MATHEMATICAL MODELING FOR LAMINAR FLOW OF POWER LAW FLUID IN POROUS MEDIA

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Abstract. *In this paper, the macroscopic equations for laminar power-law fluid flow is obtained for a porous medium starting from traditional equations (Navier-Stokes). Then, the volume averaging is applied in traditional transport equations with the power-law fluid model. This procedure leads to macroscopic transport equations set for non-Newtonian fluid.*

Keywords*: laminar flow, mathematical modeling, porous medium, power law, volumetric averaging.*

1. INTRODUCTION

In petroleum production there are many parameters that must be analyzed before deciding if an oil well is economically viable and come to decide if the oil exploitation is possible or not from a nature reservoir. One of the main parameters that are usually under consideration for financial analysis by the technical staff to evaluate the capacity of the oil production is the reservoir production oil estimative. In this case, numerical tools can be crucial to make a decision, being that, the experimental simulation usually is not enough to reproduce this process, because the complexity of the problem in question is higher.

In this context, numerical tool get an important function in the several industries and has been the focus of a lot of private and public agencies for development and researches around the world.

The hydrocarbon reservoir basically is made of porous stones staying in the under soil region, where the black oil is stored. The oil, in the beginning, produced by the source rock, migrates to reservoir rocks (porous media), by the action of the buoyancies drivers and by the capillary effects, the reservoir has a boundary formed by a low permeability that that impedes the escape of hydrocarbons from the reservoir rock.

Figure 1 shows two types of different situations of the hybrid media (one clear medium and porous medium they have been analyzed in the same domain) that occurred in secondary recovery of the oil, the first is the water injection process, in this case, the fluid leaves the clear (well) and goes to porous region (reservoir). The second it is the production in the producer well, where the fluids are coming from the porous region (reservoir) to the clear medium (well).

A mathematical model that allows numerical analysis of laminar flow for Non-Newtonian fluids in the porous or hybrid structures is desirable.

In literature, there are few references about the laminar flow of power law fluid in porous media. Then, this paper has as focus to extend the mathematical model development by Pedras and De-Lemos, (2000-2001*a)* to flow of a Non-Newtonian fluid (initially must be adopted the power law constitutive model) in porous medium using the method of local volume averaging.

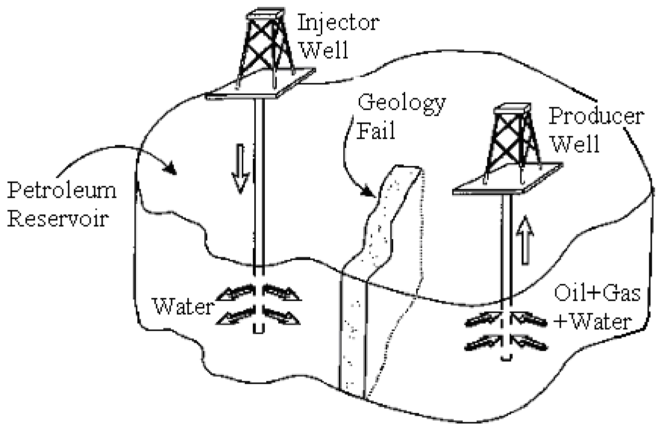

Figure 1: Secondary Oil Recovery Schema.

2. LITERATURE REVIEW

Christopher and Middleman, (1965) have used a capilar tube model to develop an application of the modified Blake-Kozeny equation to solve the laminar Non- Newtonian fluid flow in a packed and in a porous medium. The authors considered the rheological behaviour as being described by the power law model. The theory was tested and the results between analytical results were compared, the average differences between them were of 18%.

Middleman, (1965) solved the governing equations that describe the evolution of the pressure and drag forces in power-law fluid flow in a rectangular section duct.

Brea, Edwards and Wilkinson, (1976) obtained values for flow rates, pressure drop and thickness of a Non Newtonian slurry flow through a fluidized bed within uniform fixed spheres particles with constant diameters. The pressure drop and mass flow rate were obtained using a capillary tube model with rheological data from the slurry. The authors described a mathematical correlation that can be used to estimate the minimum velocity of fluidization, but it is not recommended to estimate boundary expansion characteristics on fluidized state.

Hanks and Larsen, (1979) show **a** simple algebraic solution to volumetric flow rate of the laminar non Newtonian fluid flow regime using the power law model across concentric annulus.

Liu, (1983) applied the Galerkin Finite Element Method (FEM) to determine the pressure drop and the mass flow rate to fully developed flow in arbitrary cross sections of ducts. The author finds out that the method used has a good precision to solve the pressure drop value problem. Liu, (1983) compared that with results obtained by Miller´s method and observed that the data determined by Miller got reasonable values when they were used for a fluid that has power law index close to a unity.

Dharmadhikari and Vale, (1985) proposed a calculus method to prescribe a pressure drop for inelastic fluids.

Hayes, (1990) investigated the fluid flow boundary layers hydrodynamic and thermal comportment on packed bed. The volumetric averaging of the Navier-Stokes and energy conservation was used to describe heat transfer and fluid dynamics. The authors observed that Nusselt numbers have a strong dependence in the following parameters: Reynolds, Graetz numbers and the fluid and solid thermal conductivity ratio.

Du Plessis e Maslilyah, (1991), extended the Navier-Stokes equations to input a new model to solve a laminar fluid flow across granular isotropic rigid porous medium within spatially variable permeability. The results showed versatile and useful equations and that confirm the results obtained by many empirics formulas.

Shenoy, (1993) introduced a Non Newtonian version for Forccheimer´s extension about Darcy´s law to investigate various aspects associated with convective flow in a porous medium that was saturated with power law fluid.

In Shenoy, (1994) was presented a wide literature review about Non Newtonian fluid flow and heat transfer in a porous medium.

Hayes, Afacan and Boulanger, (1995) studied Navier-Stokes applications to prescribe the pressure data in packed beds, that were constituted by spherical particles.

Hayes et al., (1996) developed a theoretical model to predict the pressure drop and velocities for power law fluid flow across the spherical uniform particles packed bed. This model was obtained from applying volumetric average on Navier-Stokes equations. The authors made comparisons with experimental data found in specific literature and concluded then it was a good agreement.

Malin, (1997) showed a numerical simulation of turbulent power law fluid flow in tubes. The results obtained for friction factor and velocity profile for fully developed flow it was compared with experimental data and it was observed a good agreement among them.

The Madhav e Malin, (1997) works showed the existence of a numerical calculus method for fully development flow in tubes. This method permits a very fast numerical simulation when compared with other usual method to solve the elliptical and parabolic problems. The authors also presented that this method was successful in the solution of the various bi-dimensional and three dimensional fully developed flows in ducts problems with or without heat transfer.

Inoue and Nakayama, (1998) investigated the viscous and inertial effects of pressure drop in Non Newtonian flow across a porous medium. The porous medium was simulated by periodic spatially array of cubes. The numerical results were used to obtain a macroscopic relationship between pressure gradient and mass flow rate.

Vijaysri, Chhabra e Eswaran, (1999) studied, theoretically, the steady flow of a power law fluid through a cylindrical rods arrangement. The authors showed results with details in terms of vortices and power law model’s viscosity modifications on the cylindrical surfaces, graphical analysis of stream function, vortices iso-lines and fluids dynamic parameters from viscous and pressure drag coefficients. They also compared the obtained results with the literature data.

Pearson and Tardy, (2002) showed some continuous transport porous medium models and also presented the necessary length scales to transport the physical phenomena from porous scales to Darcy continuous scale using averaging variables. It evaluated the influences of rheology on transport parameters in multiphase and single phase flow.

On this paper will be modelled the transport equations for power law fluid flow in porous medium using the method of local volume averaging

3. MATHEMATICAL MODELING

3.1 Microscopic Transports Equations

The mass conservation equation (or continuity equation), for steady laminar incompressible fluid flow, can be written the follow the way

$$\nabla.\mathbf{V} = \frac{\partial v_i}{\partial x_i} = 0 \tag{1}$$

The vector velocity in Cartesians coordinates is:

$$(\ \mathbf{V} = u_1\mathbf{e}_1 + u_2\mathbf{e}_2 + u_3\mathbf{e}_3\ ) \tag{2}$$

The conservation momentum equation for steady laminar incompressible fluid flow (Navier-Stokes equations) can been written as:

$$\rho\left[\frac{D\mathbf{V}}{Dt}\right] = \rho\mathbf{g} - \nabla p + \nabla.\boldsymbol{\tau} \tag{3}$$

For Generalized Newtonian Fluid, the stress tensor can be according with shear ratio as:

$$\boldsymbol{\tau} = \eta\left[\nabla\mathbf{V} + (\nabla\mathbf{V})^T\right] \tag{4}$$

Substituting the expression (4) in the momentum conservation equation (3), leads

$$\rho\left[\frac{D\mathbf{V}}{Dt}\right] = \rho\mathbf{g} - \nabla p + \nabla.\left\{\eta\left[\nabla\mathbf{V} + (\nabla\mathbf{V})^T\right]\right\} \Rightarrow \rho\left[\frac{\partial\mathbf{V}}{\partial t} + \nabla.(\mathbf{VV})\right] = \rho\mathbf{g} - \nabla\mathbf{p} + \nabla.\left\{\eta\left[\nabla\mathbf{V} + (\nabla\mathbf{V})^T\right]\right\} \tag{5}$$

Where η is apparent viscosity, that's is a function of rate of fluid stress tensor (or rate of deformation tensor); using the power-law model or Ostwald-De Waele models,

$$\eta = m\left|\sqrt{\frac{\boxed{\nabla\mathbf{V} + (\nabla\mathbf{V})^T} : \boxed{\nabla\mathbf{V} + (\nabla\mathbf{V})^T}}{2}}\right|^{n-1} \tag{6}$$

where *m* e *n* are two empirical curve-fitting parameters and are known as the fluid consistency coefficient and the flow behaviour index respectively. For a shear-thinning fluid, the index may have any value between 0 and 1. The smaller the value of *n*, the greater is the degree of shear-thinning. For a shear-thickening fluid, the index *n* will be greater than unity.

The Navier-Stokes equation in Cartesian coordinates can write in the follow form:

In direction $\mathbf{e}_1$:

$$\rho\left[\frac{\partial u_1}{\partial t} + \nabla.(u_1\mathbf{V})\right] = \rho g_{e_1} - \frac{\partial p}{\partial x_1} + \nabla.\left\{\eta_{e_1}\left[\nabla u_1 + (\nabla u_1)^T\right]\right\} \tag{7}$$

In direction $\mathbf{e}_2$:

$$\rho\left[\frac{\partial u_2}{\partial t} + \nabla.(u_2\mathbf{V})\right] = \rho g_{e_2} - \frac{\partial p}{\partial x_2} + \nabla.\left\{\eta_{e_2}\left[\nabla u_2 + (\nabla u_2)^T\right]\right\} \tag{8}$$

In direction $\mathbf{e}_3$:

$$\rho\left[\frac{\partial u_3}{\partial t} + \nabla.(u_3\mathbf{V})\right] = \rho g_{e_3} - \frac{\partial p}{\partial x_3} + \nabla.\left\{\eta_{e_3}\left[\nabla u_3 + (\nabla u_3)^T\right]\right\} \tag{9}$$

Where η_{e_1}, η_{e_2} e η_{e_3}, are given by the follow expressions:

$$\eta_{e_1} = m\left\|\nabla u_1 + (\nabla u_1)^T\right\|^{n-1} = m\left[\sqrt{2\left(\frac{\partial u_1}{\partial x_1}\right)^2 + \left(\frac{\partial u_1}{\partial x_2}\right)^2 + \left(\frac{\partial u_1}{\partial x_3}\right)^2 + \left(\frac{\partial u_2}{\partial x_1}\right)^2 + \left(\frac{\partial u_3}{\partial x_1}\right)^2}\right]^{n-1}, \tag{10}$$

$$\eta_{e_2} = m\left\|\nabla u_2 + (\nabla u_2)^T\right\|^{n-1} = m\left[\sqrt{\left(\frac{\partial u_2}{\partial x_1}\right)^2 + 2\left(\frac{\partial u_2}{\partial x_2}\right)^2 + \left(\frac{\partial u_2}{\partial x_3}\right)^2 + \left(\frac{\partial u_1}{\partial x_2}\right)^2 + \left(\frac{\partial u_3}{\partial x_2}\right)^2}\right]^{n-1}, \tag{11}$$

$$\eta_{e_3} = m\left\|\nabla u_3 + (\nabla u_3)^T\right\|^{n-1} = m\left[\sqrt{\left(\frac{\partial u_3}{\partial x_1}\right)^2 + \left(\frac{\partial u_3}{\partial x_2}\right)^2 + 2\left(\frac{\partial u_3}{\partial x_3}\right)^2 + \left(\frac{\partial u_1}{\partial x_3}\right)^2 + \left(\frac{\partial u_2}{\partial x_3}\right)^2}\right]^{n-1} \tag{12}$$

Substituting the term $\eta_{e_1} = m\left\|\nabla u_1 + (\nabla u_1)^T\right\|^{n-1}$ in momentum conservation equation in direction $\mathbf{e}_1$, gets:

$$\rho\left[\frac{\partial u_1}{\partial t}+\nabla.(u_1\mathbf{V})\right]=\rho g_{e_1}-\frac{\partial p}{\partial x_1}+\nabla.\left\{m\left\|\nabla u_1+(\nabla u_1)^T\right\|^{n-1}\left[\nabla u_1+(\nabla u_1)^T\right]\right\}=\rho g_{e_1}-\frac{\partial p}{\partial x_1}+$$
$$\left[\nabla u_1+(\nabla u_1)^T\right]\nabla\left[m\left\|\nabla u_1+(\nabla u_1)^T\right\|^{n-1}\right]+m\left\|\nabla u_1+(\nabla u_1)^T\right\|^{n-1}\nabla.\left[\nabla u_1+(\nabla u_1)^T\right] \tag{13}$$

Observe that's,

$$\nabla.\left[(\nabla u_1)^T\right]=\mathbf{e}_i\frac{\partial}{\partial x_i}.\left(\frac{\partial v_j}{\partial x_1}\mathbf{e}_j\right)=\frac{\partial}{\partial x_1}\underbrace{\frac{\partial v_i}{\partial x_i}}_{I}=0 \tag{14}$$

Because the term I in the expression (14) is null, for more information see equation (1) and find that

$$\rho\left[\frac{\partial u_1}{\partial t}+\nabla.(u_1\mathbf{V})\right]=\rho g_{e_1}-\frac{\partial p}{\partial x_1}+\left[\nabla u_1+(\nabla u_1)^T\right]\nabla\left[m\left\|\nabla u_1+(\nabla u_1)^T\right\|^{n-1}\right]+m\left\|\nabla u_1+(\nabla u_1)^T\right\|^{n-1}\nabla^2 u_1 \tag{15}$$

Admitting the viscosity coefficient, m, with a constant value, leads to:

$$\rho\left[\frac{\partial u_1}{\partial t}+\nabla.(u_1\mathbf{V})\right]=\rho g_{e_1}-\frac{\partial p}{\partial x_1}+m\left\{\left[\nabla u_1+(\nabla u_1)^T\right]\nabla\left\|\nabla u_1+(\nabla u_1)^T\right\|^{n-1}+\left\|\nabla u_1+(\nabla u_1)^T\right\|^{n-1}\nabla^2 u_1\right\} \tag{16}$$

For others directions $\mathbf{e}_2$ and $\mathbf{e}_3$, the same proceedings are valid.
In the tensorial form, momentum conservation equation is:

$$\rho\left[\frac{\partial \mathbf{V}}{\partial t}+\nabla.(\mathbf{VV})\right]=\rho\mathbf{g}-\nabla p+m\nabla.\left\{\left|\sqrt{\frac{\left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right]:\left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right]}{2}}\right|^{n-1}\left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right]\right\} \tag{17}$$

3.2 Basic Definitions

In order to facilitate the comprehension of the equations and the mathematical operations that will presented in the next sections the basic definitions, extracted from specific literature references, were repeated here. For more details, see Bear, (1972) and De-Lemos, (2006).

Representative Elementary Volume, (REV): Is the porous medium volume, where the volume averages of a certain quantity are defined. Figure (2) shows an example of this volume, [Bear, (1972)].

Porosity, ϕ : Is the ratio between the fluid volume inside the REV, ΔV_f, and ΔV.

$$\phi=\frac{\Delta V_f}{\Delta V} \tag{18}$$

Intrinsic Volume Average: Is the volume average of a certain quantity, φ , over the representative elementary volume, ΔV, weighed by the phase volume which the amount, φ belongs. For example, if φ is a fluid property, its intrinsic volume average is expressed by:

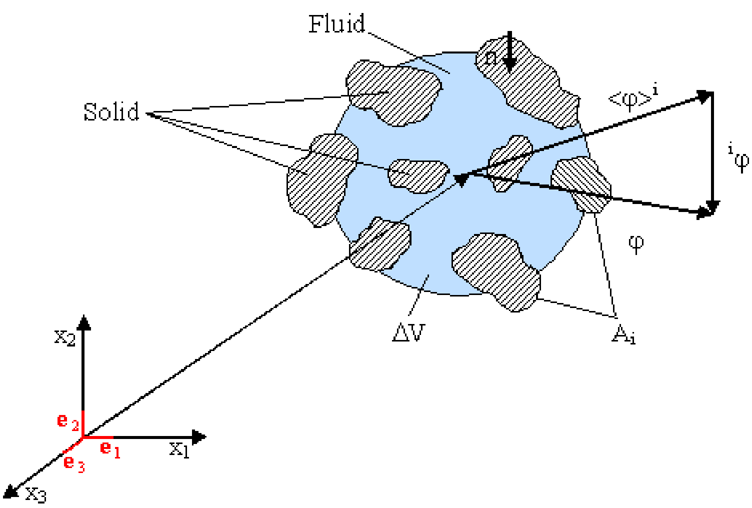

Figure 2– Representative Elementary Volume, REV.

$$\langle\varphi\rangle^i = \frac{1}{\Delta V_f}\int_{\Delta V_f}\varphi dV \tag{19}$$

Spatial Fluctuation: Is the difference between the local value (microscopic) of a certain quantity, φ, and its intrinsic volume average, $\langle\varphi\rangle^i$ [Whitaker, (1969)]., as shown in the Fig. (1)

$$^i\varphi = \varphi - \langle\varphi\rangle^i \quad => \quad \langle^i\varphi\rangle^i = 0 \tag{20}$$

Superficial Volume average: Is the volume average over ΔV of a certain quantity φ. If φ is a fluid property, one has:

$$\langle\varphi\rangle^v = \frac{1}{\Delta V}\int_{\Delta V_f}\varphi dV = \frac{\langle\varphi\rangle^i \Delta V_f}{\Delta V} = \phi\langle\varphi\rangle^i \tag{21}$$

On the other hand, if φ is a solid property (porous matrix), one has:

$$\langle\varphi\rangle^v = \frac{1}{\Delta V}\int_{\Delta V_s}\varphi dV = \frac{\langle\varphi\rangle^i \Delta V_s}{\Delta V} = (1-\phi)\langle\varphi\rangle^i \tag{22}$$

Darcy's Velocity: Is the superficial volume average of the fluid velocity:

$$\mathbf{V}_D = \langle\mathbf{V}\rangle^v = \frac{1}{\Delta V}\int_{\Delta V_f}\mathbf{V}dV = \phi\langle\mathbf{V}\rangle^i \tag{23}$$

Since Slattery, (1967) there were great progresses in volumetric average techniques applied to microscopic conservations equations, that's find a relationship between volumetric average of derivatives and derivatives of volumetric average.

Whitaker, (1969) comments that in the process of volumetric average (equation (21)) there are three types of characteristic length must been defined:

i) The microscopic length, *d*, on which occur substantial variations of the microscopic fluid velocities;

ii) The macroscopic length, *L*, on which occur substantial variations of the macroscopic fluid velocities (equation (23)) and

iii) The length, *l* associated to with representative elementary volume (REV).

The volume average of a certain quantity φ is the transformation that defines the quantity $\langle\varphi\rangle^v$ at the center of the REV.

Within three characteristics length Whitaker, (1969) shows that the relationship $\langle\langle\varphi\rangle^v\rangle^v = \langle\varphi\rangle^v$ just only true if:

$$d \ll l \ll L \tag{24}$$

From the concept of the Representative elementary volume and characteristics lengths *d*, *L* and *l* it was development by (Slattery, (1967), Whitaker, (1969), Gray and Lee, (1977)) Local Volumetric Average Theorem, their relationship are:

$$\langle\nabla\varphi\rangle^v = \nabla(\phi\langle\varphi\rangle^i) + \frac{1}{\Delta V}\int_{A_i}\mathbf{n}\varphi dS \tag{25}$$

$$\langle\nabla\cdot\boldsymbol{\varphi}\rangle^v = \nabla\cdot(\phi\langle\boldsymbol{\varphi}\rangle^i) + \frac{1}{\Delta V}\int_{A_i}\mathbf{n}\cdot\boldsymbol{\varphi} dS \tag{26}$$

$$\langle\frac{\partial\varphi}{\partial t}\rangle^v = \frac{\partial}{\partial t}(\phi\langle\varphi\rangle^i) - \frac{1}{\Delta V}\int_{A_i}\mathbf{n}\cdot(\mathbf{u}_i\varphi)dS \tag{27}$$

where A_i and $\mathbf{u}_i$ represent the area and velocity of the interface fluid/solid, respectively, and **n** is the external unit vector to the fluid and normal to the A_i. (Figure 2**).** In the development of the Eqs. (25)-(27) the only imposed restriction is the independence of ΔV in relation to the time and space (Whitaker, (1969) and Gray and Lee, (1977)). Therefore, if the medium is undeformable, then ΔV_f will be dependent only on the space and not on the time (Gray and Lee, (1977)).

3.3 Macroscopic Transport Equations

Applying the Local Volume Average Theorem (LVAT); the volumetric average of microscopic mass conservation equation for a fluid with constant mass specific ρ (Equation (1)) can has been the form as:

$$\langle\nabla\cdot\mathbf{V}\rangle^v = \nabla\cdot(\phi\langle\mathbf{V}\rangle^i) + \frac{1}{\Delta V}\int_{A_i}\mathbf{n}\cdot\mathbf{V}dS = 0 \tag{28}$$

Because the non-slip condition, the microscopic velocities, **V**, is zero on interfacial superficies A_i simply the equation (28), joint with the Dupuit-Forchheimer relationship, $\mathbf{V}_D = \phi\langle\mathbf{V}\rangle^i$, for:

$$\nabla\cdot\mathbf{V}_D=0 \tag{29}$$

The equation (29) represents the macroscopic mass conservation for an incompressible fluid.

The equation for momentum conservation, for a power law fluid model (with ρ and m keeps constants) flow in porous media can be written as:

$$\rho\left[\frac{\partial\mathbf{V}}{\partial t}+\nabla.(\mathbf{VV})\right]=\rho\mathbf{g}-\nabla p+m\nabla.\left\{\left|\sqrt{\frac{[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]:[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]}{2}}\right|^{n-1}[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]\right\} \tag{30}$$

Based on this, was developed the LVAT [Whitaker, (1969), Gray and Lee, (1977)], whose relationships are expressed by:

$$\left\langle\frac{\partial\mathbf{V}}{\partial t}\right\rangle^v=\frac{\partial}{\partial t}(\phi\langle\mathbf{V}\rangle^i)-\frac{1}{\Delta V}\int_{A_i}\mathbf{n}\cdot(\mathbf{u}_i\mathbf{V})dS \tag{31}$$

$$\langle\nabla\cdot(\mathbf{VV})\rangle^v=\nabla\cdot(\phi\langle\mathbf{VV}\rangle^i)+\frac{1}{\Delta V}\int_{A_i}\mathbf{n}\cdot(\mathbf{VV})dS \tag{32}$$

$$\langle\rho\mathbf{g}\rangle^v=\phi\rho\mathbf{g} \tag{33}$$

$$\langle\nabla p\rangle^v=\nabla(\phi\langle p\rangle^i)+\frac{1}{\Delta V}\int_{A_i}\mathbf{n}\,p\,dS \tag{34}$$

$$\begin{aligned}
&\left\langle\nabla.\left\{\left(\sqrt{\frac{[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]:[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]}{2}}\right)^{n-1}[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]\right\}\right\rangle^v=\\
&\nabla.\left\langle\left\{\left(\sqrt{\frac{[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]:[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]}{2}}\right)^{n-1}[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]\right\}\right\rangle^v+\\
&\frac{1}{\Delta V}\int_{A_i}\mathbf{n}.\left\{\left(\sqrt{\frac{[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]:[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]}{2}}\right)^{n-1}[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]\right\}dS
\end{aligned} \tag{35}$$

Where

$$\begin{aligned}
&\left\langle\left\{\left(\sqrt{\frac{[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]:[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]}{2}}\right)^{n-1}[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]\right\}\right\rangle^v=\\
&\left\langle\nabla\mathbf{V}\left(\sqrt{\frac{[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]:[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]}{2}}\right)^{n-1}+(\nabla\mathbf{V})^T\left(\sqrt{\frac{[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]:[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]}{2}}\right)^{n-1}\right\rangle^v=\\
&\left\langle\nabla\mathbf{V}\left(\sqrt{\frac{[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]:[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]}{2}}\right)^{n-1}\right\rangle^v+\left\langle(\nabla\mathbf{V})^T\left(\sqrt{\frac{[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]:[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]}{2}}\right)^{n-1}\right\rangle^v=\\
&\phi\langle\nabla\mathbf{V}\rangle^i\left\langle\left(\sqrt{\frac{[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]:[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]}{2}}\right)^{n-1}\right\rangle^i+\phi\left\langle(\nabla\mathbf{V})^T\right\rangle^i\left\langle\left(\sqrt{\frac{[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]:[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]}{2}}\right)^{n-1}\right\rangle^i=\\
&\langle\nabla\mathbf{V}\rangle^v\frac{\phi}{\phi}\left\langle\left(\sqrt{\frac{[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]:[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]}{2}}\right)^{n-1}\right\rangle^i+\left\langle(\nabla\mathbf{V})^T\right\rangle^v\frac{\phi}{\phi}\left\langle\left(\sqrt{\frac{[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]:[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]}{2}}\right)^{n-1}\right\rangle^i=\\
&\langle\nabla\mathbf{V}\rangle^v\frac{1}{\phi}\left\langle\left(\sqrt{\frac{[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]:[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]}{2}}\right)^{n-1}\right\rangle^v+\left\langle(\nabla\mathbf{V})^T\right\rangle^v\frac{1}{\phi}\left\langle\left(\sqrt{\frac{[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]:[\nabla\mathbf{V}+(\nabla\mathbf{V})^T]}{2}}\right)^{n-1}\right\rangle^v
\end{aligned} \tag{36}$$

and

$$
\begin{aligned}
&\left\langle \left(\sqrt{\frac{\left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right]:\left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right]}{2}} \right)^{n-1} \right\rangle^{v} = \\
&\phi\left\langle \left(\sqrt{\frac{\left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right]:\left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right]}{2}} \right) \right\rangle^{i}_{1} \left\langle \left(\sqrt{\frac{\left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right]:\left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right]}{2}} \right) \right\rangle^{i}_{2} \cdots \left\langle \left(\sqrt{\frac{\left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right]:\left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right]}{2}} \right) \right\rangle^{i}_{n-1} = \\
&\left\langle \left(\sqrt{\frac{\left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right]:\left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right]}{2}} \right) \right\rangle^{v} \frac{\phi^{n-2}}{\phi^{n-2}} \left[\left\langle \left(\sqrt{\frac{\left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right]:\left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right]}{2}} \right) \right\rangle^{i} \right]^{n-2} = \\
&\frac{1}{\phi^{n-2}} \left\langle \left(\sqrt{\frac{\left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right]:\left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right]}{2}} \right) \right\rangle^{v} \left[\left\langle \left(\sqrt{\frac{\left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right]:\left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right]}{2}} \right) \right\rangle^{v} \right]^{n-2} = \\
&\frac{1}{\phi^{n-2}} \left[\left\langle \left(\sqrt{\frac{\left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right]:\left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right]}{2}} \right) \right\rangle^{v} \right]^{n-1}
\end{aligned}
\tag{37}
$$

We can find,

$$
\begin{aligned}
&\left\langle \nabla\cdot\left\{ \left(\sqrt{\frac{\left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right]:\left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right]}{2}} \right)^{n-1} \left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right] \right\} \right\rangle^{v} = \\
&\nabla\cdot\left\langle \left\{ \frac{1}{\phi^{n-1}} \left[\left\langle \left(\sqrt{\frac{\left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right]:\left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right]}{2}} \right) \right\rangle^{v} \right]^{n-1} \left[\nabla\mathbf{V}_D+(\nabla\mathbf{V}_D)^T+\frac{2}{\Delta V}\int_{A_i}\mathbf{n}\mathbf{V}dS \right] \right\} \right\rangle^{v} + \\
&\frac{1}{\Delta V}\int_{A_i}\mathbf{n}\cdot\left\{ \left(\sqrt{\frac{\left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right]:\left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right]}{2}} \right)^{n-1} \left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right] \right\} dS
\end{aligned}
\tag{38}
$$

Note that's volumetric averaging from deformation rate that's was volumetric average of root square of deformation ratio can be written as follow way:

$$
\begin{aligned}
&\left\langle \sqrt{\frac{\left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right]:\left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right]}{2}} \right\rangle^{v} = \phi\left\langle \sqrt{\frac{\left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right]:\left[\nabla\mathbf{V}+(\nabla\mathbf{V})^T\right]}{2}} \right\rangle^{i} = \\
&\sqrt{\frac{\phi^2\left[\langle\nabla\mathbf{V}\rangle^i+\left\langle(\nabla\mathbf{V})^T\right\rangle^i\right]:\left[\langle\nabla\mathbf{V}\rangle^i+\left\langle(\nabla\mathbf{V})^T\right\rangle^i\right]}{2}} = \sqrt{\frac{\left[\phi\langle\nabla\mathbf{V}\rangle^i+\phi\left\langle(\nabla\mathbf{V})^T\right\rangle^i\right]:\left[\phi\langle\nabla\mathbf{V}\rangle^i+\phi\left\langle(\nabla\mathbf{V})^T\right\rangle^i\right]}{2}} = \\
&\sqrt{\frac{\left[\langle\nabla\mathbf{V}\rangle^v+\left\langle(\nabla\mathbf{V})^T\right\rangle^v\right]:\left[\langle\nabla\mathbf{V}\rangle^v+\left\langle(\nabla\mathbf{V})^T\right\rangle^v\right]}{2}} = \sqrt{\frac{\left[\nabla\mathbf{V}_D+(\nabla\mathbf{V}_D)^T\right]:\left[\nabla\mathbf{V}_D+(\nabla\mathbf{V}_D)^T\right]}{2}}
\end{aligned}
\tag{39}
$$

And knowing that is $\mathbf{V}=0$ on A_i, superficial and for an underformable porous medium, $\mathbf{u}_i=0$, the macroscopic momentum conservation can be express with:

$$
\begin{aligned}
&\rho\left[\frac{\partial\phi\langle\mathbf{V}\rangle^i}{\partial t}+\nabla\cdot\left(\phi\langle\mathbf{V}\mathbf{V}\rangle^i\right)\right]=\phi\rho\mathbf{g}-\nabla\left(\phi\langle p\rangle^i\right)+ \\
&\frac{m}{\phi^{n-1}}\nabla\cdot\left\{ \left| \sqrt{\frac{\left[\nabla\phi\langle\mathbf{V}\rangle^i+\left(\nabla\phi\langle\mathbf{V}\rangle^i\right)^T\right]:\left[\nabla\phi\langle\mathbf{V}\rangle^i+\left(\nabla\phi\langle\mathbf{V}\rangle^i\right)^T\right]}{2}} \right|^{n-1} \left[\nabla\phi\langle\mathbf{V}\rangle^i+\left(\nabla\phi\langle\mathbf{V}\rangle^i\right)^T\right] \right\}+\mathbf{R}
\end{aligned}
\tag{40}
$$

Where

$$\mathbf{R} = \frac{m}{\Delta V}\int_{A_i} \mathbf{n}.\left\{\left(\sqrt{\frac{\left[\nabla\mathbf{V} + (\nabla\mathbf{V})^T\right]:\left[\nabla\mathbf{V} + (\nabla\mathbf{V})^T\right]}{2}}\right)^{n-1}\left[\nabla\mathbf{V} + (\nabla\mathbf{V})^T\right]\right\}dS - \frac{1}{\Delta V}\int_{A_i}\mathbf{n}pdS \tag{41}$$

Represents total drag force per volume unit (superficial force weighed by volume) due the presence of solid particles, composed by both the viscous and form (or pressure) drags.

From the concept of spatially fluctuations (equation (20)), the divergent operator, in the left side of the equation (40) can be expanded as:

$$\nabla\cdot(\phi\langle\mathbf{V}\mathbf{V}\rangle^i) = \nabla\cdot[\phi(\langle\mathbf{V}\rangle^i\langle\mathbf{V}\rangle^i + \langle{}^i\mathbf{V}\,{}^i\mathbf{V}\rangle^i)] \tag{42}$$

Where the term $\nabla\cdot(\phi\langle{}^i\mathbf{V}\,{}^i\mathbf{V}\rangle^i)$ represents the hydrodynamic dispersion that is the highest order than as compare with the term $\nabla\cdot(\phi\langle\mathbf{V}\rangle^i\langle\mathbf{V}\rangle^i)$ (represents the macroscopic convective inertial force, Hsu and Cheng, (1990)) thus being neglected. In this former case, the macroscopic of momentum conservation equation in terms of Darcy velocity ($\mathbf{V}_D = \phi\langle\mathbf{V}\rangle^i$) it was reduced like:

$$\begin{aligned}&\rho\left[\frac{\partial\mathbf{V}_D}{\partial t} + \nabla.\left(\frac{\mathbf{V}_D\mathbf{V}_D}{\phi}\right)\right] = \phi\rho\mathbf{g} - \nabla\left(\phi\langle p\rangle^i\right) + \\ &\frac{m}{\phi^{n-1}}\nabla.\left\{\left|\sqrt{\frac{\left[\nabla\mathbf{V}_D + (\nabla\mathbf{V}_D)^T\right]:\left[\nabla\mathbf{V}_D + (\nabla\mathbf{V}_D)^T\right]}{2}}\right|^{n-1}\left[\nabla\mathbf{V}_D + (\nabla\mathbf{V}_D)^T\right]\right\} + \mathbf{R}\end{aligned} \tag{43}$$

In porous media, the transient of Darcy velocity, normally, it decays quickly (Whitaker, (1969), Bear, (1972), Nield and Bejan, (1992)), hardly exceeding to as one second. Since this way, the tem is neglecting in the macroscopic momentum conservation equation in almost all practical cases (Whitaker, (1969), Bear, (1972), Vafai and Tien, (1981), Vafai and Thiyagaraja, (1987), Vafai and Kim, (1990a), Vafai and Kim, (1990b), Nield and Bejan, (1992)).

The divergent on the left side of equation (43) represents the macroscopic inertial force that is always it negligible when compared with the terms on the right side of this equation (Hsu and Cheng, (1990)). This term as responsibility to increment of macroscopic hydrodynamic boundary layer, that happens in terms of scale order magnitude of length Ku_C/ν (where u_C is the non disturbing Darcy´s velocity; Vafai and Tien, (1981), yours values is small in most of the practical situations.

The term, $\frac{m}{\phi^{n-1}}\nabla.\left\{\left|\sqrt{\frac{\left[\nabla\mathbf{V}_D + (\nabla\mathbf{V}_D)^T\right]:\left[\nabla\mathbf{V}_D + (\nabla\mathbf{V}_D)^T\right]}{2}}\right|^{n-1}\left[\nabla\mathbf{V}_D + (\nabla\mathbf{V}_D)^T\right]\right\}$, in the equation (43) is the sponsor for the macroscopic boundary layers profiles almost all of the situations are negligible, the exception is in the interface regions (porous medium/impermeable wall, distinct porous media and porous medium/ fluid). Because the fact then the hydrodynamics boundary layers is confined on the fine length, the experimental observations is very complex and difficult task, for this reason, the experimental data has been limited to the bulk effects, since like pressure drop and mass flow, where the most of all cases the effect of macroscopic hydrodynamics boundary layer is negligible.

With base in the commentaries above we concluded the conservation moment equation describes the classical experiments as the conducted by Darcy, (1856), Forchheimer, (1901), Ward, (1964), it's resumed as,

$$\nabla(\phi\langle p\rangle^i) = \phi\rho\mathbf{g} + \mathbf{R} \tag{44}$$

That's for smaller gradients of ϕ,

$$\nabla\langle p\rangle^i = \rho\mathbf{g} + \frac{1}{\phi}\mathbf{R} \tag{45}$$

By another way, the modificated expanded Darcy-Forchheimer model for Power-law fluid (Shenoy, (1993)) is:

$$\nabla\langle p\rangle^i = \rho\mathbf{g} - \left(\frac{m}{K^*}\|\mathbf{V}_D\|^{n-1} - \frac{c_F\rho\|\mathbf{V}_D\|}{\sqrt{K}}\right)\mathbf{V}_D \tag{46}$$

Comparing the equations (45) e (46) the total drag force per volume unity, **R**, can be expressed by:

$$\mathbf{R} = -\left[\frac{m\phi}{K^*}\|\mathbf{V}_D\|^{n-1} - \frac{c_F\rho\phi\|\mathbf{V}_D\|}{\sqrt{K}}\right]\mathbf{V}_D \tag{47}$$

Where K^*, is modificated permeability were proposed by Inoue and Nakayama, (1998), as the follow form:

$$K^* = \frac{6}{25}\left(\frac{n\phi}{1+3n}\right)^n\left[\frac{\phi d_p}{3(1-\phi)}\right]^{n+1} \tag{48}$$

Where d_p is solid particle diameter that's composing the porous medium.

Substituting the expression of **R** in the equation (43) the macroscopic momentum equation becomes,

$$\rho\left[\frac{\partial \mathbf{V}_D}{\partial t}+\nabla.\frac{\mathbf{V}_D\mathbf{V}_D}{\phi}\right]=\phi\rho\mathbf{g}-\nabla\left(\phi\langle p\rangle^i\right)+$$
$$\frac{m}{\phi^{n-1}}\nabla.\left\{\left|\sqrt{\frac{\left[\nabla\mathbf{V}_D+(\nabla\mathbf{V}_D)^T\right]:\left[\nabla\mathbf{V}_D+(\nabla\mathbf{V}_D)^T\right]}{2}}\right|^{n-1}\left[\nabla\mathbf{V}_D+(\nabla\mathbf{V}_D)^T\right]\right\}-\left[\frac{m\phi}{K^*}\left\|\mathbf{V}_D\right\|^{n-1}+\frac{c_F\phi\rho\left\|\mathbf{V}_D\right\|}{\sqrt{K}}\right]\mathbf{V}_D \tag{49}$$

Equation (49) in Cartesians coordinates, in direction e_1, becomes the follow form:

$$\rho\left[\frac{\partial u_{1D}}{\partial t}+\nabla.\frac{u_{1D}\mathbf{V}_D}{\phi}\right]=\phi\rho g_{e_1}-\frac{\partial\left(\phi\langle p\rangle^i\right)}{\partial x_1}+\frac{m}{\phi^{n-1}}\nabla.\left\{\left[\left\|\nabla u_{1D}+(\nabla u_{1D})^T\right\|\right]^{n-1}\left[\nabla u_{1D}+(\nabla u_{1D})^T\right]\right\}$$
$$-\left[\frac{m\phi}{K^*}\left\|\mathbf{V}_D\right\|^{n-1}+\frac{c_F\phi\rho\left\|\mathbf{V}_D\right\|}{\sqrt{K}}\right]u_{1D} \tag{50}$$

Where

$$\left[\left\langle\left\|\nabla u_1+(\nabla u_1)^T\right\|\right\rangle^v\right]^{n-1}=\left[\sqrt{2\left(\frac{\partial u_{1D}}{\partial x_1}\right)^2+\left(\frac{\partial u_{1D}}{\partial x_2}\right)^2+\left(\frac{\partial u_{1D}}{\partial x_3}\right)^2+\left(\frac{\partial u_{2D}}{\partial x_1}\right)^2+\left(\frac{\partial u_{3D}}{\partial x_1}\right)^2}\right]^{n-1} \tag{51}$$

In directions $\mathbf{e}_2$, $\mathbf{e}_3$, the process of development are analogous.

4. CONCLUSING REMARKS

In this work, the equations have been derived for laminar power-law fluid flow in porous media. Derivations were carried out under the light of the method of local volume averaging. This procedure leads to macroscopic transport equations set for non-Newtonian fluid. Ultimately, it is expected that additional reseach on this new subject be stimulated by the derivations here presented.

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