MONTE CARLO METHOD APPLIED IN THE ANALYSIS OF THE PDF OF A BISTABLE FLOW ON TWO PARALLEL CIRCULAR CYLINDERS

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Abstract. *This work presents a study about the bistable phenomenon which occurs in turbulent flows impinging on two side-by-side circular cylinders. A Monte Carlo method is applied to generate random numbers, in order to obtain a probabilistic distribution functions (PDF) changed by the phenomenon under study. The random numbers are generated and then compared with time series of axial and transversal velocity, as well as the angle of deviation of the flow, obtained with the constant temperature hot wire anemometry technique in an aerodynamic channel. Discrete and continuous wavelet transforms are used, enabling the multilevel decomposition of a time signal in several bandwidth values, accordingly with the selected decomposition level and the analysis of the energy content of the time series. The definition of a pseudofrequency could be established, which can represent the probable mean frequency of the switches between the flow modes. The results show that in the changes between the flow modes the increase in the axial velocity component is accompanied by increased of the transverse component, and the PDF obtained by the Monte Carlo method for these velocity components present the predominance of two major states of energy, but with different shapes. The angle of deviation of the flow tends to have smaller fluctuations when the flow direction changes from the wide near-wake to the narrow near-wake. The wide near-wake, which has a higher frequency concentration, has a higher mean velocity than the narrow near-wake, but is less in energy. The spectrogram of the angle of deviation of the flow has an opposite behavior, where the higher energy content is distributed along a large frequency band. By analyzing this result together with the discrete reconstruction, is possible to observe that the mean feature of the signal is expressed by the fluctuations, since the low frequencies until the higher one. Comparisons between the probabilities of occurrence of the two modes show no evident correlation between the changes with time. The non-optimized Monte Carlo algorithm applied in the simulations presents low percentages of successful runs, with high computational costs. A temporal analysis of the axial velocity behavior shows that the PDF changes intermittently and its energy content is higher and more spread in frequency domain when the mean velocity increases.*

Keywords*: turbulent flow, circular cylinders, hot wires, Monte Carlo method, probability density function.*

1. INTRODUCTION

Banks of tubes or rods are found in the nuclear and process industries, being the most common geometry used in heat exchangers. Tube banks are the usual simplification for fluid flow and heat transfer in the study of shell-and-tube heat exchangers, where the coolant is forced to flow transversely to the tubes by the action of baffle plates. Geometric characterization of a tube bank is made by the P/D-ratio, being D the tube diameter and P, the pitch, which is the distance between the centerlines of adjacent tubes. Tube arrangement is also equally important in the characterization of fluid flow and heat transfer of a tube bank.

Bearman and Wadcock (1973) identified the presence of intermittences on the flow on two parallel circular cylinders, where the vortex wake that emanates from the gap between them moves from one side to another, randomly. The authors called this phenomenon of "bistability".

The need for more efficient heat exchangers leads the operating conditions of these equipments to become critical, due to the reduction of the aspect ratio of the tube banks (pitch-to-diameter ratio) and the increasing of the flow velocity. As a consequence of the reduction of the flow area in the narrow gaps between the tubes, which causes velocity fluctuations, and the constant change of the flow direction, static and dynamic loads will be increased (Endres *et al*., 1995). According to Blevins (1990), the dynamic loads of the turbulent flow over small aspect ratio tube banks are characterized by broad band turbulence, without a defined shedding frequency. For large aspect ratio tube banks, the dynamic loads are basically associated with vortex shedding process.

The cross flow passing a tube in a bank is strongly influenced by the presence of the neighboring tubes. In the narrow gap between two tubes in a row, the strong pressure gradient will influence not only the flow in that region, but the flow distribution downstream of this point, in the narrow gap between two tubes in the next row, and so on (Olinto et al., 2009).

According to Zdravkovich and Stonebanks (1988) the leading feature of flow-induced vibration in tube banks is the randomness of dynamic responses of tubes. Even if the tubes are all of equal size, have the same dynamic characteristics, are arranged in regular equidistant rows and are subjected to uniform steady flow, the dynamic response of tubes is non-uniform and random.

The Monte Carlo method is a statistical method used in stochastic simulations with various applications in areas such as physics, mathematics and biology. Simulations with this method have being applied since the 40's in digital computers, where a possible statistical approach to solving the problem of neutron diffusion in fissionable material was first presented by Metropolis and Ulam (1949), and according to Kalos and Whitlock (1986), these methods are in fact computationally effective, compared with deterministic methods when treating many-dimensional problems.

Furthermore, the study of the behavior of the bistability phenomenon in simplified geometries, as in the case of two tubes placed side-by-side, helps in understanding the parameters and variables that influence more complex geometries, such as in tube banks.

2. THE BISTABLE EFFECT

According to Sumner *et al.* (1999) the cross steady flow trough circular cylinder with same diameter (D) placed side-by-side can present a wake with different modes, depending on distances between its centers, called pitch (p). Different flow behaviors can be found for different pitch-to-diameter ratios P/D. For the case when 1.2 < P/D < 2.0 (also called intermediate pitch ratios), the flow is characterized by a wide near-wake behind one of the cylinders and a narrow near-wake behind the other, as shown schematically in Fig. 1a and Fig. 1b. This phenomenon generates two dominants vortex-shedding frequencies, each one associated with a wake: the wide wake is associated with a lower frequency and the narrow wake with a higher one.

Bistable phenomenon is characterized by a switching of this gap flow, which is biased toward the cylinder, from one side to other at irregular time intervals. For the two side-by-side cylinders case, a link between the wakes patterns, show in Figs. 1a and 1b, and a velocity measurement technique (as the hot wire anemometry technique) can be established through the Fig. 1c. This figure shows the velocity signals measured downstream the cylinders, along the tangent to their external generatrix, where one switching mode can be observed (modes A and B). Previous studies show that this pattern is independent of Reynolds number, and it is not associated to cylinders misalignment or external influences, what suggest an intrinsically flow feature.

Kim and Durbim (1988) point out that the transition between the asymmetric states is completely random and it is not associated with a natural frequency. Through a dimensional analysis, the authors concluded that the mean time between the transitions is on order 10^3 times longer than vortex shedding period, and the mean time intervals between the switches decreases with the increasing of Reynolds number. Due to the fact that Strouhal numbers are relatively independent from the Reynolds numbers (Žukauskas, 1972), they conclude that there is no correlation between the bistable feature and the vortex shedding.

Alam *et al.* (2003) presents a study about the aerodynamic characteristics of two side-by-side circular cylinders and application of wavelet analysis on the switching phenomenon.

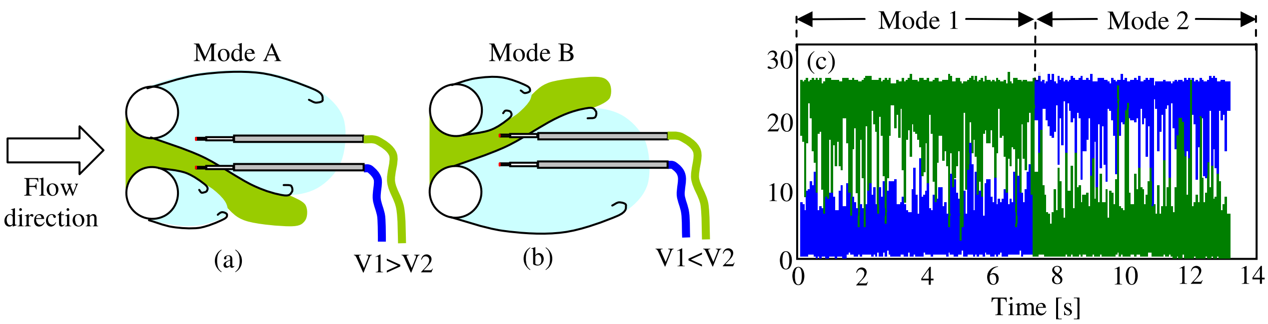

Figure 1. Bistability scheme for (a) mode A and (b) mode B, and the respective characteristic signals (c).

3. OBJECTIVES

The purpose of this paper is to present the behavior of the cross flow through two parallel circular cylinders by means of hot wire measurements, where the bistability phenomenon occurs, and describe some results of Monte Carlo simulations to obtain probabilistic distribution functions changed by this phenomenon.

4. METHODOLOGY

Time series obtained in an aerodynamic channel using hot wire anemometry are used as input to obtain probability density functions with the use of the Monte Carlo method in order to obtain a fit function changed by the phenomenon under study. In addition, a joint time-frequency domain analysis was made trough wavelet transform.

4.1. Aerodynamic channel

The velocity of the flow and its fluctuations, as well as the angle of deviation of the flow, are measured by means of the DANTEC *StreamLine* constant hot-wire anemometry system, with aid of a double hot wire probe (type DANTEC 55P71 Special), with straight/slant wires (the straight wire is placed perpendicularly to the flow, and the slant forms a 45° with the axial plane). The measurements were performed aligning the probes along the tangent to the external generatrixes of a cylinder (Fig. 2b). The aerodynamic channel used in the experiments is made of acrylic, with a rectangular test section of 0.146 m height, width of 0.193 m and 1.02 m of length (Fig. 2a). The air is impelled by a centrifugal blower of 0.64 kW, and passes through two honeycombs and two screens, which reduce the turbulence intensity to about 1% in the test section. Upstream the test section, placed in one of the side walls, there is a Pitot tube, which measures the reference velocity of the non-perturbed flow. The data acquisition is performed by a 16-bit board (NATIONAL INSTRUMENTS 9215-A) with USB interface, which converts the analogical signal to digital series. The circular cylinders, with external diameter of 25.1 mm, are made of Polyvinyl chloride (PVC), and are rigidly attached to the top wall of test section. Their both side are covered to avoid acoustic resonance phenomenon. The probe support is positioned with a 3D table placed 200 mm downstream the end of the channel (Fig. 2c). The mean error of the flow velocity determination with a hot wire was about +/- 3%. The Reynolds number of the experiment is 4.36×10^4, computed with the tube diameters and the gap velocity, and the pitch-to-diameter ratio is P/D=1.26.

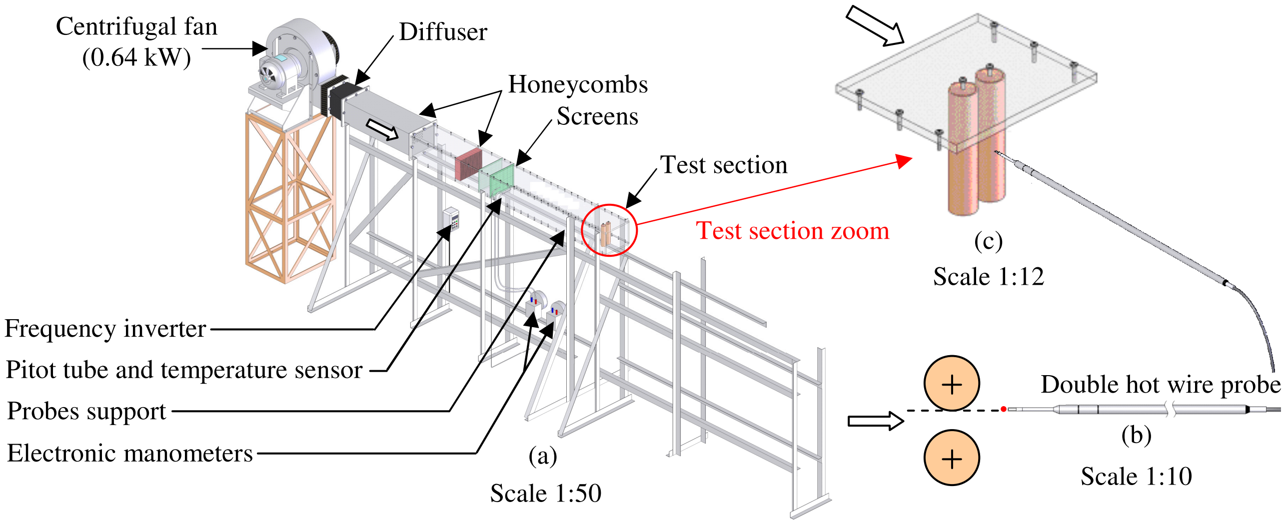

Figure 2. Schematic view of (a) the aerodynamic channel, (b) test section and (c) probe position.

4.2. Monte Carlo simulations

The Monte Carlo simulations were performed according to a function to generate random numbers, and then this random value was compared with time series, where the number of velocity intervals (bins) in the simulations was of 100, and the results are presented in form of horizontal bars. A non-optimized Monte Carlo algorithm was used in the simulations.

4.3. Wavelet analysis

The joint time-frequency domain analysis was made trough wavelet transform. Basically, a wavelet analysis is applied to time varying signals, where the stationary hypothesis cannot be maintained. Thus, discrete and continuous wavelet transforms can be used:

- *Discrete Wavelet Transform* (DWT): used to make a multilevel decomposition of a time signal in several bandwidth values, accordingly with the selected decomposition level.
- *Continuous Wavelet Transform* (CWT): used to analyze the energy content of a signal through the so called spectrogram.

In this work, Daubechies "db20" functions were used as bases of both discrete and continuous wavelet transforms. Indrusiak et al. (2005) present a more complete review of discrete and continuous wavelet transforms, applied to accelerating and decelerating turbulent flows.

5. RESULTS

The acquisition frequency of time series was of 1 kHz, and a low-pass filter of 300 Hz was used to avoid the aliasing effect (folding frequency) in spectrum. The signals of axial velocity, transversal velocity and the angle of deviation of the flow are shown in Fig. 3a, Fig. 3b and Fig. 3c, respectively. Figure 3a shows several changes between two distinct velocity levels (flow modes) from the axial component, concerning to 3.0 m/s (wide near-wake - mode 1) and 18.6 m/s (narrow near-wake - mode 2). These changes are accompanied by the transversal component (Fig. 3b), and as the flow changes direction, from the wide near-wake to the narrow near-wake mode, the angle of incidence tends to have smaller fluctuations (Fig. 3c).

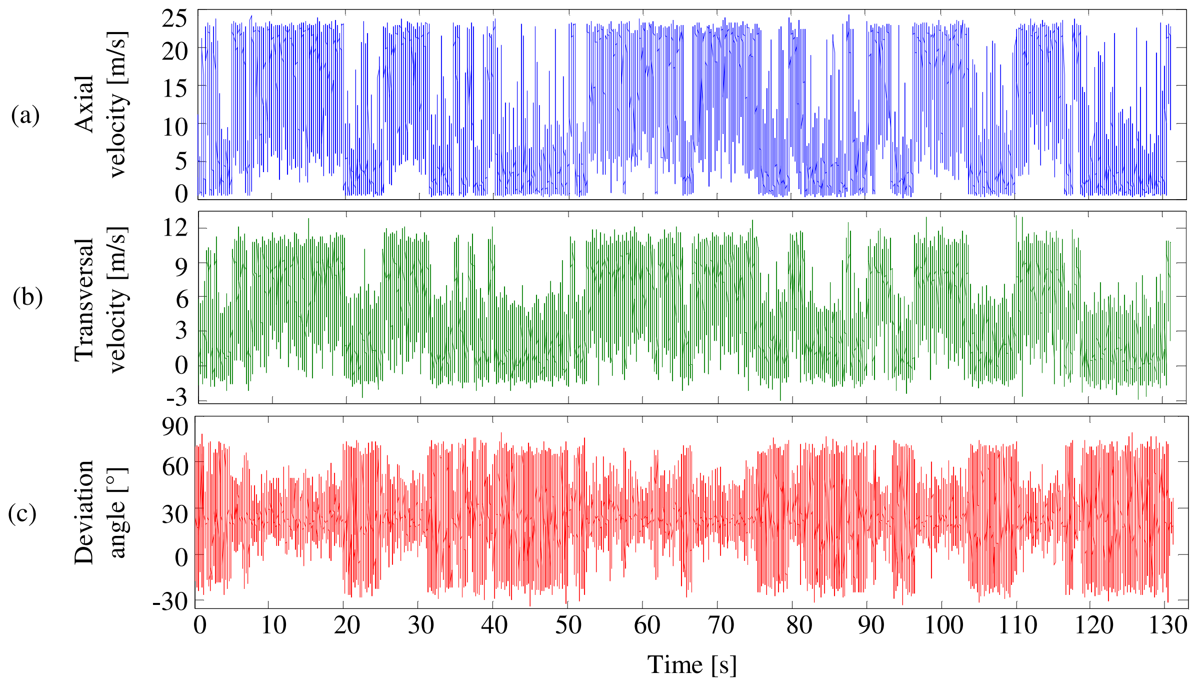

Figure 3. (a) Axial velocity signal, (b) transversal velocity signal and (c) deviation angle of the flow.

The Monte Carlo simulation was performed with a uniform function as random generator, and the results are presented in Fig. 4. A non-optimized Monte Carlo algorithm was used, where low percentages of successful runs were observed (Tab. 1), with high computational costs. The number of velocity intervals (bins) in the simulations was of 100.

From the simulations of the velocity signals two mean states of energy were observed, but with different shapes. They correspond to the mean velocity levels, referred to the wide near-wake (Fig. 4a) and to the narrow near-wake (Fig. 4b). The PDF of the deviation angle shows a concentration around a single value (Fig. 4c). Figure 5 presents the behavior of number of random points generated by Monte Carlo method with the percentage of successful runs.

An analysis in different time intervals of the axial velocity is presented in Fig. 6, where the behavior of the PDF altered by the bistability phenomenon changes intermittently, between modes 1 and 2. Comparisons between the probabilities of occurrences of the modes of this signal show that there is no evident correlation between the changes with time (Fig. 7).

Figure 8 presents a joint analysis with DWT and CWT of the time series, where a multilevel decomposition in several bandwidth values, accordingly with the selected decomposition level, can be performed together with the analysis of their energy content (spectrogram). The frequency intervals of the spectrograms vary from 10 to 150 Hz, with steps of 2 Hz. The time series were parameterized to present the same energy scale. Results show that there is an increase in energy content of velocity signals when the mean velocity is higher, with a relative spreading of frequencies (Figs. 8a and 8b), since 10 Hz until approximately 120 Hz. When the direction of the gap flow changes, to the wide near-wake, there is a decreasing in the energy content followed by a higher concentration of the frequencies (until approximately 30 Hz), what shows that the wide near-wake has a higher frequency concentration, but with less energy. The spectrogram of the axial velocity signal shows this behavior more evident, while the spectrogram of the traverse velocity signal is presented in a more discreet. The spectrogram of the angle of deviation of the flow (Fig. 8c) has an opposite behavior, where the higher energy content is distributed along a large frequency band. By analyzing this result together with the discrete reconstruction (Fig. 9), it is possible to observe that the mean feature of the signal is expressed by the fluctuations, from the lower to the higher frequencies.

As literature refers that bistability phenomenon has no natural period between the changes of mean velocity levels, becomes interesting to establish some parameter to express in a generally way such variation. A possible way to analyze this behavior is through a pseudofrequency concept, which refers a probable mean frequency of the changes, through a discrete wavelet decomposition of the signals with level 9, where the results are shown in Fig. 9. From this level of reconstruction, the approximation vectors of the DWT of the time series present frequency content from 0 to 0,976 Hz, which means that above this value the changes of direction of the flow will not be considered as “stable”, and will not be accounted to the pseudofrequency calculus of bistable phenomenon. The calculated pseudofrequency value was approximately 0.15 Hz, what means that during the 131 seconds of acquisition occurred a mean of 39 changes of the flow mode, in other words, 19.5 cycles or periods were completed, with a mean time of 6.72 seconds.

Table 1. Numerical results obtained from the Monte Carlo simulations.

Percentage of successful runs [%]			Duration of the simulation [s]		
Axial velocity	Transversal velocity	Deviation angle	Axial velocity	Transversal velocity	Deviation angle
27.7	30.7	14.9	1351	1318	1806

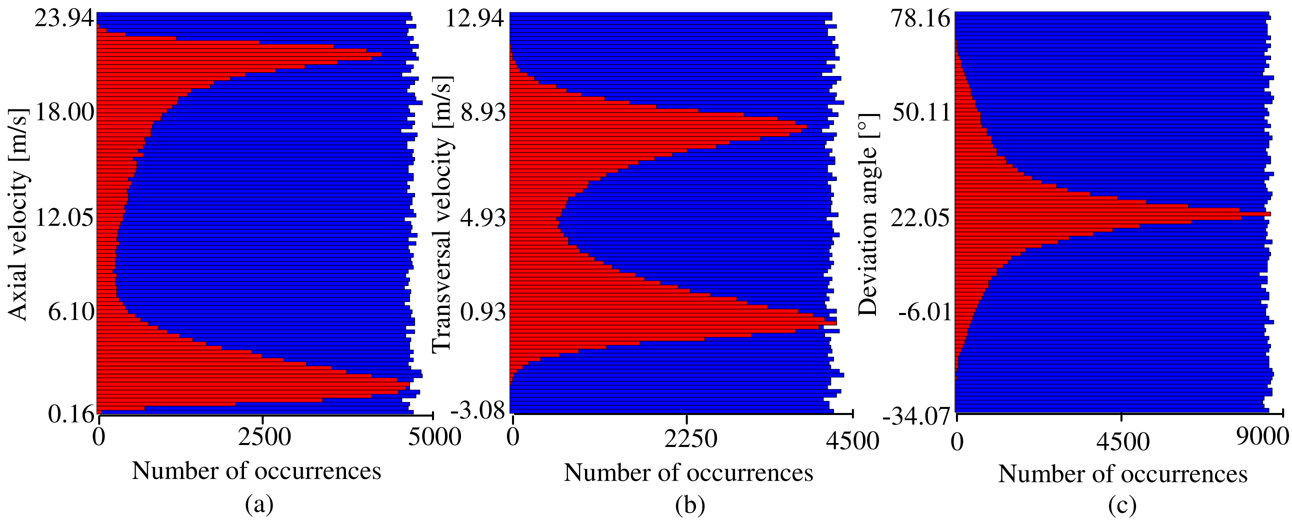

Figure 4. PDF obtained from Monte Carlo simulation with an uniform distribution function as random generator: (a) axial velocity, (b) transversal velocity and (c) deviation angle of the flow.

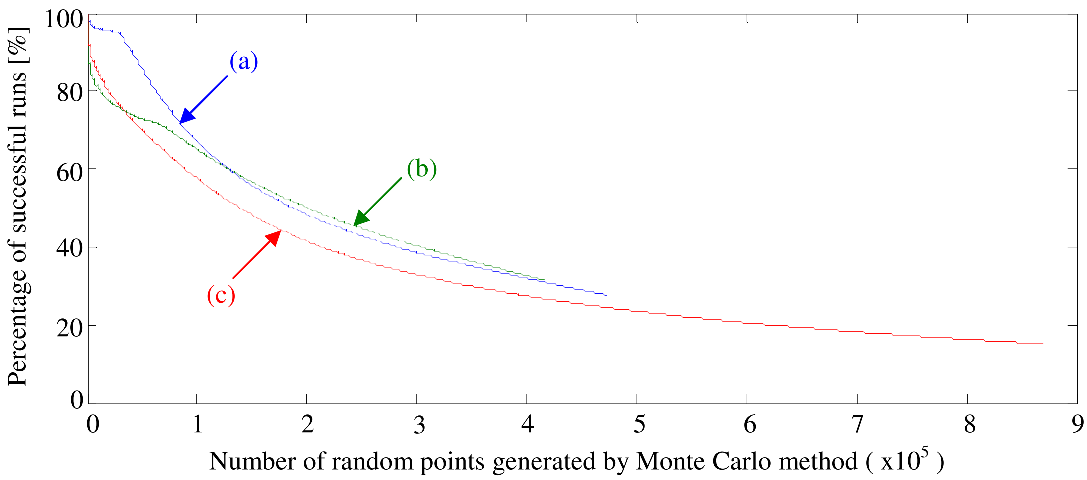

Figure 5. Evolution with the number of points in the time series of the percentage of successful runs of the Monte Carlo method: (a) axial velocity. (b) transveral velocity. (c) deviation angle.

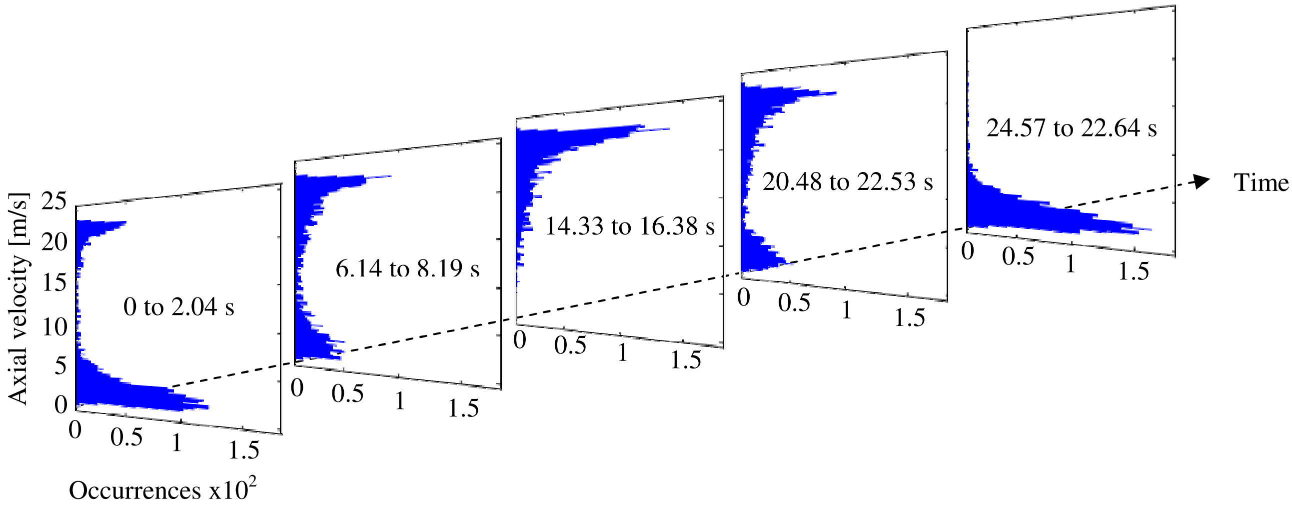

Figure 6. PDF of the axial velocity signal for various time intervals.

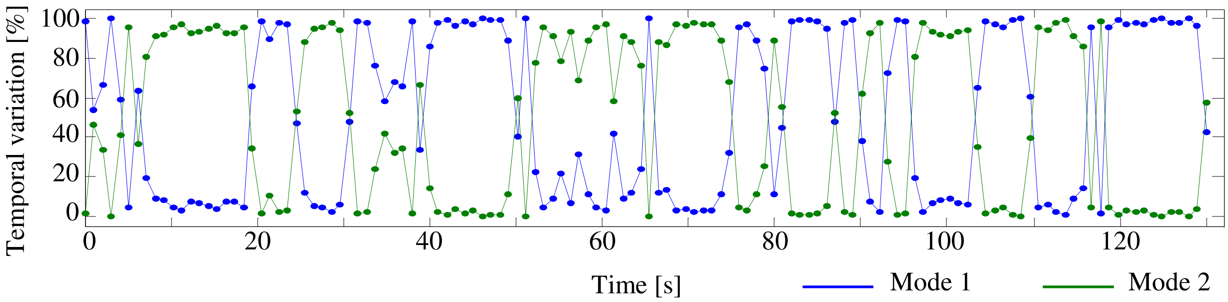

Figure 7. Temporal variation of the flow modes of the axial velocity.

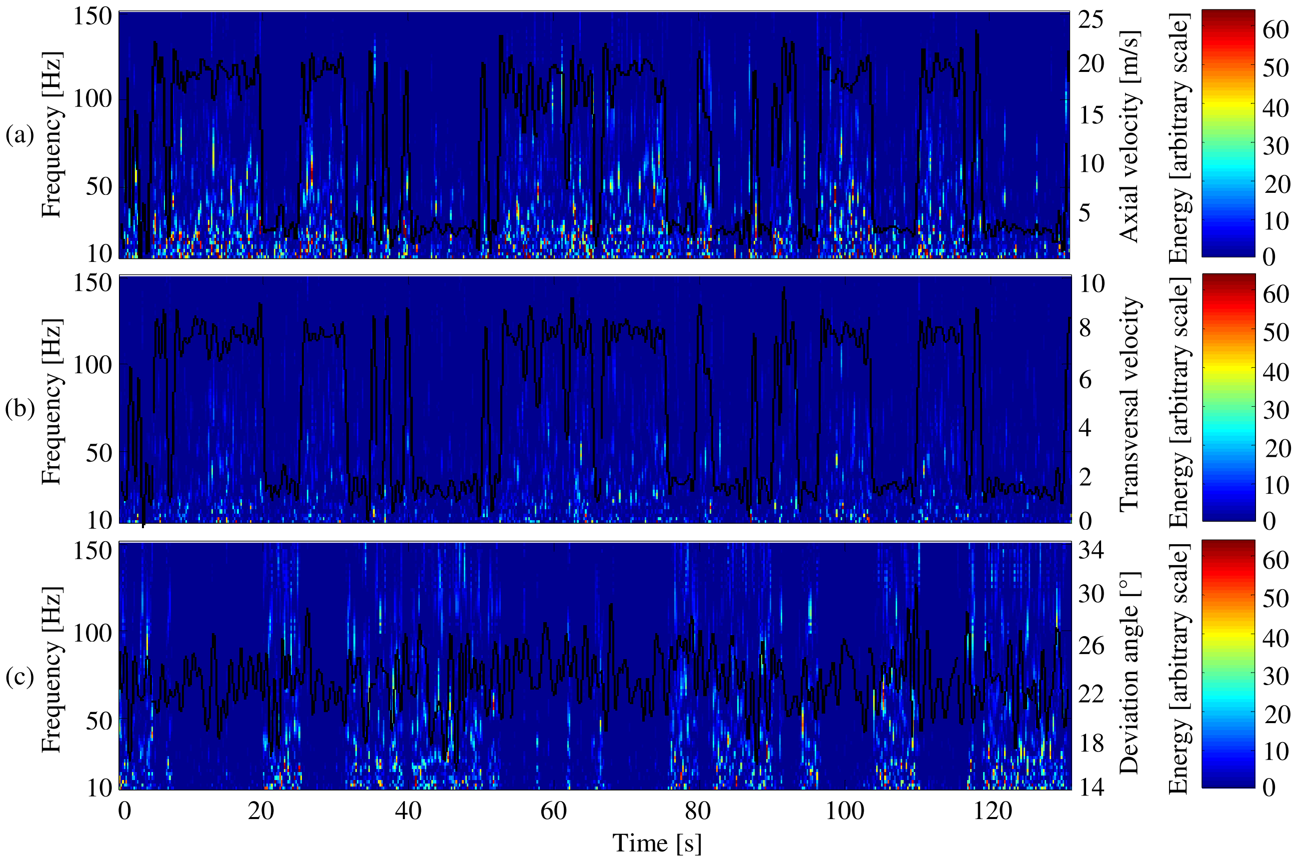

Figure 8. Joint analysis of the TCO and the TDO time series: (a) axial velocity, (b) transversal velocity and (c) deviation angle of the flow.

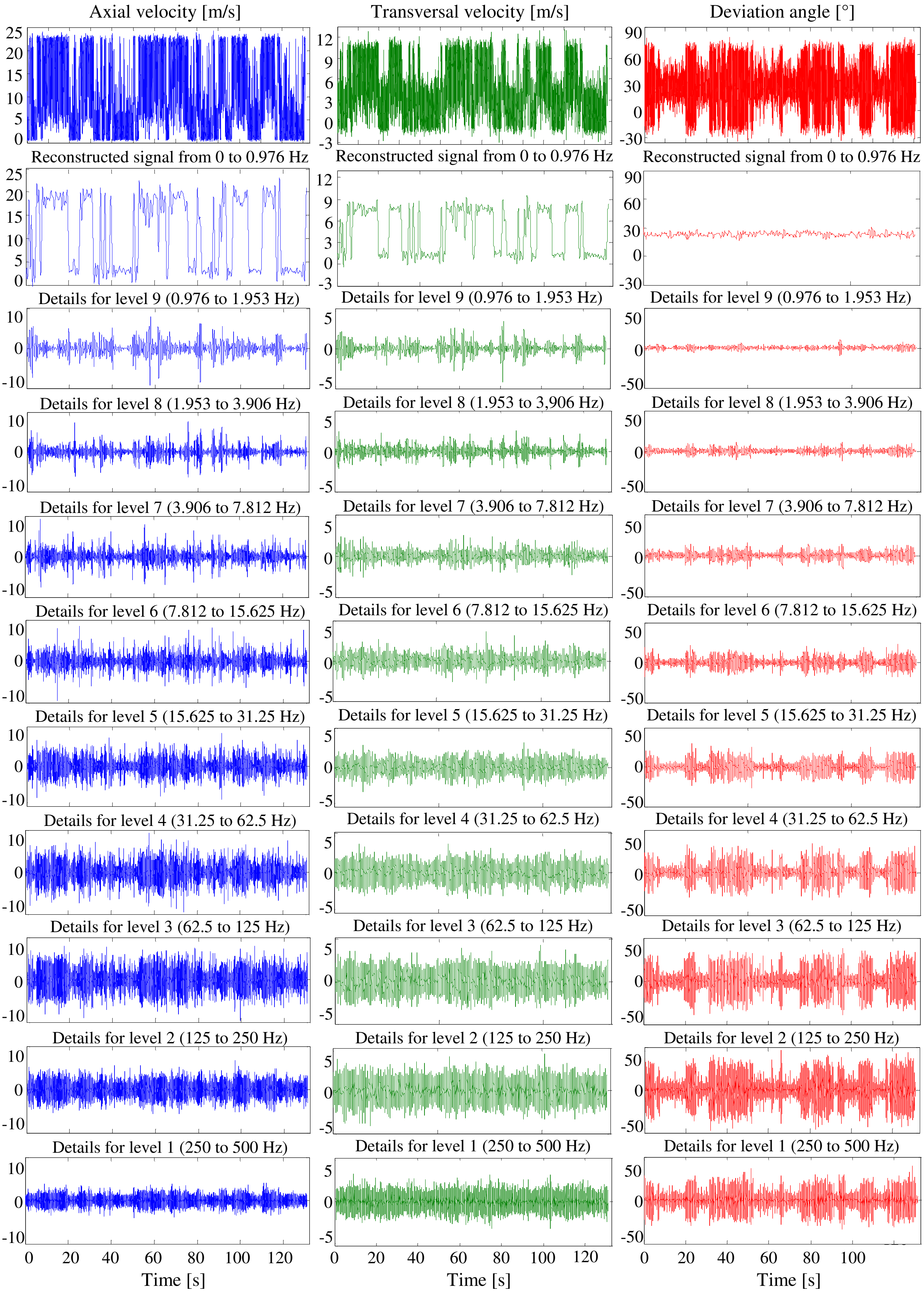

Figure 9. Reconstruction of the time series with DWT, type Db20, level 9.

6. CONCLUSIONS

This work presents a study about the bistable phenomenon which occurs in turbulent flows impinging on two side-by-side circular cylinders. As the literature has reported that the changes between two consecutive flow modes are completely random, becomes interesting the use of statistical tools for the determination of parameters of this phenomenon, as is the case of Monte Carlo method. Thereby, a Monte Carlo method is applied to generate random numbers, in order to obtain a probabilistic distribution functions (PDF) changed by the phenomenon under study.

The definition of a pseudofrequency could be established, which can represent the probable mean frequency of the switches between the flow modes. This analysis could be possible through the use of wavelet transforms, which shown to be indispensable tools for the analysis of transient turbulent signals.

The results of hot wire anemometry technique show that in the changes between the flow modes the increase in the axial velocity component is accompanied by increased of the transverse component, and the PDF obtained by the Monte Carlo method for these velocity components present the predominance of two major states of energy, with different shapes. The angle of deviation of the flow tends to have smaller fluctuations when the flow direction changes from the wide near-wake to the narrow near-wake, and their PDF presents a concentration around a single value. The wide near-wake, which has a higher frequency concentration, has a higher mean velocity than the narrow near-wake, but is less in energy. From the spectrograms of the velocity signals, the energy content is higher and more spread in frequency domain when the mean velocity increases. The spectrogram of the angle of deviation of the flow has an opposite behavior, where the higher energy content is distributed along a large frequency band. By analyzing this result together with the discrete reconstruction, is possible to observe that the mean feature of the signal is expressed by the fluctuations, since the low frequencies until the higher one. Comparisons between the probabilities of occurrence of the two modes show no evident correlation between the changes with time.

Future works contemplating new simulations and scenarios are intended, in order to determine best fitting functions to the probabilistic distributions obtained from experimental time series, with the objective to maximize the percentage of successful runs with the Monte Carlo method, or also using copulas, which are functions that describe dependencies among random variables and provide a way to create probability distributions to model dependent multivariate data. In addition, time series obtained inside banks of tubes can be performed, and the present methodology can be applied. This can better elucidate the bistable flow phenomenon that can be an addition excitation mechanism on the tubes.

7. ACKNOWLEDGEMENTS

Authors thank the financial support by The National Council for Scientific and Technological Development (CNPq).

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