the equations of continuity for the two phases, Eq. (1) and Eq. (2), and momentum, Eq. (3). Taking into account the slip ratio between the phases in liquid-liquid stratified flow, there are two different situations.

3.1.1. The velocity of the oil is greater than the velocity of water (slip ratio, *S*, is greater than 1):

$$S = \frac{V_1}{V_2} > 1 \tag{8}$$

It is assumed that the stability condition of the MOC (Courant-Friedrich-Lewy) is that the interfacial wave speed is lower than the *in situ* speed of the oil phase. Thus, one can consider that the oil flow is quasi-permanent, so Eq. (1) can be:

$$\int_{V_{1s}}^{V_1}\frac{\partial V_1}{V_1} = -\int_{A}^{A_1}\frac{\partial A_1}{A_1} \tag{9}$$

Where, V_{1s} represents the superficial velocity of the oil, V_1 is the *in situ* velocity of oil, and A is the cross-sectional area of the pipe.

$$V_1 = \frac{V_{1s}A}{A_1} \tag{10}$$

Substituting Eq. (10) in Eq. (3), and dividing all terms by the density of water, we have:

$$\frac{\partial V_2}{\partial t} + V_2\frac{\partial V_2}{\partial z} + G_1\frac{\partial h(z)}{\partial z} - \frac{\sigma}{\rho_2}\frac{\partial^3 h(z)}{\partial z^3} = E_1 \tag{11}$$

Where:

$$G_1 = \frac{(\rho_2-\rho_1) g \cos\theta}{\rho_2} - \frac{\rho_1 {V_{1s}}^2 A^2 \acute{A_2}}{\rho_2 {A_1}^3} \tag{12}$$

$$E_1 = -\frac{fe}{\rho_2} = \frac{\tau_i S_i\left(\frac{1}{A_2}+\frac{1}{A_1}\right) - \frac{\tau_{2w} S_2}{A_2} + \frac{\tau_{1w} S_1}{A_1} - (\rho_2-\rho_1) g \sin\theta}{\rho_2} \tag{13}$$

Considering the long-wave approximation, where the surface tension terms are negligible. One can rewrite Eq. (11) as follows:

$$\frac{\partial V_2}{\partial t} + V_2 \frac{\partial V_2}{\partial z} + G_1 \frac{\partial h(z)}{\partial z} + E_1 = 0 \tag{14}$$

Thus, the system of two partial differential equations, modeling the liquid-liquid stratified flow, can be reduced to:

$$\begin{cases} \frac{\partial h}{\partial t} + \frac{A_2}{\acute{A_2}} \frac{\partial V_2}{\partial z} + V_2 \frac{\partial h}{\partial z} = 0 \\ \frac{\partial V_2}{\partial t} + V_2 \frac{\partial V_2}{\partial z} + G_1 \frac{\partial h}{\partial z} + E_1 = 0 \end{cases} \tag{15}$$

Following the methodology of Crowley (1992), by applying the method of characteristics the system of two partial differential equations, Eq. (15), is reduced to a system of two ordinary differential equations, Eq. (16), with each equation being valid along a characteristic direction, C_L and C_H:

$$\begin{cases} \frac{dh}{dt} - \sqrt{\frac{H_1}{G_1}} \frac{dh}{dt} + E_1 \sqrt{\frac{H_1}{G_1}} = 0, along\ C_{L1} = V_2 - \sqrt{H_1 G_1} = \frac{dz_1}{dt} \\ \frac{dh}{dt} + \sqrt{\frac{H_1}{G_1}} \frac{dh}{dt} - E_1 \sqrt{\frac{H_1}{G_1}} = 0, along\ C_{H1} = V_2 + \sqrt{H_1 G_1} = \frac{dz_2}{dt} \end{cases} \tag{16}$$

Where C_{L1} and C_{H1} represent the lower and higher characteristic velocities, respectively, and for simplicity:

$$H_1 = \frac{A_2}{\acute{A_2}} \tag{17}$$

The system, Eq. (16), can be solved numerically by using the finite difference method, as proposed by Barnea (1994b) and Trallero (1995), as follows:

$$\frac{z_{i,k+1} - z_{i+1,k}}{t_{i,k+1} - t_{i+1,k}} = C_{L1_{i+1,k}} \tag{18}$$

$$\frac{z_{i,k+1} - z_{i,k}}{t_{i,k+1} - t_{i,k}} = C_{H1_{i,k}} \tag{19}$$

$$h_{i,k+1} - h_{i+1,k} - \sqrt{\frac{H_1}{G_1}}_{i+1,k} \left(V_{2_{i,k+1}} - V_{2_{i+1,k}}\right) + E_{1_{i+1,k}} \sqrt{\frac{H_1}{G_1}}_{i+1,k} \left(t_{i,k+1} - t_{i+1,k}\right) = 0 \tag{20}$$

$$h_{i,k+1} - h_{i,k} - \sqrt{\frac{H_1}{G_1}}_{i,k} \left(V_{2_{i,k+1}} - V_{2_{i,k}}\right) + E_{1_{i,k}} \sqrt{\frac{H_1}{G_1}}_{i,k} \left(t_{i,k+1} - t_{i,k}\right) = 0 \tag{21}$$

For a given initial condition, along $k = 1, \cdots, n$, the variables are $z_{i,k}$, $t_{i,k}$, $h_{i,k}$ and $V_{2\ i,k}$. The values in the points $k+1$ and $i = 1, \cdots, n$, for the variables $z_{i,k+1}$, $t_{i,k+1}$, $h_{i,k+1}$ and $V_{2\ i,k+1}$ are calculated from Eqs (18), (19), (20) and (21). The simulation begins with the initial condition of equilibrium, in which it is imposed a solitary wave of finite amplitude. Hence, the propagation of this wave over space and time is calculated. Two types of propagation are possible, depending on values of C_{L1}. When $C_{L1} > 0$ there is a supercritical flow and the disturbance propagates only downstream. But, when $C_{L1}<0$ there is a subcritical flow and the disturbance propagates also upstream, being reflected at the inlet of the pipe. In this case, some additional calculations are needed to check the growth of the disturbance in the upstream direction. Thus, for $C_{L1}<0$ the condition of constant flow rate of the permanent phase is used and the equation of negative characteristic velocity is used to calculate the boundary conditions at the point where $z = 0$:

$$t_{0,k+1} = t_{1,k} - \frac{z_{1,k}}{C_{L1_{1,k}}} \tag{22}$$

$$h_{0,k+1} = h_{1,k} + \sqrt{\frac{H_1}{G_1}}_{1,k} \left(\frac{V_{2S}A}{A_{2_{0,k+1}}} - V_{2_{1,k}}\right) + E_{1_{1,k}} \sqrt{\frac{H_1}{G_1}}_{1,k} \left(t_{0,k+1} - t_{1,k}\right) \tag{23}$$

where $A_{2_{0,k+1}}$ is calculated interactively.

3.1.2. The velocity of the water is greater than the velocity of oil (slip ratio, *S*, is lower than 1):

$$S = \frac{V_1}{V_2} < 1 \tag{24}$$

It is assumed that the stability condition of the MOC (Courant-Friedrich-Lewy) is that the interfacial wave speed is lower than the in situ velocity of the water phase. In this case, the phase considered in quasi-permanent regime is the water phase, thus:

$$V_2 = \frac{V_{2S}A}{A_2} \tag{25}$$

The steps are the same as those shown for the case of slip greater than 1. Therefore, the system of ordinary differential equations is given by:

$$\begin{cases} \frac{dh}{dt} - \sqrt{\frac{H_2}{G_2}}\frac{dh}{dt} - E_2\sqrt{\frac{H_2}{G_2}} = 0, along\ C_{L1} = V_1 - \sqrt{H_2G_2} = \frac{dz_1}{dt} \\ \frac{dh}{dt} + \sqrt{\frac{H_2}{G_2}}\frac{dh}{dt} + E_2\sqrt{\frac{H_2}{G_2}} = 0, along\ C_{H1} = V_1 + \sqrt{H_2G_2} = \frac{dz_2}{dt} \end{cases} \tag{26}$$

3.2. Core-Annular Flow Pattern

The MOC is applied to deal with Eqs. (5), (6) and Eq. (7), which are valid for the core-annular flow pattern. Here, only one case of slip greater than one is considered, because the oil core is always the fastest in upward vertical flow, or:

$$S = \frac{V_1}{V_2} > 1 \tag{27}$$

Again, it is assumed that the stability condition of the MOC (Courant-Friedrich-Lewy) is that the interfacial wave speed is lower than the *in-situ* speed of the oil phase. Thus, one can consider that the oil flow is quasi-permanent, so Eq. (5) can be integrated as:

$$\int_{V_{1s}}^{V_1} \frac{\partial V_1}{V_1} = -\int_1^{\varepsilon_1} \frac{\partial \varepsilon_1}{\varepsilon_1} \tag{28}$$

where V_{1s} represents the superficial velocity of the oil phase and V_1 is the *in-situ* velocity of the oil. Thus:

$$V_1 = \frac{V_{1s}}{\varepsilon_1} \tag{29}$$

Substituting Eq. (29) in Eq. (7):

$$\frac{\partial V_2}{\partial t} + V_2\frac{\partial V_2}{\partial z} + G_1\frac{\partial \varepsilon_1}{\partial z} - \frac{D\sigma}{4\rho_2\sqrt{\varepsilon_1}}\frac{\partial^3 \varepsilon_1}{\partial z^3} = E_1 \tag{30}$$

where:

$$G_1 = -\frac{\rho_1 {V_{1s}}^2}{\rho_2 {\varepsilon_1}^3} \tag{31}$$

$$E_1 = \frac{\tau_i S_i}{A\varepsilon_1(1-\varepsilon_1)\rho_2} - \frac{4\tau_w}{D(1-\varepsilon_1)\rho_2} - \frac{(\rho_1-\rho_2)g}{\rho_2} \tag{32}$$

The set of partial differential equations becomes:

$$\begin{cases} \frac{\partial \varepsilon_1}{\partial t} + (1-\varepsilon_1)\frac{\partial V_2}{\partial z} + V_2\frac{\partial \varepsilon_1}{\partial z} = 0 \\ \frac{\partial V_2}{\partial t} + V_2\frac{\partial V_2}{\partial z} + G_1\frac{\partial \varepsilon_1}{\partial z} - \frac{D\sigma}{4\rho_2\sqrt{\varepsilon_1}}\frac{\partial^3 \varepsilon_1}{\partial z^3} - E_1 = 0 \end{cases} \quad (33)$$

Applying the method of characteristics to the system of PDEs, Eq. (33), and neglecting the higher-order derivatives it arises the following system of ordinary differential equations:

$$\begin{cases} \frac{d\varepsilon_1}{dt} - \frac{(1-\varepsilon_1)}{\sqrt{G_1(\varepsilon_1-1)}}\frac{dV_2}{dt} + E_1\frac{(1-\varepsilon_1)}{\sqrt{G_1(\varepsilon_1-1)}} = 0, along\ C_{L1} = V_2 - \sqrt{G_1(\varepsilon_1-1)} = \frac{dz_1}{dt} \\ \frac{d\varepsilon_1}{dt} + \frac{(\varepsilon_1-1)}{\sqrt{G_1(\varepsilon_1-1)}}\frac{dV_2}{dt} - E_1\frac{(\varepsilon_1-1)}{\sqrt{G_1(\varepsilon_1-1)}} = 0, along\ C_{H1} = V_2 + \sqrt{G_1(\varepsilon_1-1)} = \frac{dz_2}{dt} \end{cases} \quad (34)$$

Where C_{L1} and C_{H1} represent the lower and higher characteristic velocities, respectively. The ODEs system, Eq. (34), can be solved numerically by using the finite difference method, as follows, proposed by Trallero, 1995:

$$\frac{z_{i,k+1}-z_{i+1,k}}{t_{i,k+1}-t_{i+1,k}} = C_{L1_{i+1,k}} \quad (35)$$

$$\frac{z_{i,k+1}-z_{i,k}}{t_{i,k+1}-t_{i,k}} = C_{H1_{i,k}} \quad (36)$$

$$\varepsilon_{1_{i,k+1}} - \varepsilon_{1_{i+1,k}} - \frac{(1-\varepsilon_1)}{\sqrt{G_1(\varepsilon_1-1)}}_{i+1,k}\left(V_{2_{i,k+1}} - V_{2_{i+1,k}}\right) + E_{1_{i+1,k}}\frac{(1-\varepsilon_1)}{\sqrt{G_1(\varepsilon_1-1)}}_{i+1,k}\left(t_{i,k+1} - t_{i+1,k}\right) = 0 \quad (37)$$

$$\varepsilon_{1_{i,k+1}} - \varepsilon_{1_{i,k}} - \frac{(1-\varepsilon_1)}{\sqrt{G_1(\varepsilon_1-1)}}_{i,k}\left(V_{2_{i,k+1}} - V_{2_{i,k}}\right) + E_{1_{i,k}}\frac{(1-\varepsilon_1)}{\sqrt{G_1(\varepsilon_1-1)}}_{i,k}\left(t_{i,k+1} - t_{i,k}\right) = 0 \quad (38)$$

It is important to note that in the case of core-annular flow there is not subcritical flow, because the interfacial wave only propagates in the direction of the flow (Rodriguez and Bannwart, 2008, Rodriguez and Bannwart, 2006). The flow is always supercritical.

4. PRELIMINARY ANALYSIS

4.1. Stratified Flow Pattern

A first qualitative analysis of the propagation of interfacial waves in liquid-liquid stratified flow obtained via MOC can be seen in Fig. (3). It is basically a subcritical flow with a solitary wave propagating up and downstream.

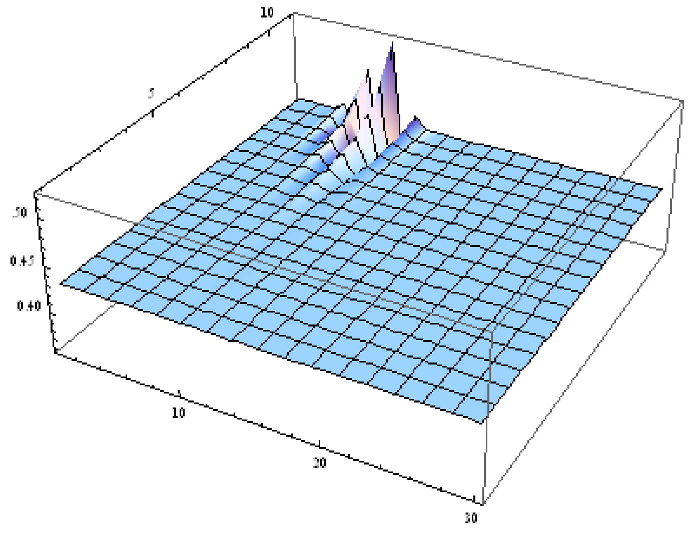

Figure 3. Propagation of a generic solitary wave in liquid-liquid stratified flow (horizontal axes are space and time and vertical axis is the water height).

Some problems were found in the simulation of the propagation of interfacial waves in the stratified flow pattern. The matters are probably due to the neglecting of the interfacial tension terms and because of the type of wave used as initial condition. In the case of liquid-liquid flow the interfacial tension term may be of higher relevance and of the same order of the viscosity dissipation. Therefore, both terms may be significant for stabilizing the flow (Rodriguez and Bannwart (2008)). Thus, it is important to include such terms in the simulation. However, to the best of our knowledge, it has not been accomplished so far. On the other hand, the matter of shape and speed of the initial wave can be easily overcome by using data from the literature (AL-Wahaibi and Angeli (2007)). It should be pointed out that an experiment is being conducted in the present moment in the inclinable oil-water flow loop of the LETeF (EESC - USP) in order to obtain the geometrical and kinetic properties of the interfacial wave of a stratified oil-water flow pattern.

4.2. Core-Annular Flow Pattern

The phenomenon of the propagation of the interfacial wave is not yet very well understood in core-annular flow. On the other hand, some importants features of the wave are already known as its knematic nature and that it propagates only downstream. Based on those features, a study with the MOC is being done. For this analysis the experimental data of oil holdup, interfacial wave profile, velocity and amplitude acquired by Rodriguez (2002) (or Rodriguez and Bannwar, 2006) are being used. Being aware that the interfacial wave of the core-annular flow only propagates in the direction of the flow and that it is stable one can admit that the characteristic directions in which the set of two PDEs can be reduced to a set of two ODEs have to be in a way that alows the wave to propagate as said before. The characteiristic velocities have to be real for the system to be hyperbolic so that it can be solved by the MOC, and they have to be positive. The first approach adopts Eq. (34) without the interfacial tension term. So, the characteristic velocities (C_L and C_H) are calculated as:

$$C_L = V_2 - \sqrt{G(\varepsilon_1 - 1)}$$
$$C_H = V_2 + \sqrt{G(\varepsilon_1 - 1)} \quad (39)$$

Where:

$$G = -\frac{\rho_1 V_{1s}^2}{\rho_2} \quad (40)$$

Figure 4 shows the value of the characteristic velocities. The focus of the analysis is the points that have negative lower characteristic velocities within the oil holdup range between 0.5 and 0.8. According to Rodriguez and Bannwart (2006), these points are related to a stable core-annular flow with an interfacial wave propagating downstream, which is in disagreement with the present predictions.

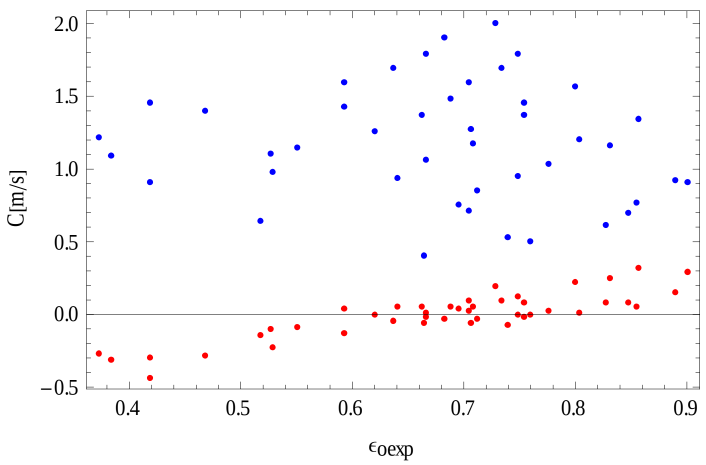

Figure 4. Characteristic Velocities neglecting all the interfacial tension terms *versus* the experimental holdup of oil. Blue points are C_H and red points are C_L.

It is proposed to add to the analysis one of the interfacial tension terms. Firstly, the destabilizing term (Rodriguez (2008)), which is related to the first order derivative. Again, it is used Eq. (34), but now with the term G_1 given by:

$$G_1 = -\frac{\rho_1 V_{1s}^2}{\rho_2 \varepsilon_1^3} - \frac{2\sigma}{D\varepsilon_1 \rho_2 \sqrt{\varepsilon_1}} \quad (41)$$

The influence of the interfacial-tension destabilizing term is not significant, giving an average variation of only 0.3% in the characteristic velocities (Table 1). However, it is important to note is that although the variation is small it causes the lower characteristic velocities to be even more negative. Such a result suggests that the inclusion of a destabilizing term implies in a trend towards subcritical flow, which would be against the experimental observations. Therefore, the question that arises is: would the inclusion of the interfacial-tension stabilizing term produce a trend towards supercritical flow and allow a better prediction?

Table 1. Lower characteristic velocity (C_L), with and without the interfacial shear stress destabilizing term. (Uws and Uos are, respectively, the superficial velocity of water and oil).

Run	Uws [m/s]	Uos [m/s]	Oil holdup	C_L without the destabilizing term [m/s]	C_L with the destabilizing term [m/s]	Variation
2	0.115	0.227	0.52	-0.148	-0.153	3,26%
30	0.18	0.76	0.70	-0.057	-0.058	1,75%
40	0.29	1.0	0.68	-0.027	-0.028	3,7%

Figure 5 shows the characteristic velocities with the inclusion of the destabilizing interfacial tension term as a function of the oil holdup. For the experimental point chosen the predicted oil holdup that is related to both positive characteristic velocities is 13.3% higher than that measured by Rodriguez and Bannwart, 2006.

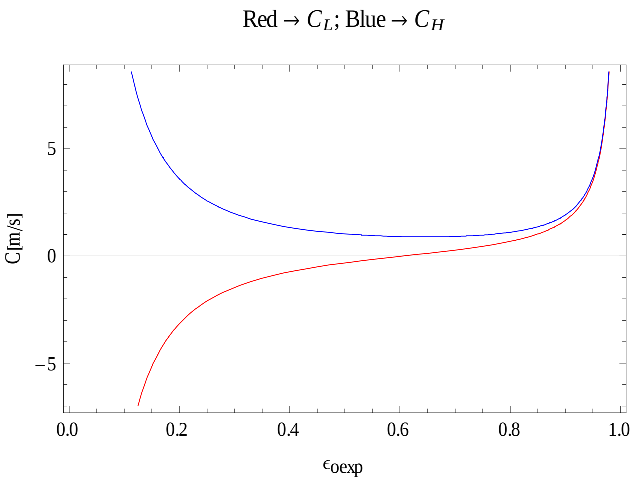

Figure 5. Characteristic velocities with the destabilizing interfacial tension term as function of the oil holdup. Experimental data: Superficial velocity: of water (0.17m/s); of oil (0.35 m/s), experimental oil holdup (0.52).

Looking at the governing equations there is only one term neglected in the MOC, which is the interfacial tension term, *i.e.*, the stabilizing one. So, it is plausible to propose that this term may be of high importance, as suggested by Rodriguez *et al.* (2008) results obtained from a linear stability analysis. Therefore, it is proposed that the inclusion of the stabilizing interfacial tension term in the formulation would allow the simulation of supercritical interfacial waves and, ultimately, better model predictions. However, the inclusion of the stabilizing interfacial tension term in the MOC formulation is not trivial and has not been observed in the literature so far. In order to do so, three different approaches are proposed. First, the simplest way would be to model the interfacial wave as a sinusoidal one. Let's say, for simplicity, a sine. The derivatives could be rewritten as:

$$\begin{aligned} &\varepsilon_1(z)=\sin z \\ &\frac{\partial \varepsilon_1(z)}{\partial z}=\cos z \\ &\frac{\partial^2 \varepsilon_1(z)}{\partial z^2}=-\sin z \\ &\frac{\partial^3 \varepsilon_1(z)}{\partial z^3}=-\cos z \end{aligned} \tag{42}$$

then:

$$K * \frac{\partial \varepsilon_1(z)}{\partial z} = \frac{\partial^3 \varepsilon_1(z)}{\partial z^3} \tag{43}$$

where K is a constant, in this case, equal to -1. Accordingly, the stabilizing interfacial tension term could be readily included in the formulation as long as the value of K is a known quantity. Therefore, it is proposed that:

$$\frac{\partial^3 \varepsilon_1}{\partial z^3} \cong \frac{K_1}{\varepsilon_1 A}\frac{\partial \varepsilon_1}{\partial z} \quad (44)$$

where K_1 is a constant that as a first approach should be empirically evaluated. Notice that to maintain the coherence of units it is necessary to divide de constant by an area (the area of the core was chosen for simplicity).

Another suggested way to include the effect of the stabilizing interfacial tension term is to use another form of the characteristic directions so that they will not be straight anymore, but curves. However, the MOC would have to be adapted. The third way is to deduce a closure equation to model this term, as it is usually done for the viscous dissipation (Wylie and Streeter (1993)).

In order to assess the magnitude of the proposed stabilizing interfacial tension term, a ratio between it, Eq. (45), and the gravitational term, (Eq. (46), is plotted in Fig. 6 as a function of the oil-water input ratio. The value of the constant was estimated through the use of the holdup data of Rodriguez and Bannwart (2006). One can infer that many points are of the same order of magnitude or higher than the gravitational term. Therefore, it is proposed that the stabilizing interfacial tension term should be included in the hydrodynamic stability analysis of the oil-water core annular flow pattern.

$$Tension = \frac{D\sigma}{4\rho_2\sqrt{\varepsilon_1}}\frac{K_1}{\varepsilon_1 A} \quad (45)$$

$$Gravitational = \frac{g(\rho_1-\rho_2)}{\rho_2} \quad (46)$$

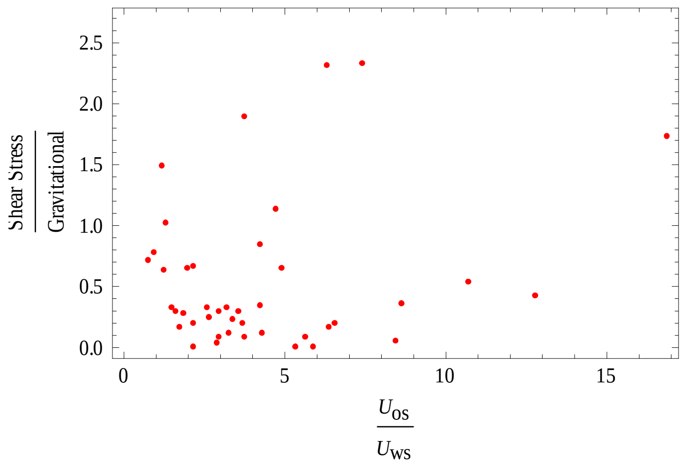

Figure 7. Ratio between the proposed stabilizing interfacial tension term and the gravitational term as a function of the oil-water input ratio.

5. CONCLUSIONS

The stability of the core annular and liquid-liquid stratified flow patterns, in many ways, is an open problem. One of them is the spatial propagation of the interfacial wave. This paper suggests manners to find out how the wave propagation occurs and the terms related to this. The paper has shown that the Method Of Characteristic (MOC) is a way to estimate this propagation of instabilities, as shown in literature.

For liquid-liquid stratified flow, the model predicts a subcritical wave propagation, *i.e.*, it propagates up and downstream, which is against the experimental observations. It could be due to the lack of the interfacial tension terms in the formulation.

Based on the idea that the interfacial wave in the core-annular flow is kinematic and that it propagates in the direction of the flow only, the paper suggests that the stabilizing interfacial tension term is important and of the same order of magnitude of the gravitational term. It is proposed a way to model the stabilizing interfacial tension term via MOC, and other two approaches are proposed but not tested yet.

Acquiring new interfacial wave experimental data of liquid-liquid stratified flow is in order. Also, finding the best way to model the stabilizing interfacial tension term is necessary and, to the best of our knowledge, has not been reported in the literature so far.

6. ACKNOWLEDGEMENTS

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